Nonlinear Analysis of Solids and Structures

Midterm Quiz, 27MAR2009

1. We want to study the error of the infinitesimal deformation theory. Let us consider a simple shear deformation corresponding to a shear angle θ.
(a) Draw the deformed shapes corresponding to the finite and infinitesimal deformation theories. (3 pt)
(b) When the infinitesimal deformation theory is used, find the angle θ that gives 1% of error ($= \frac{|\text{exact}-\text{approx.}|}{\text{exact}}$) in volume change. (5pt)

2. At a material point, we have a stress
$$\boldsymbol{\sigma} = \beta \begin{bmatrix} 1.1 & 0 & 1 \\ 0 & 1.3 & -0.1 \\ 1 & -0.1 & 0.9 \end{bmatrix} \mathrm{N/m^2} \quad \text{with a load factor } \beta.$$
(a) Decompose the stress tensor into volume and deviator parts. (2pt)
(b) Considering a Von-Mises material, find the range of the load factor where the stress state is admissible. (5pt)

3. The elastic potential of an isotropic linear thermoelastic material is given
$$\psi(\boldsymbol{\epsilon}, \theta) = \frac{\lambda}{2}(\mathrm{tr}\,\boldsymbol{\epsilon})^2 + G\,\mathrm{tr}(\boldsymbol{\epsilon}\cdot\boldsymbol{\epsilon}) - (3\lambda + 2G)\,\alpha\,(\mathrm{tr}\boldsymbol{\epsilon})\theta - \frac{1}{2}\frac{C\theta^2}{T_0},$$
where T_0 and T are absolute temperatures in the initial and current configurations, respectively, θ is the temperature variation ($\theta = T - T_0$) and λ, G, α and C are material constants. Derive the relation between stress and strain, and express the resulting material law in a tensor form. (5pt)

4. As shown in the figure below, we have an isotropic linear elastic material of height H (the shaded area in the figure) in a frictionless rigid pit. A positive temperature variation θ is uniformly applied in the material ($T > T_0$). Assume that the deformation is infinitesimal and gravity is ignored.

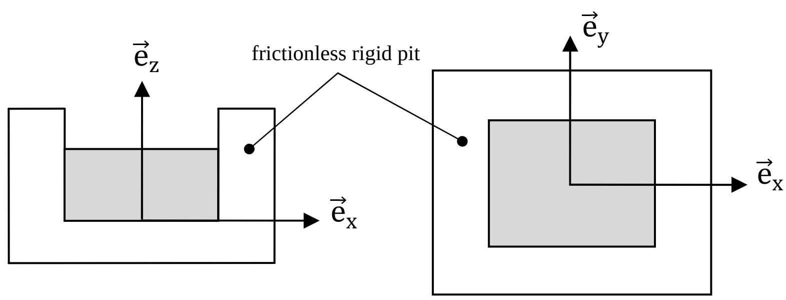


Figure. A material in a frictionless rigid pit

(a) Find the solutions (stress, strain and displacement fields) using "displacement method." (5pt)
(b) In order to prevent the deformation, we will apply a uniform pressure ($-p\vec{e}_z$) on the top surface of the material. Find the magnitude of the pressure. (5pt)

Nonlinear Analysis of Solids and Structures

Midterm Quiz, 25MAR2010

1. (10 pt.) We study the 2D bending deformation of a body as shown in Fig. 1. The current position (x, y, z) is a function of the initial position (X, Y, Z)

$$x = (R+Y)\sin\theta$$
$$y = (R+Y)\cos\theta - R \qquad \text{with} \quad \theta = X/R.$$
$$z = Z$$

(a) Calculate the deformation tensor.
(b) Calculate the Green-Lagrange strain tensor.
(c) When $Y/R << 1.0$, specify the condition for the infinitesimal deformation.
(d) Calculate the infinitesimal strain tensor.

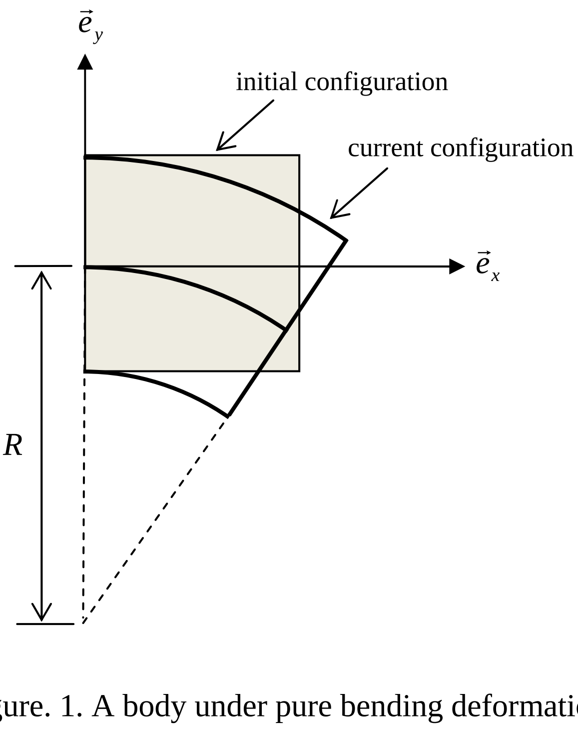


Figure. 1. A body under pure bending deformation

2. (10 pt.) A cylinder is subjected to the uniform normal pressures P and Q as shown in Fig. 2. The self-weight is neglected. Considering two different ratios between those pressures (P/Q), two experiments have been performed to find the critical loads (Q_{crit}) at which the used material fails and the results are given.

	P/Q	Q_{crit}
Test 1	1.0	300 MPa
Test 2	2.0	200 MPa

(a) Plot the stress states in the principal stress space.
(b) Plot the stress states in the Mohr-plane.
(c) Among the tension-cutoff, Tresca, Mohr-Columb and Von-Mises strength criterions, find proper strength criterions for the material?
(d) Specify the strength criterions. (Material parameters should be given.)

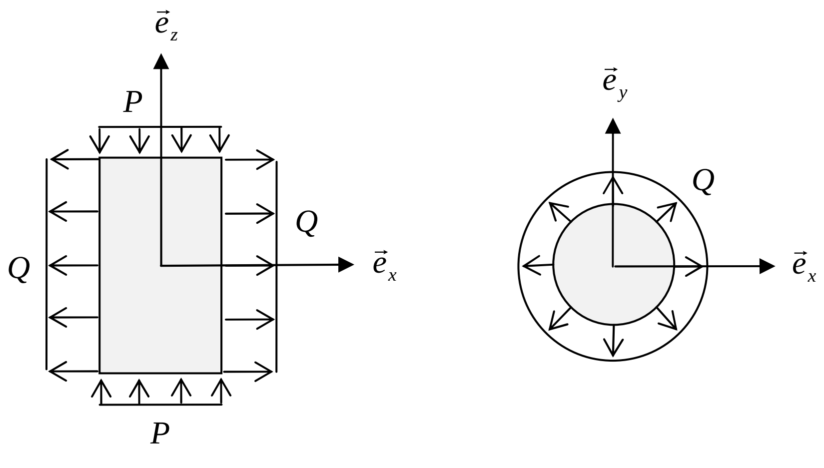


Figure. 2. A cylinder under external pressure

3. (10 pt.) We have a body (square section of $L \times L$ in the x-y plane) of infinite length in the $\vec{e}_z$ -direction. The body is subjected to the external pressure (P) as shown in Fig. 3. The self-weight is neglected.

   (a) Specify the governing equations and the boundary conditions.
   (b) Find the admissible stress field.
   (c) Find the solutions (stress, strain and displacement fields). Assume that the deformation is infinitesimal and we have an isotropic linear elastic material.

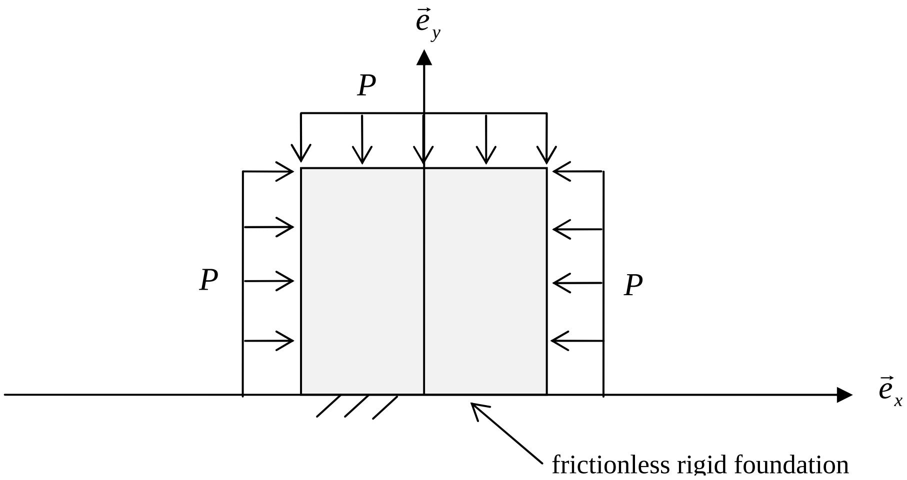


Figure. 3. A body of infinite length under lateral pressure

Nonlinear Analysis of Solids and Structures

Midterm Quiz, 30MAR2011

1. (10 pt.) A solid ball is shrunken without rotation and shape change in deep water of depth h as shown in Figure. The initial and current radii are R and r, respectively.

   (a) Find the current position in terms of the initial position.
   (b) Calculate the deformation tensor.
   (c) Find the change of an initial material surface $(\vec{e}_x - \vec{e}_y) d\,{}^0a / \sqrt{2}$ at (*R/2,0,0*) after deformation.
   (d) Calculate the Green-Lagrange strain tensor.
   (e) Calculate the gradient of the displacement vector, $d\vec{u} / d\,{}^0\vec{x}$.
   (f) Find the small strain tensor after specifying the condition for the infinitesimal deformation.

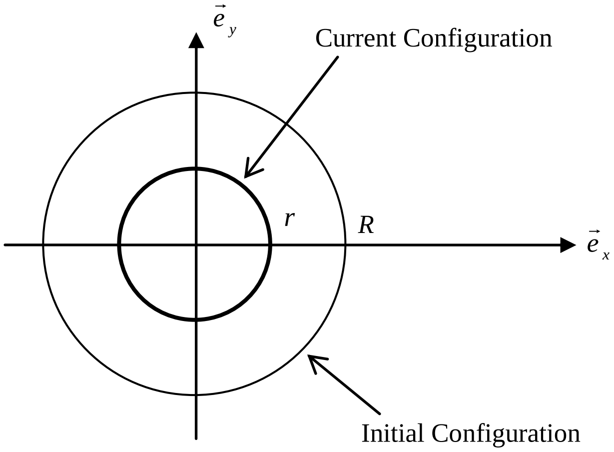


Figure. A solid ball in deep water. A uniform hydrostatic pressure is assumed to be applied to the whole surface of the ball.

2. (10 pt.) The ball is made by an isotropic linear elastic material, which follows the Mohr-Columb strength criterion defined by φ and c.

   (a) Specify the governing equations with boundary conditions.
   (b) Find the displacement and stress fields. You should show the step-by-step procedure of the displacement method.
   (c) Calculate the depth of the water when the ball fails (γ : specific weight of water).
   (d) Find the depth of the water when *r* becomes *0.5R.*

3. (10 pt.) The ball in the water is uniformly heated. Note that the elastic potential of an isotropic linear thermo-elastic material is given

$$\psi(\underline{\underline{\varepsilon}}, \theta) = \frac{\lambda}{2}(tr\,\underline{\underline{\varepsilon}})^2 + G\,tr(\underline{\underline{\varepsilon}}\,\underline{\underline{\varepsilon}}) - \alpha\theta(3\lambda + 2G)tr\,\underline{\underline{\varepsilon}} - \frac{C\theta^2}{2T_0} \quad \text{with } \theta = T - T_0,$$

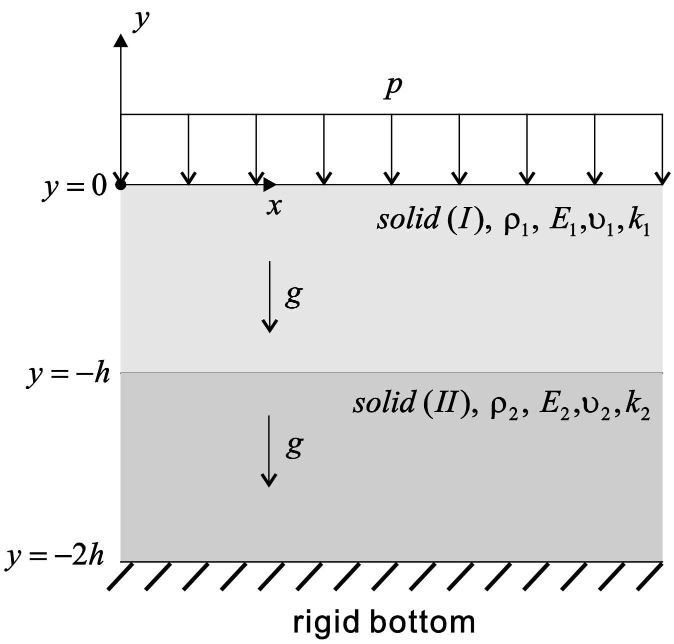
where T_0 and T are absolute temperatures in the initial and current configurations, respectively, θ is the temperature variation and, λ , *G*, α and *C* are material constants.

   (a) Find the material law including the temperature effect.
   (b) When the ball finally resumes its initial radius, find the temperature variation.
   (c) Find the temperature variation which leads the failure of the ball with the failure criterion given in Problem 2.

Advanced Analysis of Solids and Structures

Midterm Quiz (30pt.), 30 MAR 2012

We have two solid layers of infinite length in the x and z directions. The solids are subjected to the gravitational field in $-y$ direction and an external pressure p on the top surface as shown in Figure. Both solids fails following "von-Mises strength criterion" with a strength parameter k. We want to find stress, strain and displacement fields by using "displacement method" and "stress method."



$g = 10 m/s^2$
$h = 10 m$

$\rho_1 = 8000 kg/m^2$
$E_1 = 200 GPa$
$\upsilon_1 = 0$
$k_1 = 8.0 \times 10^6 Pa$

$\rho_2 = 0.0 kg/m^2$
$E_2 = 100 GPa$
$\upsilon_2 = 0$
$k_2 = 9.0 \times 10^6 Pa$

Figure. Two solid layers

(a) Specify all the governing equations with boundary conditions (5pt.)

(b) Solve the problem by using "displacement method" (10pt.)

(c) Solve the problem by using "stress method" (10pt.)

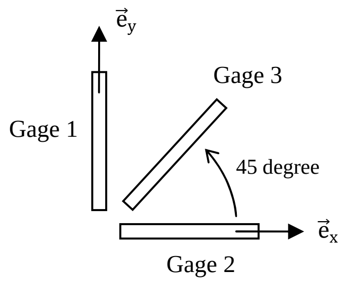
(d) As the pressure p increase, which solid layer fails first? At that moment, what is the critical pressure p_{cr} (5pt.)

** In problems (b) and (c), stress, strain and displacement fields should be calculated.

Nonlinear Analysis of Solids and Structures

Final Quiz, 22MAY2009

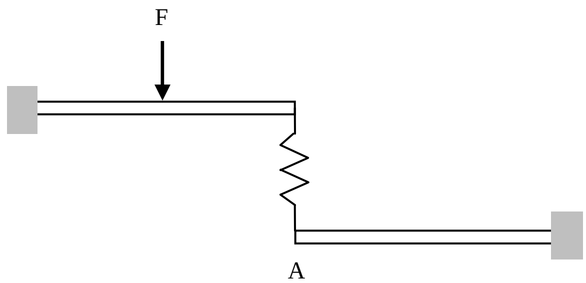
1. We installed a set of strain gages at a certain point on a hull plate (t<<L,B) of a container ship and read the values. Young's modulus is 200GPa and Poisson's ratio is 0.3.



Gage #	Strain
1	0.001
2	-0.002
3	-0.0005

(a) Find all the strain components (or tensor) (5pt).
(b) Find all the stress components (or tensor) (5pt).

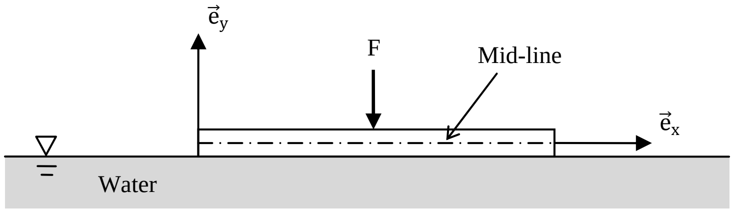
2. Two cantilever beams of length L are connected by a spring. The vertical load F is applied at the center of the left cantilever. (L=10m, F=48kN, E=200GPa, I=0.005m^4, spring constant k=1000kN/m)



(a) Calculate the displacement at point A (5pt).
(b) Find all the reaction forces (3pt).
(c) Plot BMD (\Bending Moment Diagram) for the beams (2pt).

3. A long elastic barge of length L and unit width is floating on the water (specific weight γ_w). The barge can be considered as a beam. A load F is applied at the center of the barge and the total self-weight W is uniformly distributed. The bending stiffness is EI.

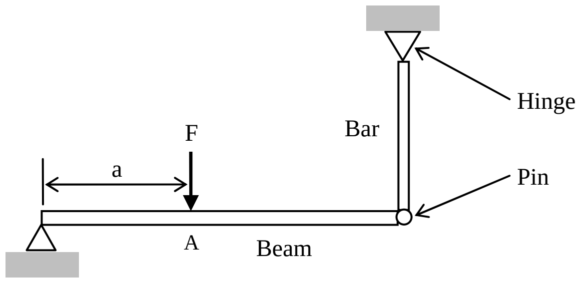
   (a) Plot the expected displacement of the mid-line (1pt).
   (b) Assume that the barge has a sine-shape deformation. Using "Principle of Virtual Work", approximate the displacement from the initial configuration shown (7pt).
   (c) Plot the approximated BMD (2pt).



< Initial configuration >

(Hint) $\int_0^L \sin\frac{\pi x}{L}\,dx = \frac{2L}{\pi}, \quad \int_0^L \left(\sin\frac{\pi x}{L}\right)^2 dx = \frac{L}{2}$

4. We have a structural system which consists of a beam (length L, Young's modulus E, second moment of area I, and plastic moment M_o) and a bar (length L/2, Young's modulus E, area A, and plastic axial force P_o) . A load F is applied as shown.



Assuming that an elastic material is used for the structural members,
(a) Find the displacement at point A (3pt).
(b) Plot BMD and AFD (Axial Force Diagram) (2pt).

When the material used is elastic-perfectly plastic,
(c) Which loading position will make the both members simultaneously fail (3pt)?
(d) Discuss the failure mechanism of this structure depending on the loading position (2pt).

Nonlinear Analysis of Solids and Structures

Final Quiz (50pt.), 18MAY2010

1. A body is deformed as shown in Fig. 1. Calculate $\int_V \det F \, dV$, where **F** is the deformation tensor and V is the original volume. (10pt)

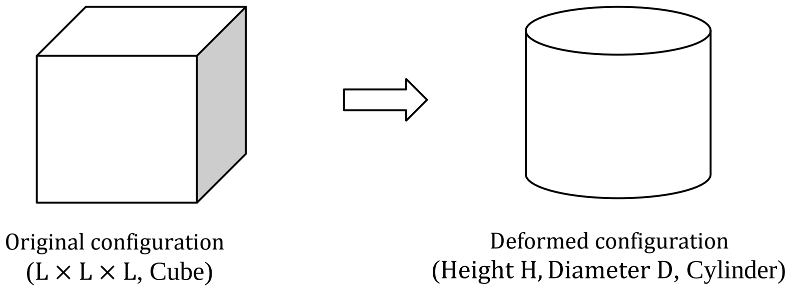


Fig. 1. A deformation

2. As shown in Fig. 2, a beam structure (bending stiffness = EI) of length L is supported by a spring (stiffness = k) at mid-length and hinges at both ends.

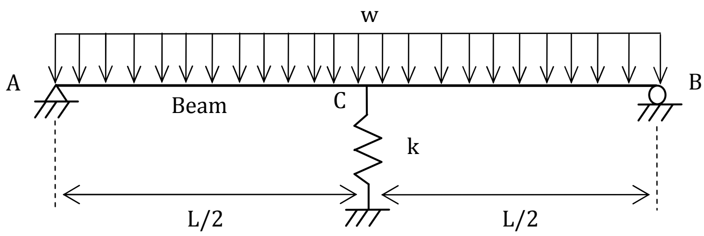


Fig. 2. A beam structure

Using the Euler-Bernoulli beam theory,

(a) Calculate the exact deflection at point C. (5pt.)
(b) Let us suppose that the displaced shape of the beam is given by $\nu = a \sin \frac{\pi x}{L}$. Approximate the deflection at point C and compare with exact solution. (Any energy method can be used.) (5pt.)
(c) Discuss on the accuracy of the approximated solution depending on the magnitude of k. (5pt.)

3. We have a structural system which consists of a beam (Young's modulus = E, second moment of inertia = I and plastic moment = M_0) and a bar (Young's modulus = E, Area = A and plastic axial force = P_0). A load F is applied as shown in Fig. 3.

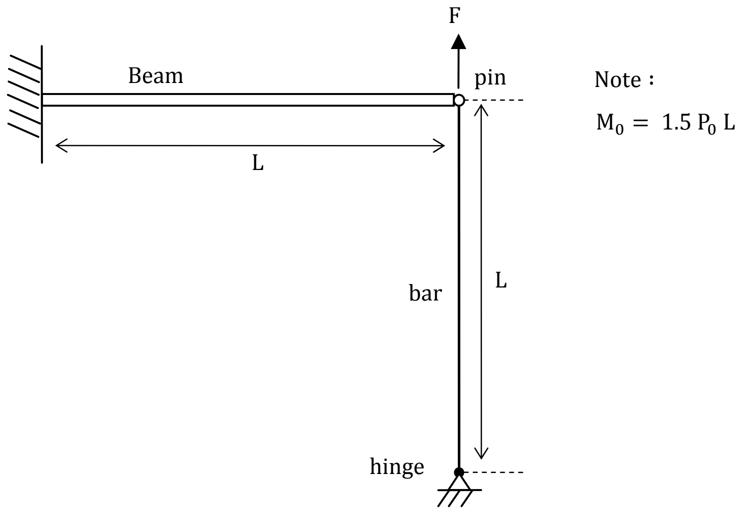


Fig. 3. A structural system

(a) Using the incremental analysis, calculate the load-displacement history and determine the critical load F_{crit}. (10pt.)
(b) Using the limit analysis, determine the critical load F_{crit}. (5pt.)

4. We have a 2D truss structure which consists of two bar members with a circular section ($\overline{AB}$ and $\overline{BC}$). A linear elastic material (Young's modulus = E) is considered.

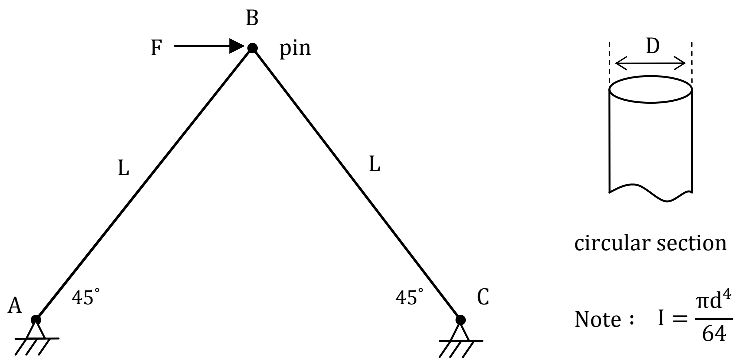


Fig. 4. A truss structure

(a) Calculate the reaction forces, displacements and axial forces. (5pt.)
(b) Determine the critical load F_{crit}. (5pt.)

Nonlinear Analysis of Solids and Structures

Final exam, Spring, 2011-5-25 (50 pt.)

1. (15 pt.) A cylinder is subjected to an axial load P and a torsional moment M. Considering a Von-Mises material ($\sqrt{J_2} - k \leq 0$), we want to find the range of the loads where the stress state is admissible. When both P and M are applied, determine (P_{crit}, M_{crit}) and plot it on the 2D plane defined by P and M.

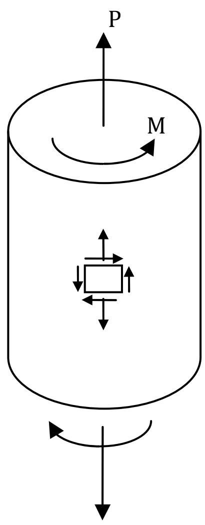


Fig. 1. A cylinder problem
(Sectional area = A, outer radius = R and polar moment of inertia = J)

2. (15 pt.) As shown in Fig. 2, a half-circular arch structure is subjected to a load F at the center. Draw AFD (Axial Force Diagram), SFD (Shear Force Diagram) and BMD (Bending Moment Diagram).

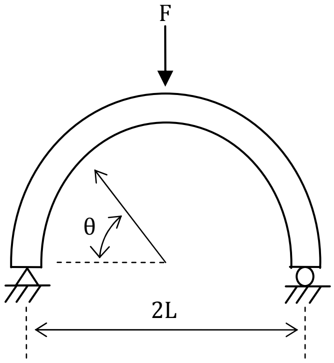


Fig. 2. A half-circular arch structure

3. (20 pt.) We have a structural system which consists of a beam (Young's modulus = E, second moment of inertia = I, sectional area = A and yield stress = σ_y) and a rigid bar of length L. Considering the failure envelop in Fig. 4, determine the critical load and find the optimal position x which maximizes the critical load F_{crit}.

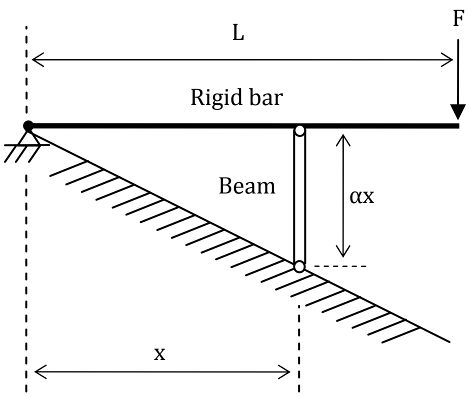


Fig. 3. A structural system

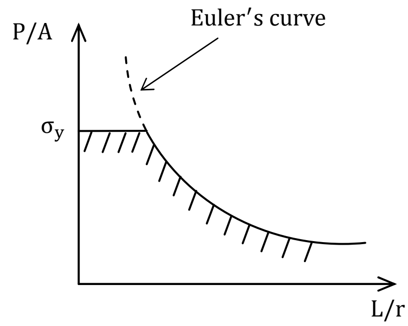
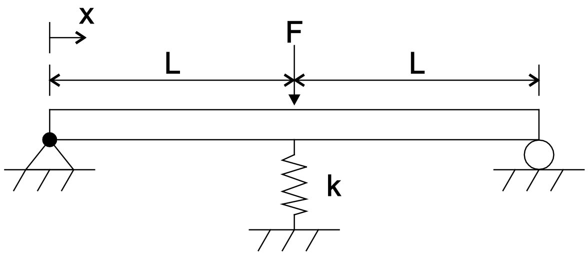


Fig. 4. Failure envelop

Advanced Analysis of Solids and Structures

Final exam, Spring, 2012-5-25 (50 pt.)

1. (20 pt.) Let us consider a simply supported beam (bending rigidity: EI) with additional spring support (spring constant: $k = EI / L^3$) at $x = L$.



(a) 10pt. Using the principle of virtual work, find the approximated deflection of the beam at $x = L$. In the calculation, use the following real and virtual displacement functions

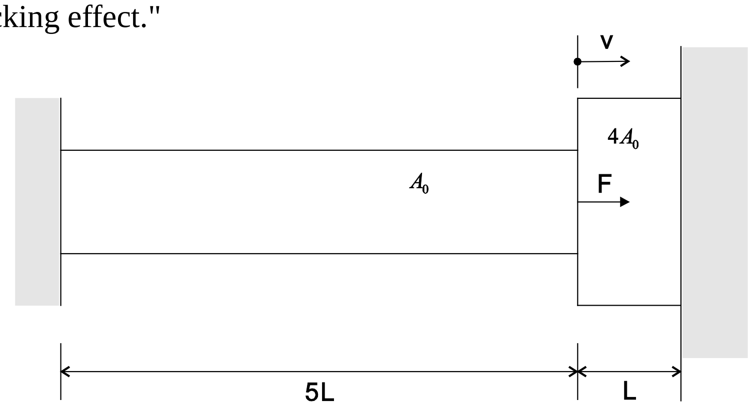
$$v = A\sin\frac{\pi}{2L}x, \qquad \delta v = \delta A\sin\frac{\pi}{2L}x.$$

(b) 5pt. Repeat the calculation as in (a) using the following functions

$$v = A\sin\frac{\pi}{L}x, \qquad \delta v = \delta A\sin\frac{\pi}{L}x.$$

(c) 5pt. The analytical solution for the deflection at $x = L$ is $-\frac{FL^3}{7EI}$. Evaluate the errors of the solutions calculated in (a) and (b), and discuss on the results..

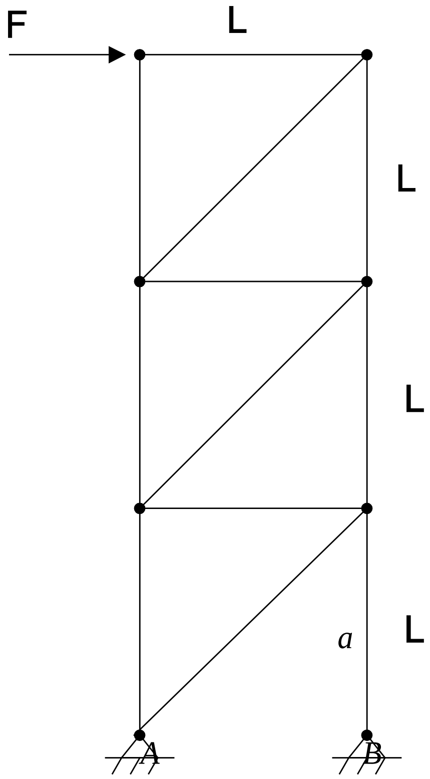
2. (15 pt.) We have two connected bars with the same elastic-perfectly plastic material of E (elastic modulus) and σ_0 (yield stress). However, each bar has different cross-sectional area. In this problem, we do not consider "bucking effect."



(a) 10pt. Using the incremental analysis, calculate the load(F)-displacement(v) history and determine the critical load F_{crit} .

(b) 5pt. Calculate the critical load using the limit analysis.

3. (15 pt.) Let us consider a 2D truss structure. All the members are have same E (elastic modulus), A (cross-sectional area), and I (2nd moment of area).



(a) 5pt. Calculate reaction forces at supports A and B.

(b) 10pt. When we assume that this structure collapses due to the buckling of the bar member a, determine the critical load F_{crit}.