유한요소 구조동역학 모델의 차수감소기법

On the finite element model reduction methods in computational dynamics

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2014. 5. 16.

KAIST

Contents

Introduction of FE model reduction

❖ Computational efficiency

Supercomputing cluster

Parallel algorithm

Computing power

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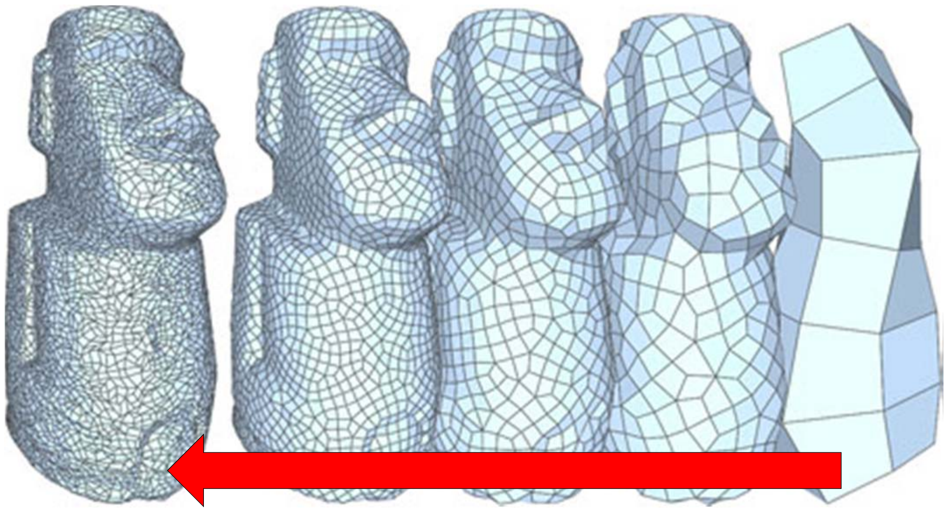
Algorithm ↑



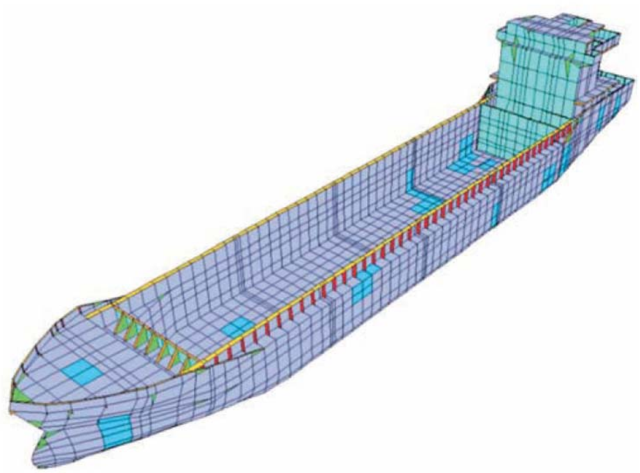
Finite element model

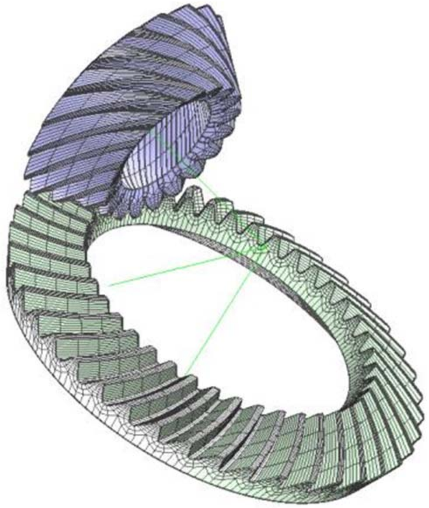
Model size ↑

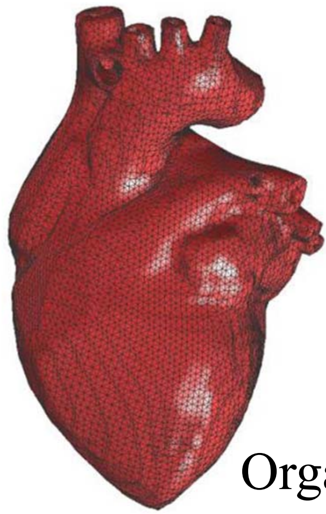
Complexity ↑

Reducing the computational cost is still important issue.

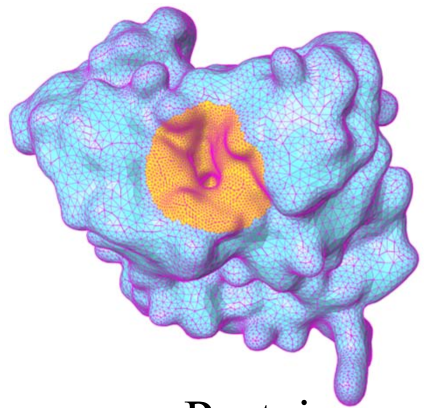
❖ What is model reduction?







Organ



Protein structure

Original FE model

$$\boxed{\mathbf{M}_g,\ \mathbf{K}_g}$$

DOFs: 10,000~1,000,000

(1~5%)

$\longrightarrow$

Model reduction

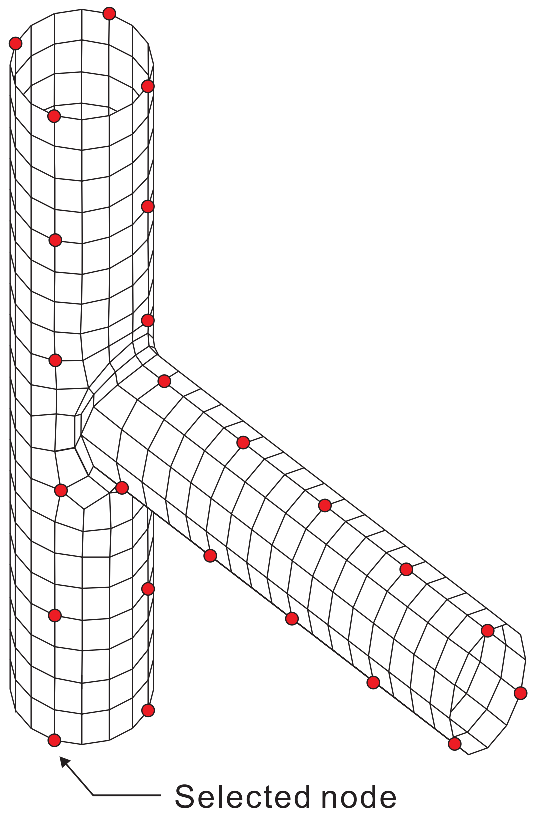
Reduced model

$$\boxed{\overline{\mathbf{M}}_p,\ \overline{\mathbf{K}}_p}$$

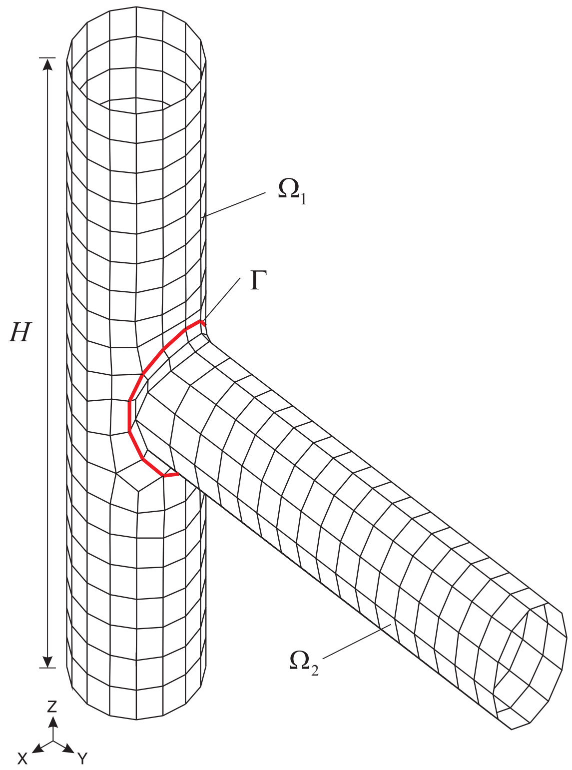
DOFs: 100~500

Goal: Smaller model size with less loss of accuracy

❖ Key concepts of model reduction

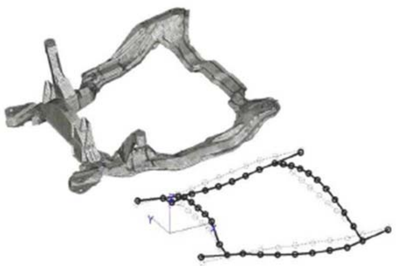


1. DOFs based reduction



2. Mode based reduction
(with substructuring)
= Component mode synthesis
= Dynamic substructuring
= Domain decomposition method

❖ Usages of model reduction

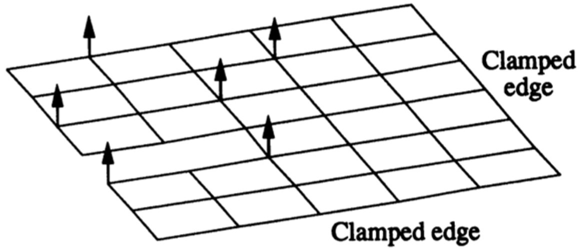


(a) Subframe

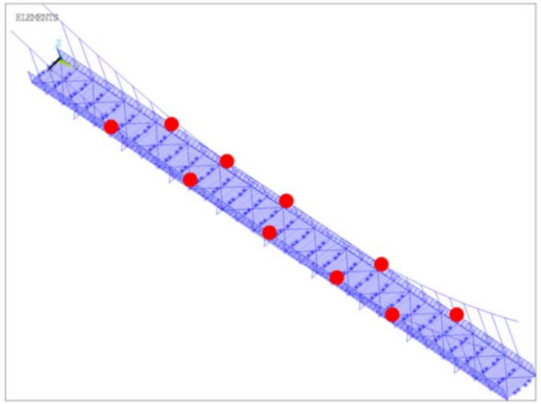
(b) Suspension

(c) Trimmed body

Eigenvalue solver and structural analysis

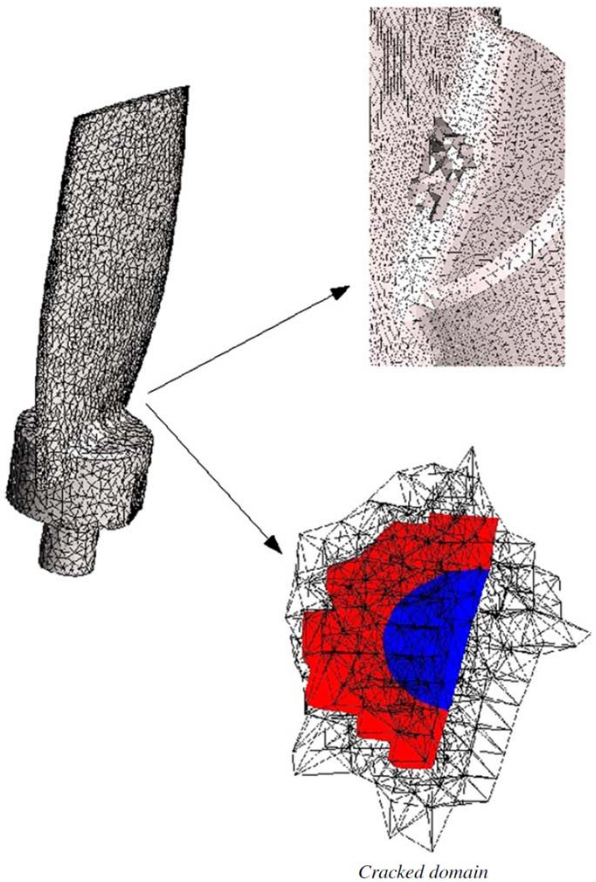


6 measurement locations chosen by Guyan reduction.

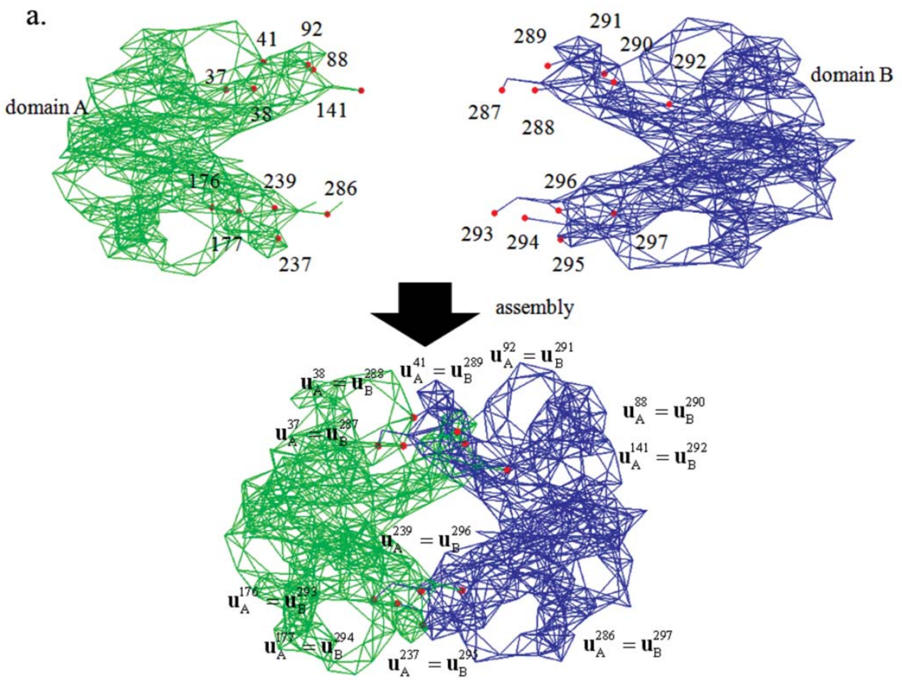


Health monitoring and

measurement positioning



FEM / X-FEM coupled modeling



Elastic network modeling
for Nanomechanics and Biomechanics

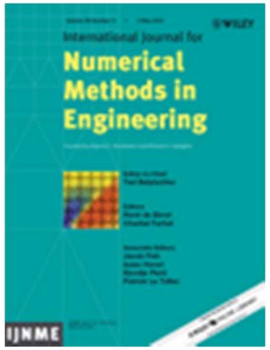
❖ Related major journals

A keyword: component mode synthesis

I.F. 0.99
410 편

I.F. 2.617
207 편

I.F. 1.509
355 편



I.F. 2.068
206 편

Top publications - Structural Engineering   Learn more

Publication	h5-index	h5-median
1. Computer Methods in Applied Mechanics and Engineering	53	66
2. International Journal for Numerical Methods in Engineering	44	57
3. Composite Structures	41	54
4. Engineering Structures	37	45
5. Computers & Structures	34	45
6. Computational Mechanics	31	49
7. Structural and Multidisciplinary Optimization	30	46
8. Earthquake Engineering & Structural Dynamics	28	36
9. Journal of Structural Engineering	27	36
10. Journal of Constructional Steel Research	26	33
11. Structural Safety	25	36
12. Thin-Walled Structures	24	28
16. Structural Control and Health Monitoring	20	34
17. Journal of Engineering Mechanics	20	29
18. Journal of Composites for Construction	20	24

ISI Journal Citation Reports © Ranking: 2012: 8/90 (Engineering Multidisciplinary); 14/93 (Mathematics Interdisciplinary Applications)

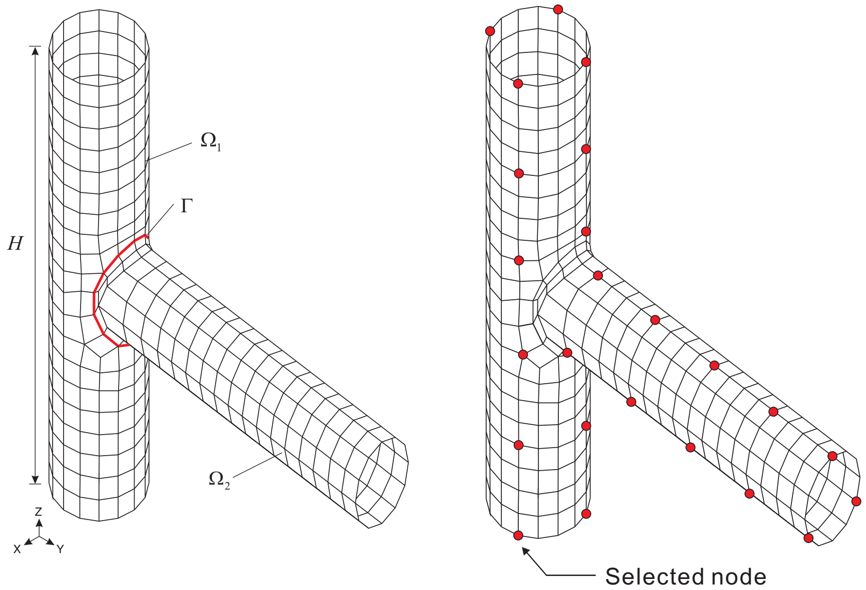
Related research progresses in this thesis have been accepted and submitted these journals.

❖ Major issues of FE model reduction

1. Model reduction for more precise reduced-order modeling

2. How do we evaluate the solution accuracy of the reduced problem?

3. What substructural modes / DOFs might be selected?



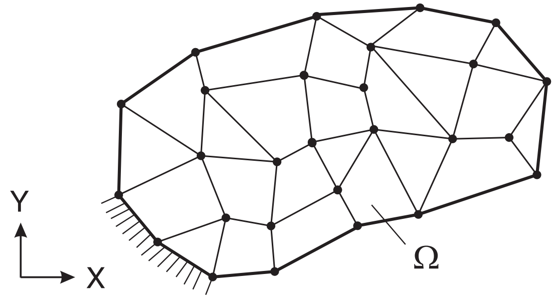
Today, we will handle these three issues.

Topic 1. Enhanced CMS methods

❖ General description of CMS methods

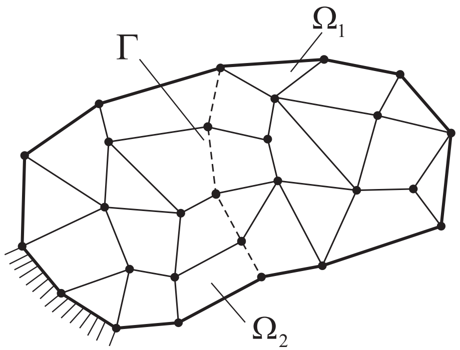
1. Global model and its eigenvalue problem

$$\mathbf{K}_g(\boldsymbol{\varphi}_g)_i = \lambda_i \mathbf{M}_g(\boldsymbol{\varphi}_g)_i$$

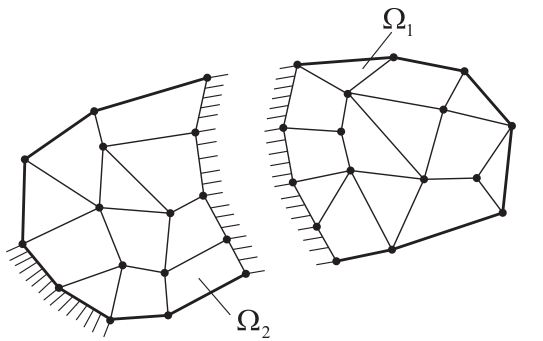


Global (non-partitioned) structure

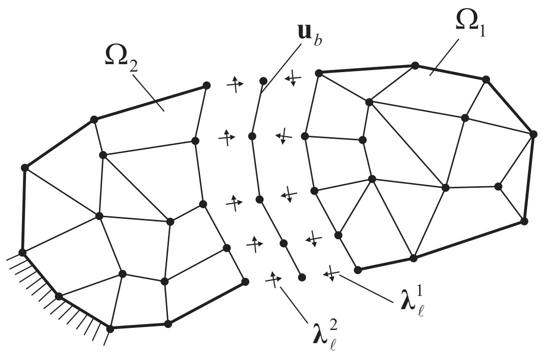
2. Partitioning and interface handling



Partitioned structure



Fixed interface
(e.g. CB method, AMLS method)



Free interface
(e.g. F-CMS method)

3. eigenvalue problem of Substructures $\mathbf{K}^{(k)}(\boldsymbol{\varphi}^{(k)})_i = \lambda_i^{(k)} \mathbf{M}^{(k)}(\boldsymbol{\varphi}^{(k)})_i$

❖ General description of CMS methods

4. Approximation of the global eigenvector using dominant substructural modes

$$(\boldsymbol{\varphi}_g)_i \approx (\overline{\boldsymbol{\varphi}}_g)_i = \overline{\mathbf{T}}_0(\overline{\boldsymbol{\varphi}}_p)_i$$

✓ Formulation details of T_0 differ depending on methodologies.

5. Reduced model

$$\bar{\mathbf{M}}_p = \bar{\mathbf{T}}_0^T \mathbf{M}_g \bar{\mathbf{T}}_0, \quad \bar{\mathbf{K}}_p = \bar{\mathbf{T}}_0^T \mathbf{K}_g \bar{\mathbf{T}}_0$$

6. Reduced eigenvalue problem

$$\overline{\mathbf{K}}_p(\overline{\boldsymbol{\varphi}}_p)_i = \lambda_i \overline{\mathbf{M}}_p(\overline{\boldsymbol{\varphi}}_p)_i \qquad \boxed{\bar{N}_p \ll N_g}$$

Computational cost ↓

Key of the improved model reduction method is the more precise approximation of the global eigenvector!

❖ Related researches

1) Craig-Bampton (CB) method: Craig and Bampton (1968) Most popular

2) Hybrid method: MacNeal (1971)

3) Dual CB method: Rixen (2004)

4) Flexibility based CMS (F-CMS) method: KC Park and YH Park (2004) Improved accuracy

5) AMLS method: Bennighof (2004) Computer-aid formulation of the CB method

6) Enhanced CB method: JG Kim and PS Lee (2014)

7) Enhanced AMLS method: JG Kim, SH Boo and PS Lee (2014)

<u>Derivation procedure will be presented by the CB method.</u>

❖Original CB method

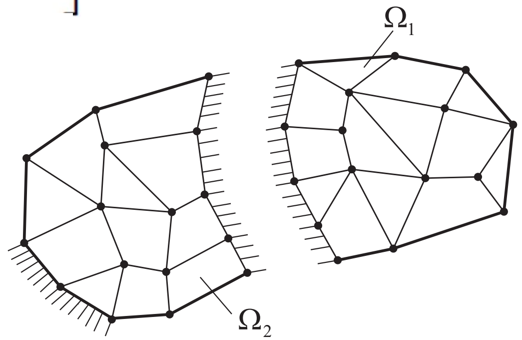
➢ Transformation matrix

$$N_g \times N_g \qquad\qquad N_g \times \bar{N}_p$$

$$\mathbf{T}_0 = \begin{bmatrix} \boldsymbol{\Phi}_d & \boldsymbol{\Phi}_r & -\mathbf{K}_s^{-1}\mathbf{K}_c \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_b \end{bmatrix} \longrightarrow \bar{\mathbf{T}}_0 = \begin{bmatrix} \boldsymbol{\Phi}_d & -\mathbf{K}_s^{-1}\mathbf{K}_c \\ \mathbf{0} & \mathbf{I}_b \end{bmatrix}$$

Substructural eigenvector matrix

Interface constraint mode



$$\mathbf{u}_g \approx \bar{\mathbf{u}}_g = \bar{\mathbf{T}}_0 \bar{\mathbf{u}}_p$$

➢ Reduced model

$$\bar{\mathbf{M}}_p = \bar{\mathbf{T}}_0^T \mathbf{M}_g \bar{\mathbf{T}}_0, \quad \bar{\mathbf{K}}_p = \bar{\mathbf{T}}_0^T \mathbf{K}_g \bar{\mathbf{T}}_0$$

For better approximation of the global eigenvector, we have focused on the residual modal effect.

❖Enhanced CB method

➢ **Enhanced transformation matrix**

$$\mathbf{T}_0 = \begin{bmatrix} \boxed{\begin{matrix} \mathbf{\Phi}_d & \mathbf{\Phi}_r \\ \mathbf{0} & \mathbf{0} \end{matrix}} & \boxed{\begin{matrix} -\mathbf{K}_s^{-1}\mathbf{K}_c \\ \mathbf{I}_b \end{matrix}} \end{bmatrix} \longrightarrow \bar{\mathbf{T}}_1 = \bar{\mathbf{T}}_0 + \bar{\mathbf{T}}_r,$$

Substructural eigenvector matrix

Interface constraint mode

$$\bar{\mathbf{T}}_r = \begin{bmatrix} \mathbf{0} & \lambda \mathbf{F}_{rs}\left[-\mathbf{M}_s\mathbf{K}_s^{-1}\mathbf{K}_c + \mathbf{M}_c\right] \\ \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$\mathbf{F}_{rs} = \mathbf{K}_s^{-1} - \mathbf{\Phi}_d \mathbf{\Lambda}_d^{-1} \mathbf{\Phi}_d^T$$

✓ No calculation of residual modes

➡ $\mathbf{u}_g \approx \bar{\mathbf{u}}_g = \bar{\mathbf{T}}_1 \bar{\mathbf{u}}_p$

✓ More precisely approximated global eigenvector

Since λ is unknown, it might be handle to employ it for the model reduction method.

❖Enhanced CB method

- **Handling technique of λ**

$$\boxed{\lambda\bar{\mathbf{u}}_p = \bar{\mathbf{M}}_p^{-1}\bar{\mathbf{K}}_p\bar{\mathbf{u}}_p}$$

$$\bar{\mathbf{T}}_r = \begin{bmatrix} \mathbf{0} & \mathbf{F}_{rs}\left[-\mathbf{M}_s\mathbf{K}_s^{-1}\mathbf{K}_c + \mathbf{M}_c\right] \\ \mathbf{0} & \mathbf{0} \end{bmatrix}\bar{\mathbf{M}}_p^{-1}\bar{\mathbf{K}}_p$$

redefined without unknown

- **Reduced model**

Original CB

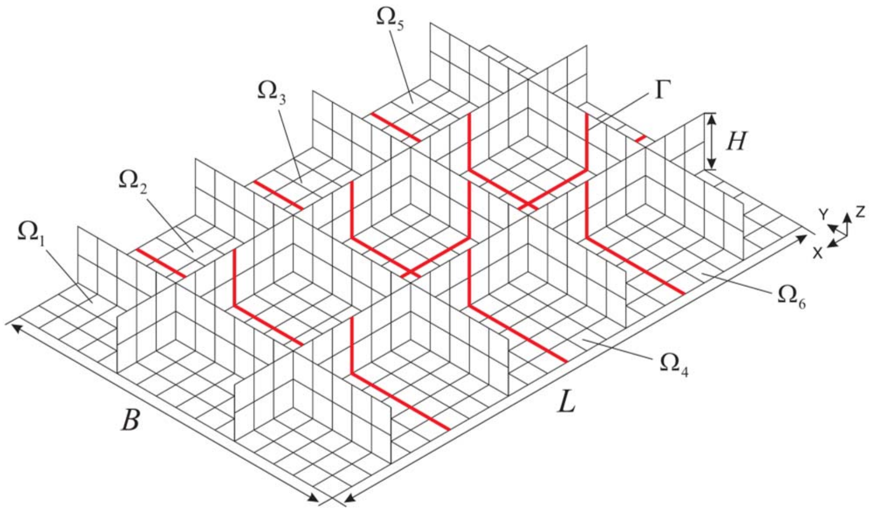
Additional terms in E-CB

$$\tilde{\mathbf{M}}_p = \bar{\mathbf{T}}_1^T\mathbf{M}_g\bar{\mathbf{T}}_1 = \boxed{\bar{\mathbf{T}}_0^T\mathbf{M}_g\bar{\mathbf{T}}_0} + \boxed{\bar{\mathbf{T}}_r^T\mathbf{M}_g\bar{\mathbf{T}}_0 + \bar{\mathbf{T}}_0^T\mathbf{M}_g\bar{\mathbf{T}}_r + \bar{\mathbf{T}}_r^T\mathbf{M}_g\bar{\mathbf{T}}_r},$$

$$\tilde{\mathbf{K}}_p = \bar{\mathbf{T}}_1^T\mathbf{K}_g\bar{\mathbf{T}}_1 = \boxed{\bar{\mathbf{T}}_0^T\mathbf{K}_g\bar{\mathbf{T}}_0} + \boxed{\bar{\mathbf{T}}_r^T\mathbf{K}_g\bar{\mathbf{T}}_0 + \bar{\mathbf{T}}_0^T\mathbf{K}_g\bar{\mathbf{T}}_r + \bar{\mathbf{T}}_r^T\mathbf{K}_g\bar{\mathbf{T}}_r}.$$

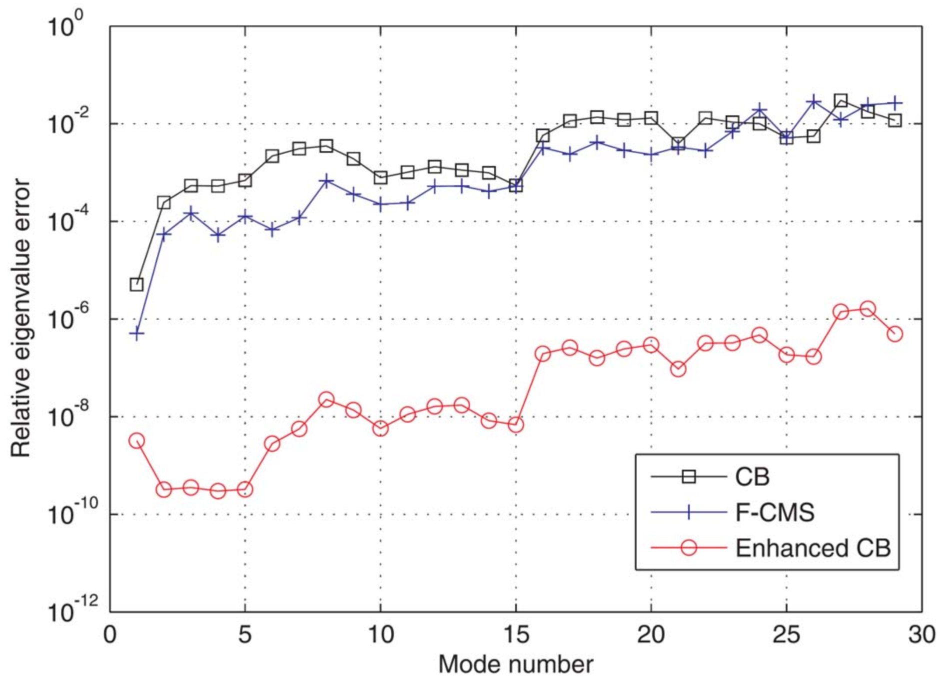
Better approximation. Same matrix size.

❖ Numerical result



<Stiffened plate problem>

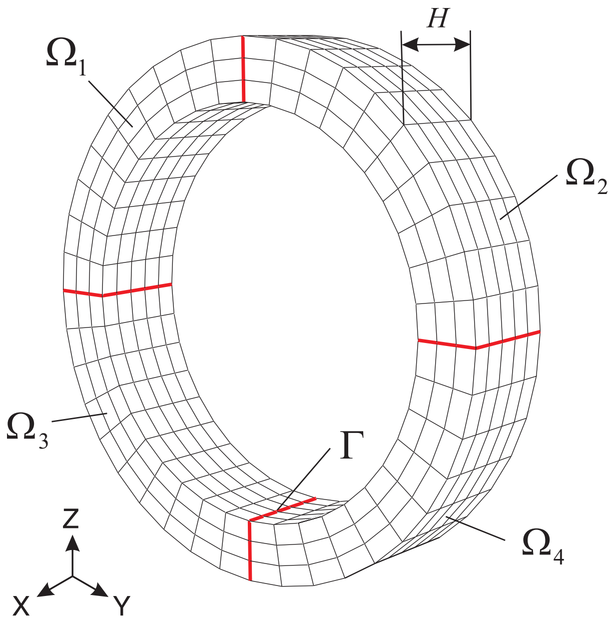
$$\xi_j = \frac{\bar{\lambda}_i}{\lambda_i} - 1$$

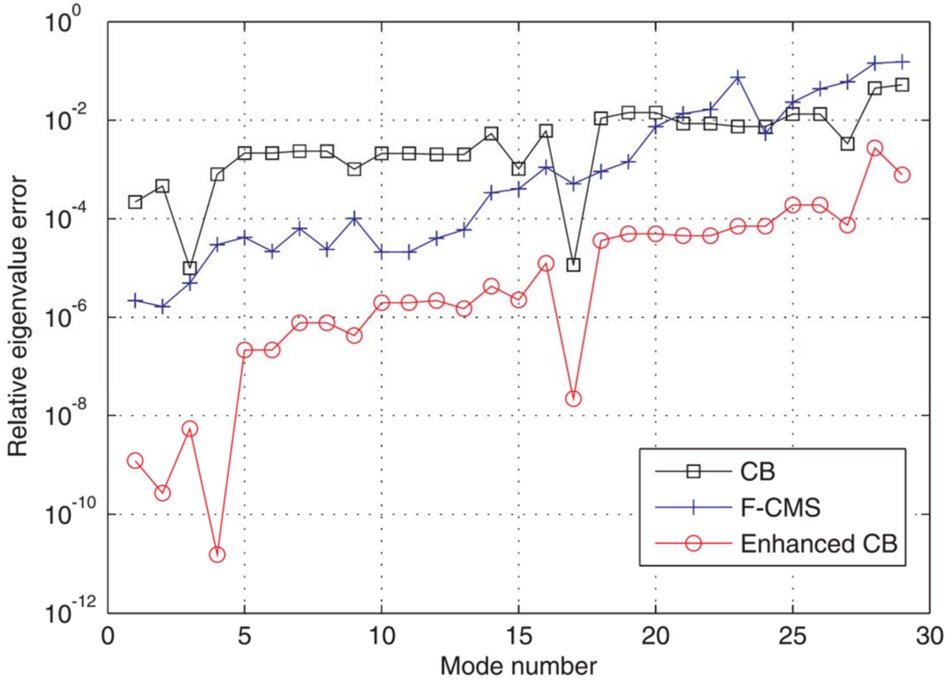


$$N_d = 50$$

The present formulation generally shows 1,000 times better solution accuracy in this example.

❖ Numerical result





<Solid bearing problem>

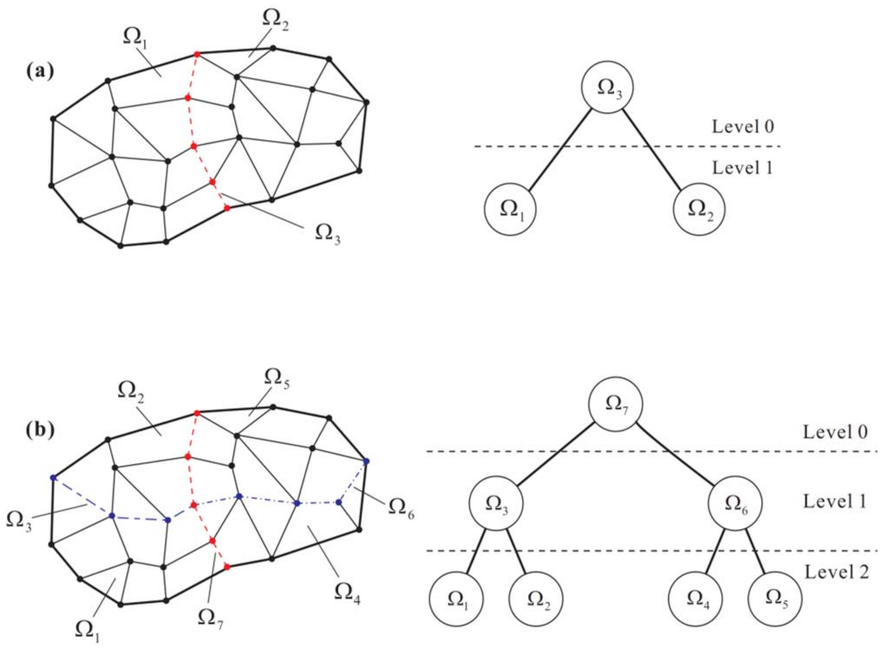
$$N_d = 40$$

$$\xi_j = \frac{\bar{\lambda}_i}{\lambda_i} - 1$$

❖ Enhanced AMLS method

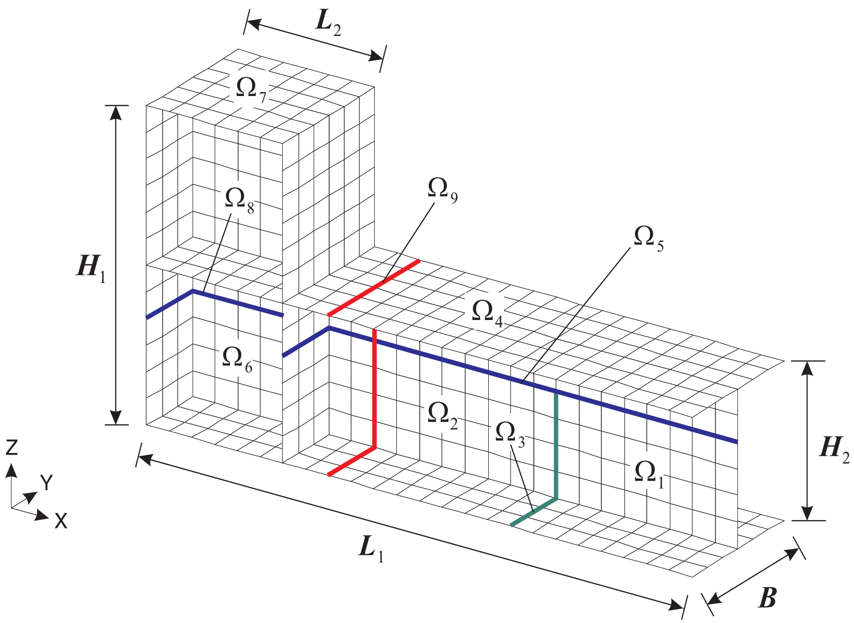
➢ Automated multi-level substructuring (AMLS) method (*Bennighof, 2004*)

: Computer-aid formulation based on the CB method, Hierarchical partitioning, Interface reduction, Much more complex formulation

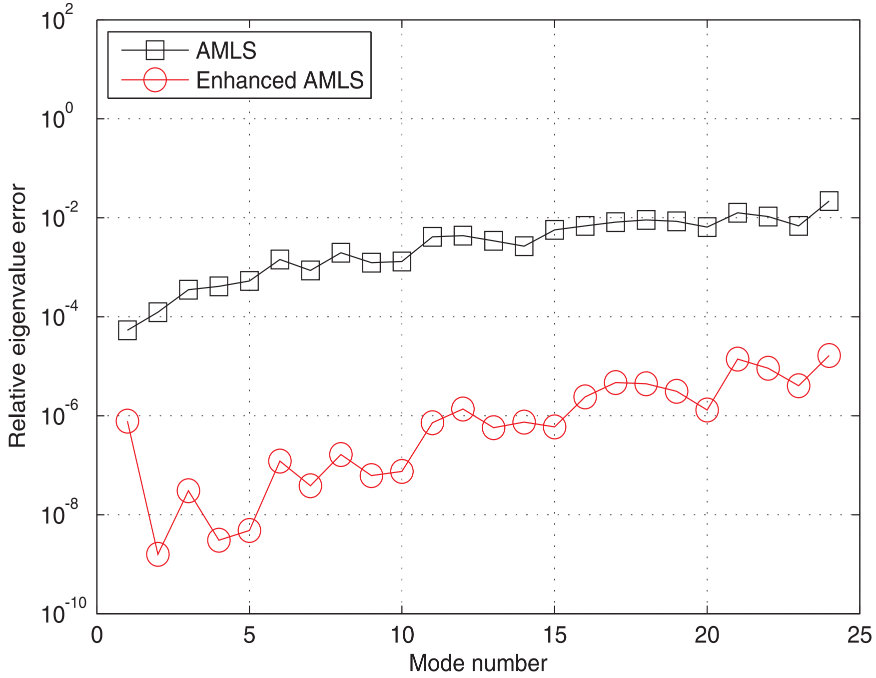


AMLS	Enhanced AMLS
$\overline{\mathbf{T}}_0$	$\overline{\mathbf{T}}_0 + \overline{\mathbf{T}}_r$
$\overline{\mathbf{M}}_p$	$\overline{\mathbf{M}}_p + \overline{\mathbf{T}}_r^T \mathbf{M}_g \overline{\mathbf{T}}_0 + \overline{\mathbf{T}}_0^T \mathbf{M}_g \overline{\mathbf{T}}_r + \overline{\mathbf{T}}_r^T \mathbf{M}_g \overline{\mathbf{T}}_r$
$\overline{\mathbf{K}}_p$	$\overline{\mathbf{K}}_p + \overline{\mathbf{T}}_r^T \mathbf{K}_g \overline{\mathbf{T}}_0 + \overline{\mathbf{T}}_0^T \mathbf{K}_g \overline{\mathbf{T}}_r + \overline{\mathbf{T}}_r^T \mathbf{K}_g \overline{\mathbf{T}}_r$
$\overline{N}_p$	$\overline{N}_p$

❖ Numerical result



<Bench corner problem>



$$N_d = 50$$

❖ Computational cost

➢ Comparison of computational cost

	DOFs		Computation time (sec)	
	N_g	$\bar{N}_p$	AMLS	Enhanced AMLS
Rectangular plate (Freq. cut-off, $N_d = 30$)	1365	135	2.685E-01	2.711E-01
Cylindrical solid (Freq. cut-off, $N_d = 70$)	1740	250	2.862E-01	2.950E-01
Bench corner structure (Freq. cut-off, $N_d = 50$)	3508	147	7.992E-01	8.155E-01

Verification of computational efficiency

❖ Closure

1. **We proposed two enhanced CMS method: the enhanced CB and AMLS methods.**

2. **This concept can be applied for other model reduction methods such as the dual CB method and the F-CMS method.**

3. **Using the proposed enhanced method, most existing related techniques may be upgraded.**

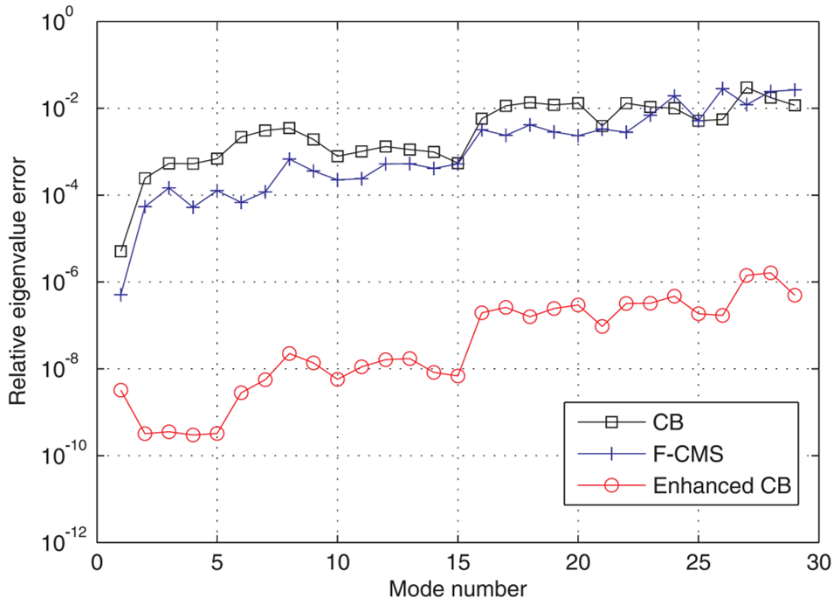
JG Kim, PS Lee, An enhanced Craig-Bampton method, International Journal for Numerical Methods in Engineering, submitted.

JG Kim, SH Boo, PS Lee. A new automated multi-level substructuring method, Computer Methods in Applied Mechanics and Engineering, in preparation (in May).

JG Kim, PS Lee, KC Park. An enhanced flexibility based component mode synthesis, ongoing research.

Topic 2. Error estimator for model reduction

❖ Major difficulty



$$\xi_j = \frac{\bar{\lambda}_i}{\lambda_i} - 1$$

Unless we have the reference solution…?

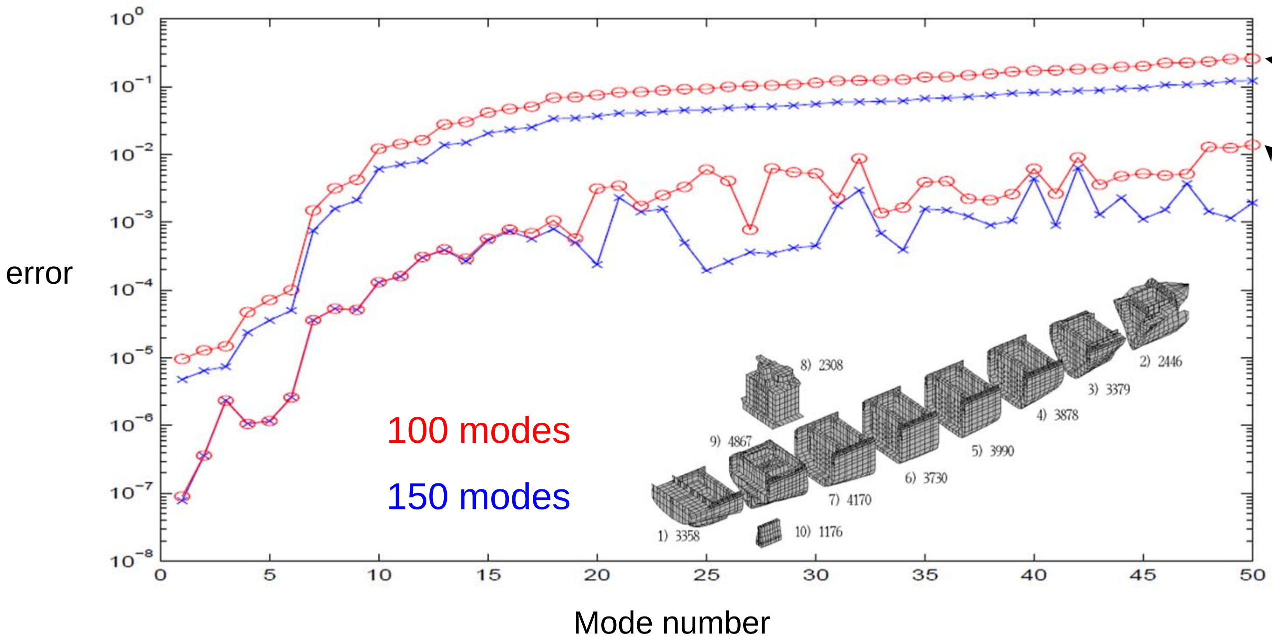
Problem: How do we estimate the solution accuracy and reliability of the reduced problem efficiently? *<u>This is major difficulty of model reduction techniques!</u>*

A disadvantage of reduction techniques such as the Iron-Guyan procedure and component mode synthesis is that there is no guarantee that the eigenvalues and eigenvectors of the reduced problem will be good approximations of those of the original problem….. *T.J.R. Hughes*

❖ Related researches

- Mode based reduction method, Elssel and Voss (2007)

  : an upper bound of relative eigenvalue error, CB and AMLS methods



$$\eta_i' = \frac{\bar{\lambda}_i}{|\lambda_r - \bar{\lambda}_i|}$$

$$\xi_j = \frac{\bar{\lambda}_i}{\lambda_i} - 1$$

Indirect estimation

- DOFs based reduction method: No error estimator → iterative method

To overcome this difficulty, we here propose an accurate error estimator for model reduction. It employed CB, AMLS, F-CMS and Guyan reduction!

❖ Enhanced transformation matrix

➢ **In the CB method**

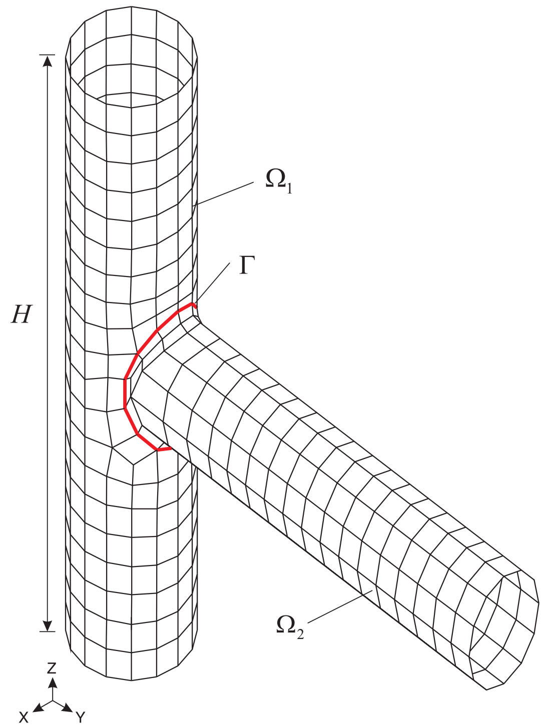
$$\mathbf{T}_0 = \begin{bmatrix} \boldsymbol{\Phi}_d & \boldsymbol{\Phi}_r & -\mathbf{K}_s^{-1}\mathbf{K}_c \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_b \end{bmatrix}$$

Substructural eigenvector matrix
Interface constraint mode

$$\longrightarrow \bar{\mathbf{T}}_1 = \bar{\mathbf{T}}_0 + \bar{\mathbf{T}}_r,$$

$$\bar{\mathbf{T}}_r = \begin{bmatrix} \mathbf{0} & \lambda\mathbf{F}_{rs}\left[-\mathbf{M}_s\mathbf{K}_s^{-1}\mathbf{K}_c + \mathbf{M}_c\right] \\ \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$\mathbf{F}_{rs} = \mathbf{K}_s^{-1} - \boldsymbol{\Phi}_d\boldsymbol{\Lambda}_d^{-1}\boldsymbol{\Phi}_d^T$$



This is pre-requisite to develop an accurate error estimator.

❖ Brief derivation procedure of error estimator

Step 1. **Start from global eigenvalue problem.**

$$\frac{1}{\lambda_i}(\boldsymbol{\varphi}_g)_i^T \mathbf{K}_g (\boldsymbol{\varphi}_g)_i = (\boldsymbol{\varphi}_g)_i^T \mathbf{M}_g (\boldsymbol{\varphi}_g)_i$$

Step 2. **Decompose the original eigenvector.**

$$(\boldsymbol{\varphi}_g)_i = (\overline{\boldsymbol{\varphi}}_g)_i + (\delta\boldsymbol{\varphi})_i$$

Step 3. **Represent the approximated original eigenvector using $\mathbf{T}_1$.**

$$(\overline{\boldsymbol{\varphi}}_g)_i = \overline{\mathbf{T}}_1 (\overline{\boldsymbol{\varphi}}_p)_i, \qquad \overline{\mathbf{T}}_1 = \overline{\mathbf{T}}_0 + \overline{\mathbf{T}}_r$$

Step 4. **submit and rearrange.**

$$\boxed{\frac{\overline{\lambda}_i}{\lambda_i} - 1} \approx \boxed{\begin{aligned} & 2(\overline{\boldsymbol{\varphi}}_p)_i^T \mathbf{T}_0^{\,T}\left[\mathbf{M}_g - \frac{1}{\lambda_i}\mathbf{K}_g\right]\mathbf{T}_r(\overline{\boldsymbol{\varphi}}_p)_i \\ & + (\overline{\boldsymbol{\varphi}}_p)_i^T \mathbf{T}_r^{\,T}\left[\mathbf{M}_g - \frac{1}{\lambda_i}\mathbf{K}_g\right]\mathbf{T}_r(\overline{\boldsymbol{\varphi}}_p)_i \end{aligned}}$$

Relative eigenvalue error can be directly approximated by matrix and vector operations.

❖ Specific derivation

From the global eigenvalue problem

$$\Rightarrow \quad \frac{1}{\lambda_i}(\boldsymbol{\varphi}_g)_i^T \mathbf{K}_g (\boldsymbol{\varphi}_g)_i = (\boldsymbol{\varphi}_g)_i^T \mathbf{M}_g (\boldsymbol{\varphi}_g)_i$$

$$(\boldsymbol{\varphi}_g)_i = (\overline{\boldsymbol{\varphi}}_g)_i + (\delta\boldsymbol{\varphi})_i$$

Then,

$$\Rightarrow \underline{\frac{1}{\lambda_i}(\overline{\boldsymbol{\varphi}}_g)_i^T \mathbf{K}_g (\overline{\boldsymbol{\varphi}}_g)_i - (\overline{\boldsymbol{\varphi}}_g)_i^T \mathbf{M}_g (\overline{\boldsymbol{\varphi}}_g)_i - \frac{1}{\lambda_i}(\delta\boldsymbol{\varphi})_i^T \mathbf{K}_g (\delta\boldsymbol{\varphi})_i + (\delta\boldsymbol{\varphi})_i^T \mathbf{M}_g (\delta\boldsymbol{\varphi})_i = 0}$$

Four scalar terms

The approximated global eigenvector using the enhanced transformation matrix

$$(\overline{\boldsymbol{\varphi}}_g)_i = \overline{\mathbf{T}}_1 (\overline{\boldsymbol{\varphi}}_p)_i, \qquad \overline{\mathbf{T}}_1 = \overline{\mathbf{T}}_0 + \overline{\mathbf{T}}_r$$

❖ Specific derivation

$$\Rightarrow \frac{1}{\lambda_i}(\overline{\boldsymbol{\varphi}}_p)_i^T[\mathbf{T}_0+\mathbf{T}_r]^T\mathbf{K}_g[\mathbf{T}_0+\mathbf{T}_r](\overline{\boldsymbol{\varphi}}_p)_i-(\overline{\boldsymbol{\varphi}}_p)_i^T[\mathbf{T}_0+\mathbf{T}_r]^T\mathbf{M}_g[\mathbf{T}_0+\mathbf{T}_r](\overline{\boldsymbol{\varphi}}_p)_i$$

$$-\frac{1}{\lambda_i}(\delta\boldsymbol{\varphi})_i^T\left[\mathbf{K}_g-\lambda_i\mathbf{M}_g\right](\delta\boldsymbol{\varphi})_i=0$$

$$\boxed{(\overline{\boldsymbol{\varphi}}_p)_i^T\mathbf{T}_0^T\mathbf{M}_g\mathbf{T}_0(\overline{\boldsymbol{\varphi}}_p)_i}=(\overline{\boldsymbol{\varphi}}_p)_i^T\overline{\mathbf{M}}_p(\overline{\boldsymbol{\varphi}}_p)_i=\delta_{ij}$$

$$\boxed{(\overline{\boldsymbol{\varphi}}_p)_i^T\mathbf{T}_0^T\mathbf{K}_g\mathbf{T}_0(\overline{\boldsymbol{\varphi}}_p)_i}=(\overline{\boldsymbol{\varphi}}_p)_i^T\overline{\mathbf{K}}_p(\overline{\boldsymbol{\varphi}}_p)_i=\overline{\lambda}_j\delta_{ij}$$

Using mass-orthonormality and stiffness-orthogonality conditions

$$\Rightarrow \frac{\overline{\lambda}_i}{\lambda_i}-1=2(\overline{\boldsymbol{\varphi}}_p)_i^T\mathbf{T}_0^T\left[\mathbf{M}_g-\frac{1}{\lambda_i}\mathbf{K}_g\right]\mathbf{T}_r(\overline{\boldsymbol{\varphi}}_p)_i+(\overline{\boldsymbol{\varphi}}_p)_i^T\mathbf{T}_r^T\left[\mathbf{M}_g-\frac{1}{\lambda_i}\mathbf{K}_g\right]\mathbf{T}_r(\overline{\boldsymbol{\varphi}}_p)_i$$

$$\boxed{+\frac{1}{\lambda_i}(\delta\boldsymbol{\varphi})_i^T\left[\mathbf{K}_g-\lambda_i\mathbf{M}_g\right](\delta\boldsymbol{\varphi})_i}$$

Relative eigenvalue error

This equation has two unknowns: $\lambda_i,\ (\delta\boldsymbol{\varphi})_i$

❖ Specific derivation

Using assumptions: $(\boldsymbol{\varphi}_g)_i \approx (\overline{\boldsymbol{\varphi}}_g)_i$

$$\frac{1}{\lambda_i}(\overline{\boldsymbol{\varphi}}_g)_i^T \mathbf{K}_g (\overline{\boldsymbol{\varphi}}_g)_i \approx 1 \text{ and } \frac{1}{\lambda_i}(\overline{\boldsymbol{\varphi}}_g)_i^T \mathbf{K}_g (\overline{\boldsymbol{\varphi}}_g)_i >> \frac{1}{\lambda_i}(\delta\boldsymbol{\varphi})_i^T \mathbf{K}_g (\delta\boldsymbol{\varphi})_i$$

$$(\overline{\boldsymbol{\varphi}}_g)_i^T \mathbf{M}_g (\overline{\boldsymbol{\varphi}}_g)_i \approx 1 \text{ and } (\overline{\boldsymbol{\varphi}}_g)_i^T \mathbf{M}_g (\overline{\boldsymbol{\varphi}}_g)_i >> (\delta\boldsymbol{\varphi})_i^T \mathbf{M}_g (\delta\boldsymbol{\varphi})_i$$

$$\Rightarrow \underline{\frac{\overline{\lambda}_i}{\lambda_i} - 1 = 2(\boldsymbol{\varphi}_p)_i^T \mathbf{T}_0^T \left[\mathbf{M}_g - \frac{1}{\lambda_i}\mathbf{K}_g\right] \mathbf{T}_r (\boldsymbol{\varphi}_p)_i + (\boldsymbol{\varphi}_p)_i^T \mathbf{T}_r^T \left[\mathbf{M}_g - \frac{1}{\lambda_i}\mathbf{K}_g\right] \mathbf{T}_r (\boldsymbol{\varphi}_p)_i}$$

$$\cancel{+\frac{1}{\lambda_i}(\delta\boldsymbol{\varphi})_i^T \left[\mathbf{K}_g - \lambda_i \mathbf{M}_g\right] (\delta\boldsymbol{\varphi})_i}$$

$$\therefore \frac{\overline{\lambda}_i}{\lambda_i} - 1 \approx \boxed{2(\boldsymbol{\varphi}_p)_i^T \mathbf{T}_0^T \left[\mathbf{M}_g - \frac{1}{\lambda_i}\mathbf{K}_g\right] \mathbf{T}_r (\boldsymbol{\varphi}_p)_i + (\boldsymbol{\varphi}_p)_i^T \mathbf{T}_r^T \left[\mathbf{M}_g - \frac{1}{\lambda_i}\mathbf{K}_g\right] \mathbf{T}_r (\boldsymbol{\varphi}_p)_i}$$

❖ Another specific derivation

The approximated global eigenvector can be represented by a linear combination of the exact global eigenvectors:

$$(\bar{\boldsymbol{\varphi}}_g)_i = \sum_{k=1}^{N_g} \alpha_k (\boldsymbol{\varphi}_g)_k$$

Using assumptions: $(\boldsymbol{\varphi}_g)_i \approx (\bar{\boldsymbol{\varphi}}_g)_i$

$$\alpha_i \approx 1 \quad |\alpha_i| >> |\alpha_i - 1|,\ |\alpha_1|,\ |\alpha_2| \cdots |\alpha_{i-1}|,\ |\alpha_{i+1}|, \cdots, |\alpha_{N_g}|$$

This assumption can be numerically proved.

$$\Rightarrow \quad \frac{\bar{\lambda}_i}{\lambda_i} - 1 = 2(\bar{\boldsymbol{\varphi}}_p)_i^T \bar{\mathbf{T}}_0^{\mathrm{T}} \left[\mathbf{M}_g - \frac{1}{\lambda_i}\mathbf{K}_g \right] \bar{\mathbf{T}}_r (\bar{\boldsymbol{\varphi}}_p)_i$$

$$+ (\bar{\boldsymbol{\varphi}}_p)_i^T \bar{\mathbf{T}}_r^T \left[\mathbf{M}_g - \frac{1}{\lambda_i}\mathbf{K}_g \right] \bar{\mathbf{T}}_r (\bar{\boldsymbol{\varphi}}_p)_i + \cancel{\left[\sum_{\substack{k=1 \\ k \neq i}}^{N_g} \alpha_k^2 \left(\frac{\lambda_k}{\lambda_i} - 1 \right) \right]}$$

❖ Error estimator

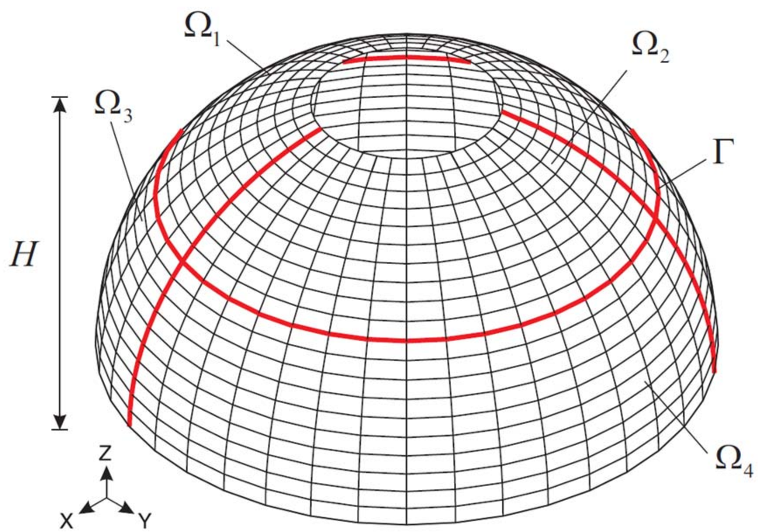
$$\frac{\bar{\lambda}_i}{\lambda_i} - 1 \approx 2(\overline{\boldsymbol{\varphi}}_p)_i^T \mathbf{T}_0{}^T \left[\mathbf{M}_g - \frac{1}{\lambda_i}\mathbf{K}_g \right] \mathbf{T}_r (\overline{\boldsymbol{\varphi}}_p)_i + (\overline{\boldsymbol{\varphi}}_p)_i^T \mathbf{T}_r{}^T \left[\mathbf{M}_g - \frac{1}{\lambda_i}\mathbf{K}_g \right] \mathbf{T}_r (\overline{\boldsymbol{\varphi}}_p)_i$$

$$\therefore \quad \mu_i = 2(\overline{\boldsymbol{\varphi}}_p)_i^T \overline{\mathbf{T}}_0^{\mathrm{T}} \left[\mathbf{M}_g - \frac{1}{\bar{\lambda}_i}\mathbf{K}_g \right] \overline{\mathbf{T}}_r (\overline{\boldsymbol{\varphi}}_p)_i + (\overline{\boldsymbol{\varphi}}_p)_i^T \overline{\mathbf{T}}_r^T \left[\mathbf{M}_g - \frac{1}{\bar{\lambda}_i}\mathbf{K}_g \right] \overline{\mathbf{T}}_r (\overline{\boldsymbol{\varphi}}_p)_i$$

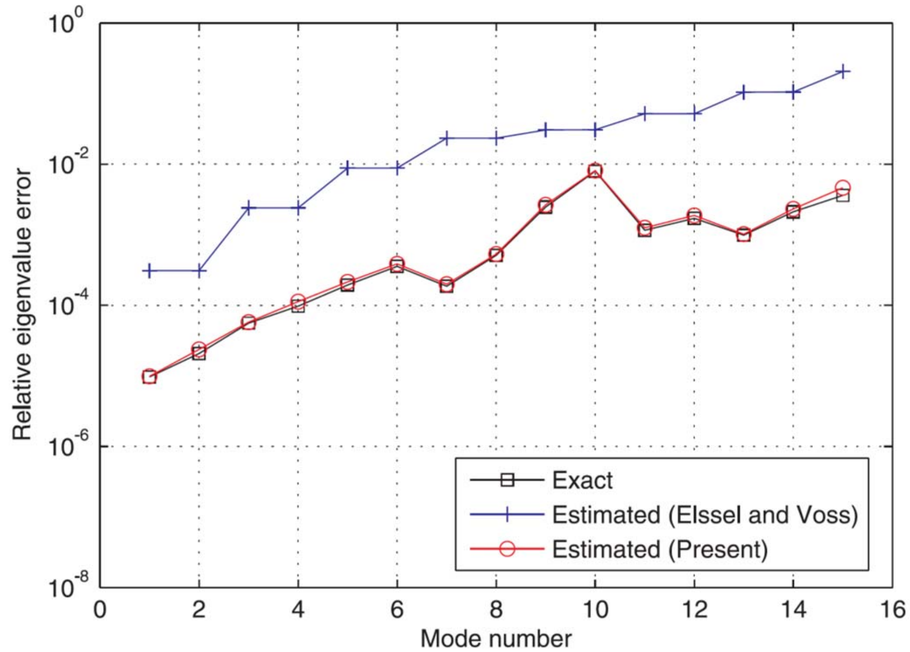
General form of error estimator

Note : Application key of this error estimator is how to derive the enhanced transformation matrix!

❖ Numerical test in the CB method



<Hemisphere shell problem>

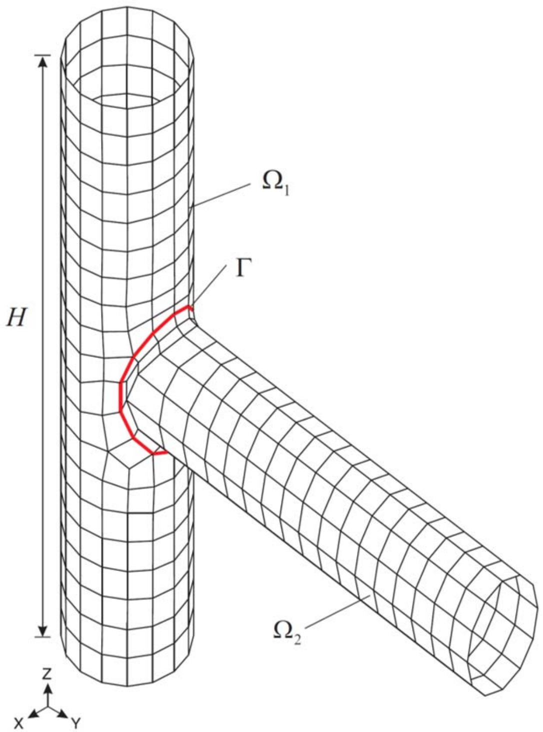


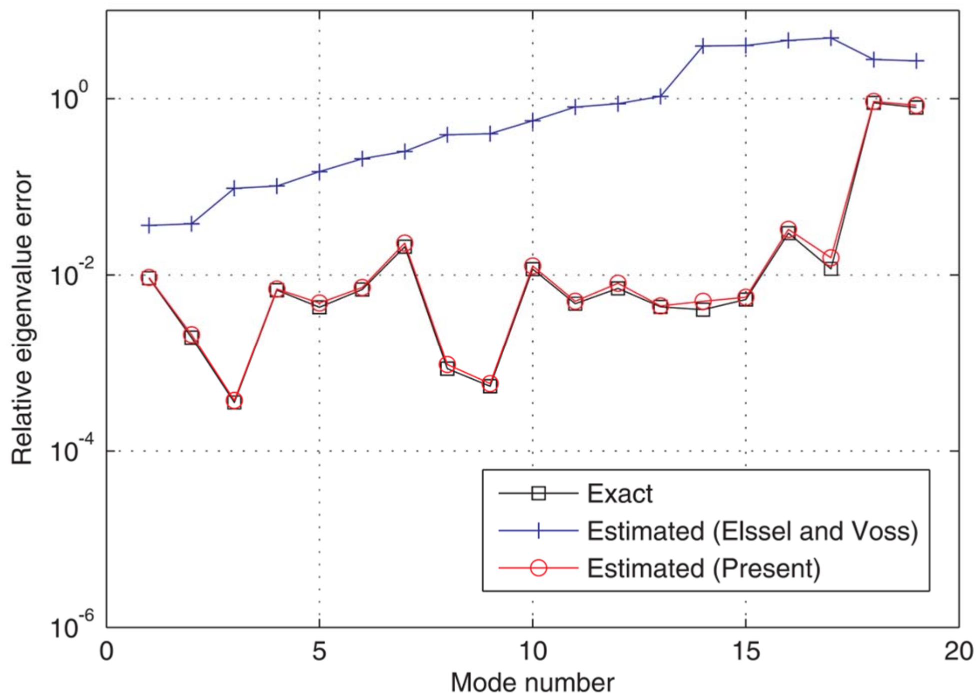
$$\boxed{\xi_j = \frac{\bar{\lambda}_i}{\lambda_i} - 1}$$

$$\eta_i = 2(\bar{\boldsymbol{\phi}}_p)_i^T \bar{\mathbf{T}}_0^T \left[\mathbf{M}_g - \frac{1}{\bar{\lambda}_i}\mathbf{K}_g\right] \bar{\mathbf{T}}_r (\bar{\boldsymbol{\phi}}_p)_i + (\bar{\boldsymbol{\phi}}_p)_i^T \bar{\mathbf{T}}_r^T \left[\mathbf{M}_g - \frac{1}{\bar{\lambda}_i}\mathbf{K}_g\right] \bar{\mathbf{T}}_r (\bar{\boldsymbol{\phi}}_p)_i,$$

$$\bar{\mathbf{T}}_r = \bar{\lambda}_i \begin{bmatrix} \mathbf{0} & \mathbf{F}_{rs}\left[-\mathbf{M}_s \mathbf{K}_s^{-1} \mathbf{K}_c + \mathbf{M}_c\right] \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$$

❖ Numerical test in the CB method





<Shaft-shaft interaction problem>

Verification of performance

❖ Computational cost

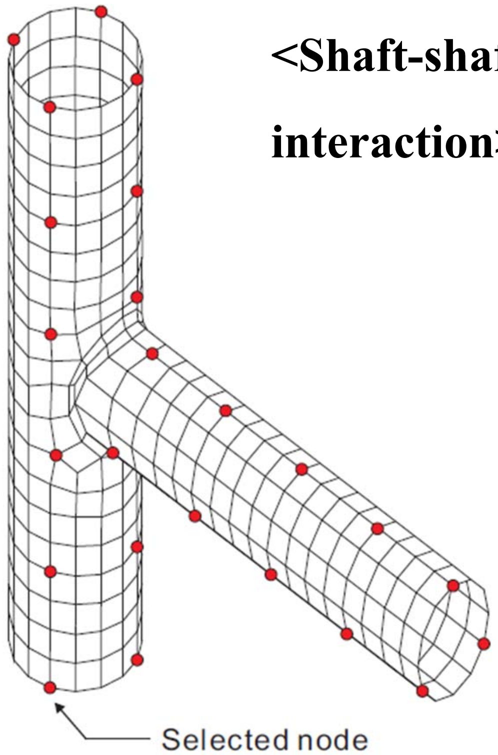
Table 6: Computation times for the exact and estimated relative eigenvalue errors.

	DOFs		Computation time (sec)		
	N_g	$\bar{N}_p$	Exact	Estimated using Eq. 43 (Elssel and Voss)	Estimated using Eq. 41 (Present)
Rectangular plate (Freq. cut-off, $N_d = 15$)	273	36	2.160E-01	1.548E-05	1.936E-03
Shaft-shaft interaction (Freq. cut-off, $N_d = 20$)	2,775	90	3.735E+00	1.897E-05	2.186E-02
Hemisphere shell (Freq. cut-off, $N_d = 25$)	4,200	425	1.722E+01	3.597E-05	4.722E-02
Stiffened plate (Freq. cut-off, $N_d = 25$)	3,351	423	5.141E+00	9.131E-06	2.413E-02

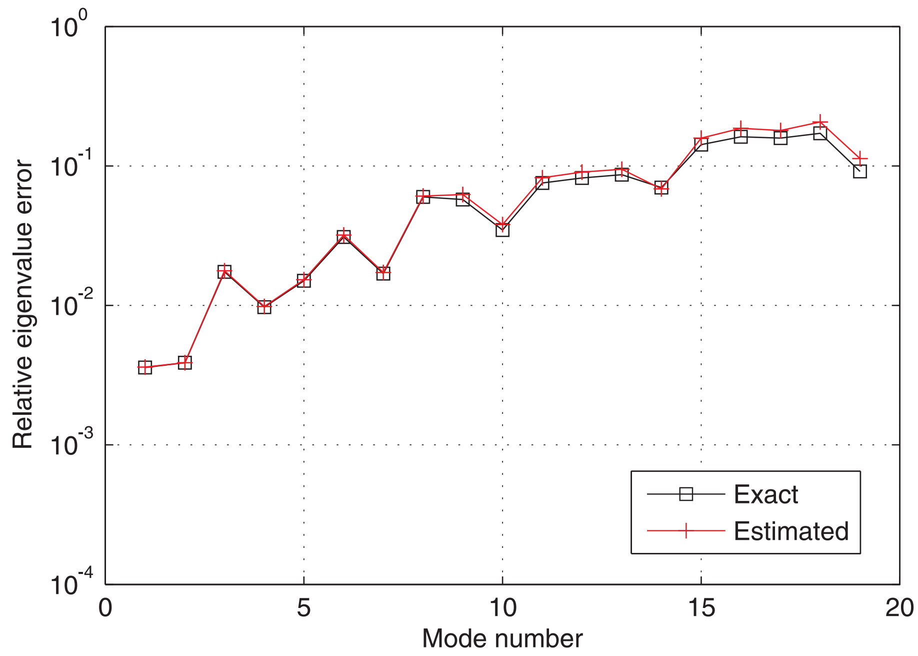
$$\xi_j = \frac{\bar{\lambda}_i}{\lambda_i} - 1$$

Verification of computational efficiency

❖ Numerical example in Guyan reduction



<Shaft-shaft interaction>



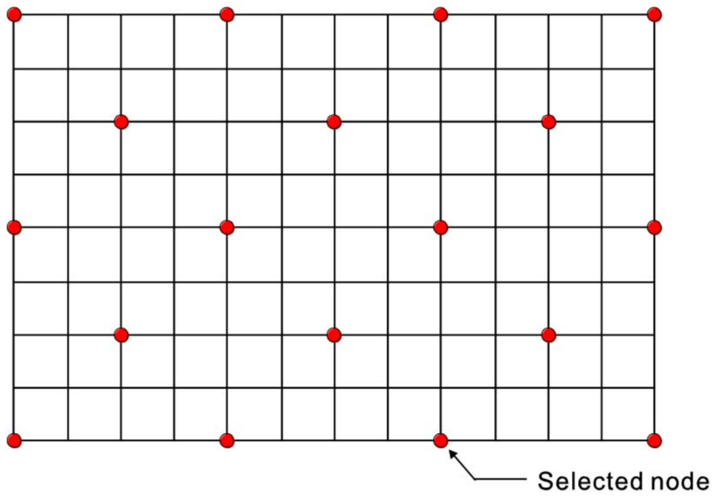
$$\xi_j = \frac{\bar{\lambda}_i}{\lambda_i} - 1$$

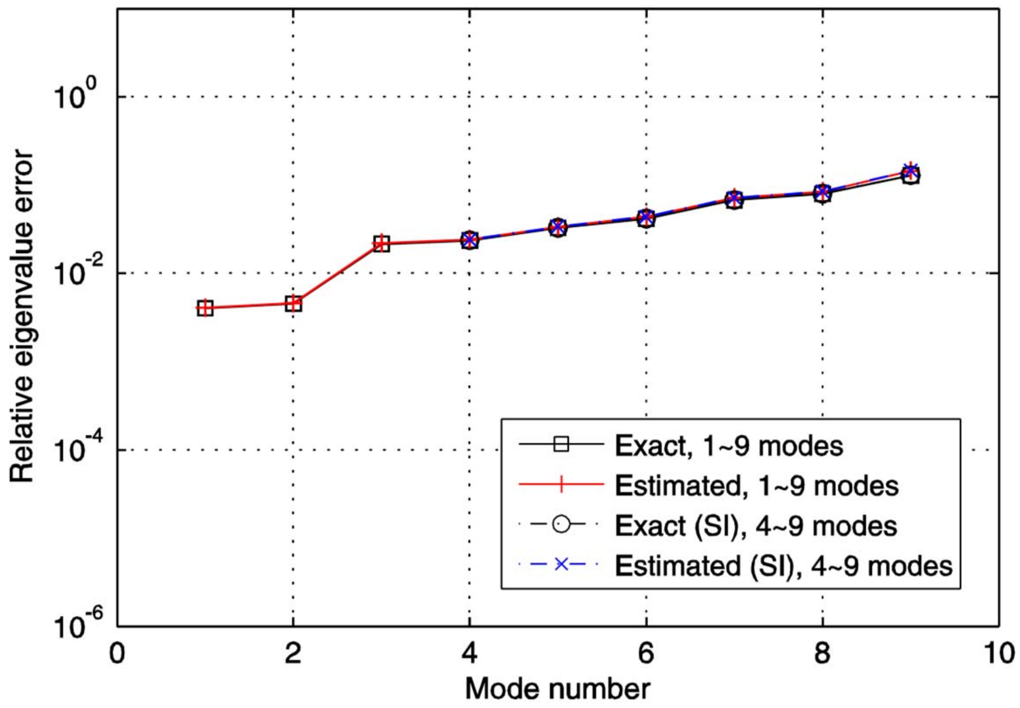
$$\eta_i = 2(\boldsymbol{\varphi}_1)_i^T \mathbf{T}_G{}^T \left[\mathbf{M} - \frac{1}{\bar{\lambda}_i}\mathbf{K}\right]\mathbf{T}_r(\boldsymbol{\varphi}_1)_i + (\boldsymbol{\varphi}_1)_i^T \mathbf{T}_r{}^T \left[\mathbf{M} - \frac{1}{\bar{\lambda}_i}\mathbf{K}\right]\mathbf{T}_r(\boldsymbol{\varphi}_1)_i$$

$$\mathbf{T}_r = \begin{bmatrix} \mathbf{0} \\ \bar{\lambda}_i\left[\mathbf{K}_{22}^{-1}\mathbf{M}_{21} - \mathbf{K}_{22}^{-1}\mathbf{M}_{22}\mathbf{K}_{22}^{-1}\mathbf{K}_{21}\right] + \bar{\lambda}_i{}^2\mathbf{K}_{22}^{-1}\mathbf{M}_{22}\mathbf{K}_{22}^{-1}\mathbf{M}_{21} \end{bmatrix}$$

❖ Numerical example in Guyan reduction

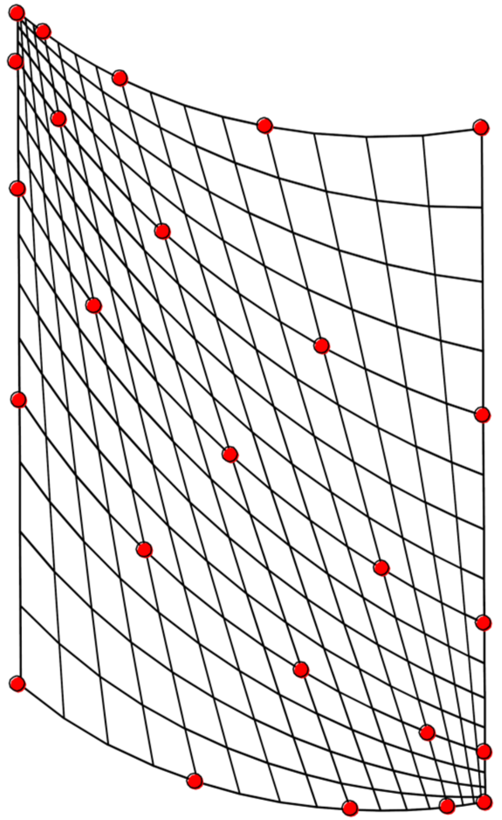
➢ **Shift and invert spectral transform**

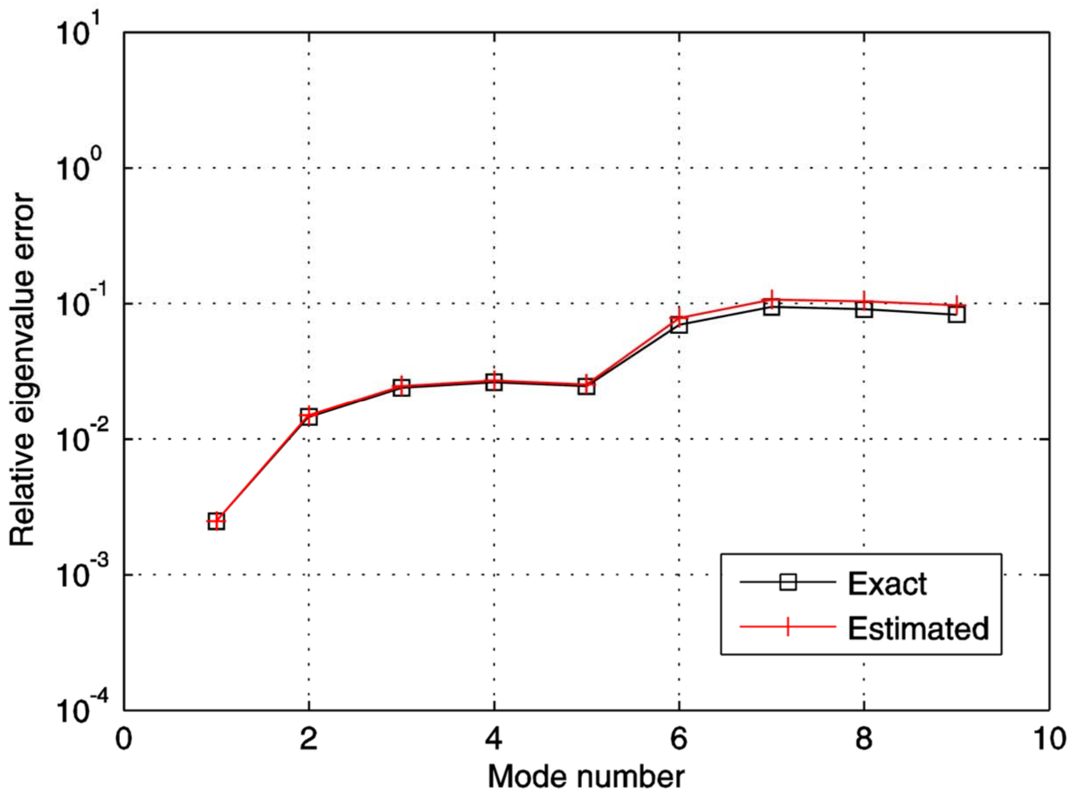




❖ Numerical example in Guyan reduction

➢ **Distorted mesh**





❖ Closure

1. **We developed the general error estimator, and it employed in CB, AMLS, F-CMS methods and Guyan reduction.**

2. **This concept can be applied for other model reduction methods.**

3. **Error estimators of enhanced methods might be developed.**

JG Kim, GH Lee, PS Lee. Estimating relative eigenvalue errors in the Craig-Bampton method, Computers and Structures, 139 (2014) 54-64.

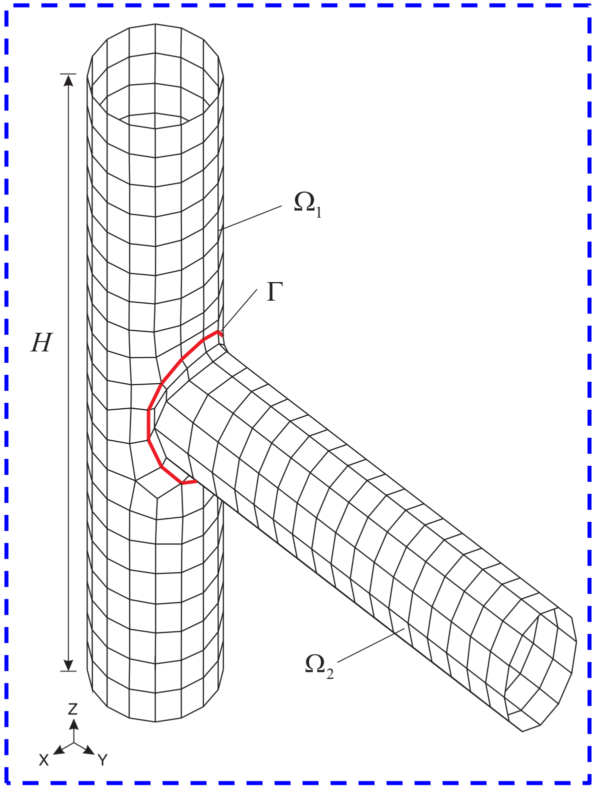
JG Kim, PS Lee. An accurate error estimator for Guyan reduction, Computer Methods in Applied Mechanics and Engineering, in press.

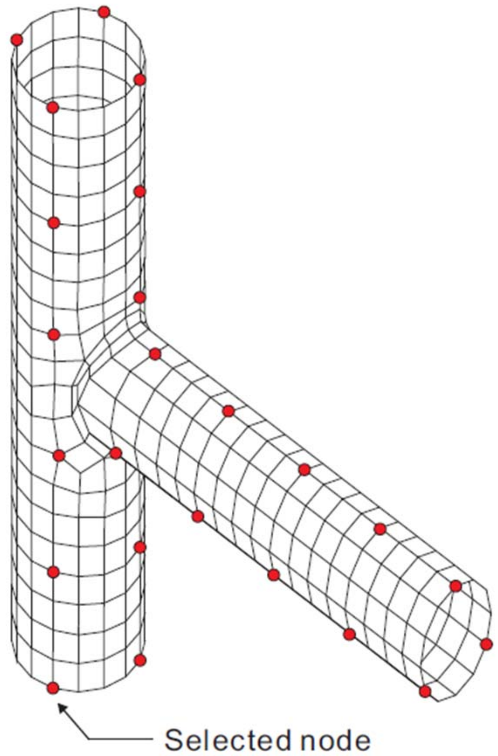
JG Kim, PS Lee. An error estimation method for the flexibility-based component mode synthesis method, AIAA journal, submitted.

SH Boo, JG Kim and PS Lee. An error estimator for the automated multi-level substructuring method, International Journal for Numerical Methods in Engineering, in preparation (in May).

Topic 3. Mode selection method for CMS methods

❖ Problem and key idea





Problem : What kinds of substructural modes / DOFs might be selected?

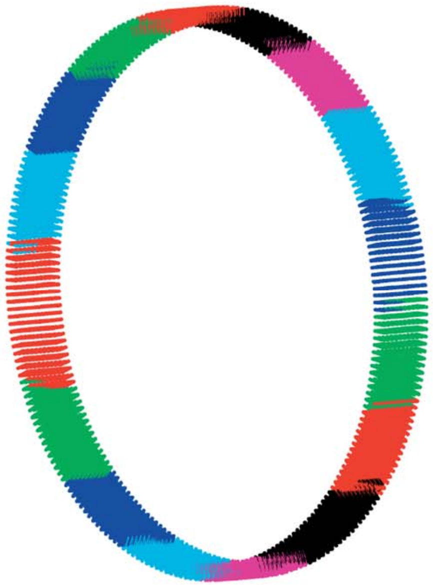
❖ Related researches

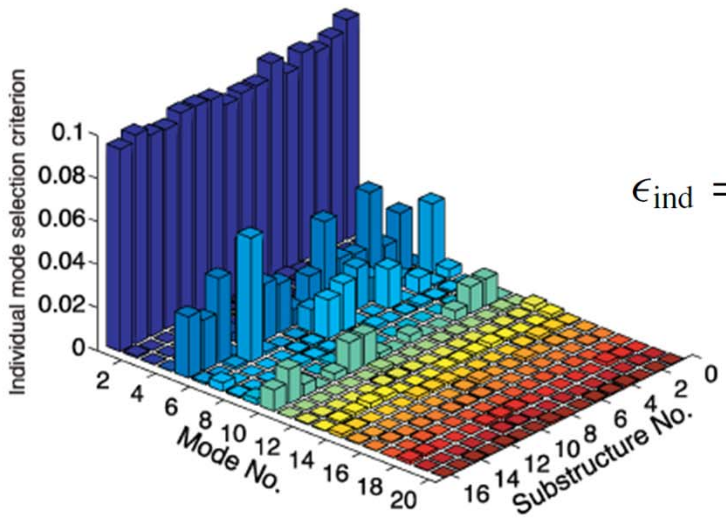
- Frequency cut-off

  : Hurty (1967), Collins et al. (1972), Craig and Chang (1977), Yang et al. (2005)

- Interface modal contribution based method

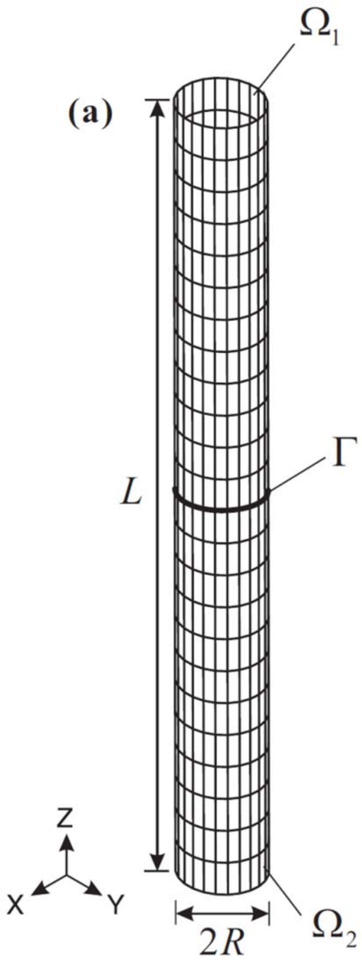
  : Kammer and Triller (1994, 1996), Barbone et al. (2003), Givoli et al. (2004), Park and Park (2004), Liao et al. (2007)

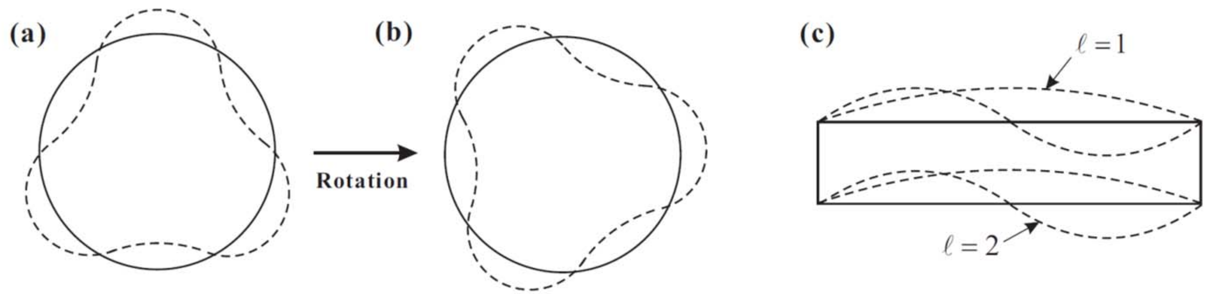




$$\epsilon_{\text{ind}} = 1 - \|\mathcal{F}_{rs}|_k\| / \|\boldsymbol{F}_{bb}\|$$

❖ Limitation of interface mode based selection





Circumferential mode

(○)

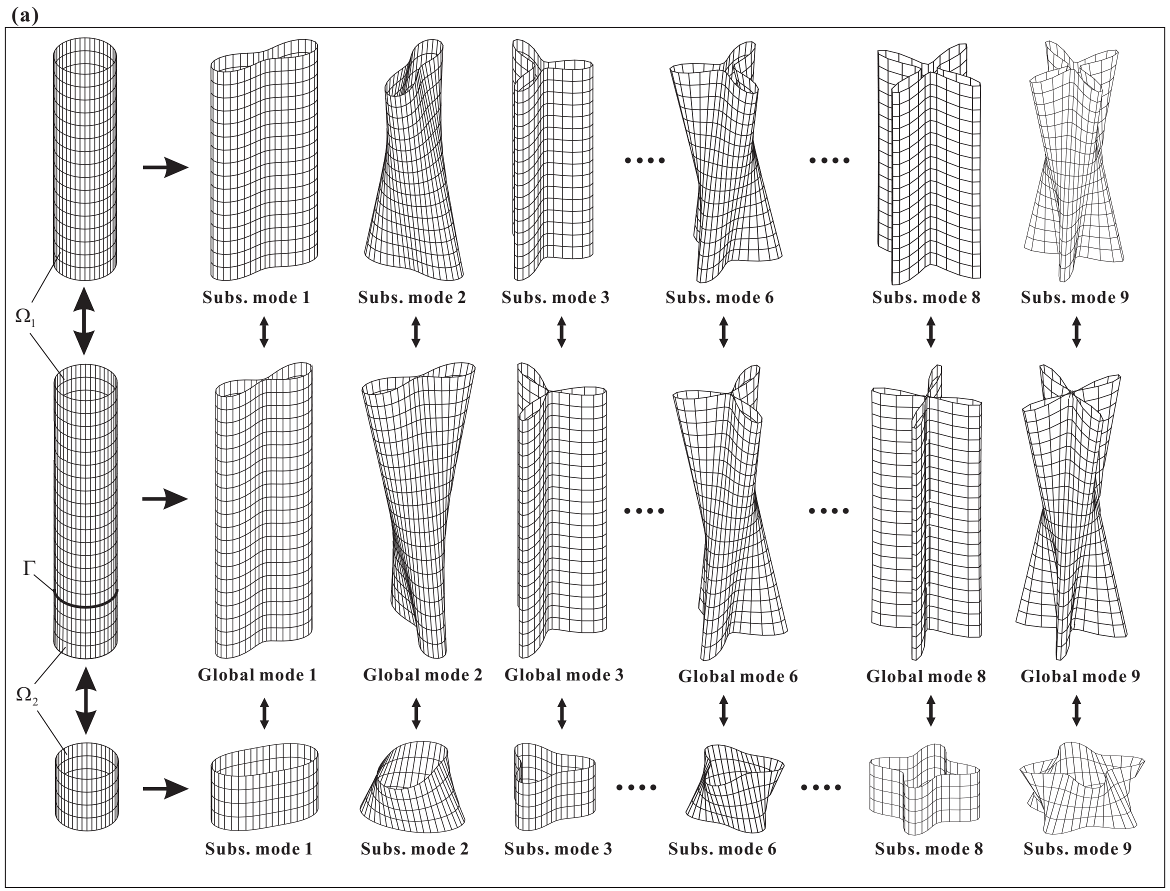
Longitudinal mode

(X)

Still, frequency cut-off is selected .

❖ Limitation of frequency cut-off

Lower global mode: Lower substructural modes are important.

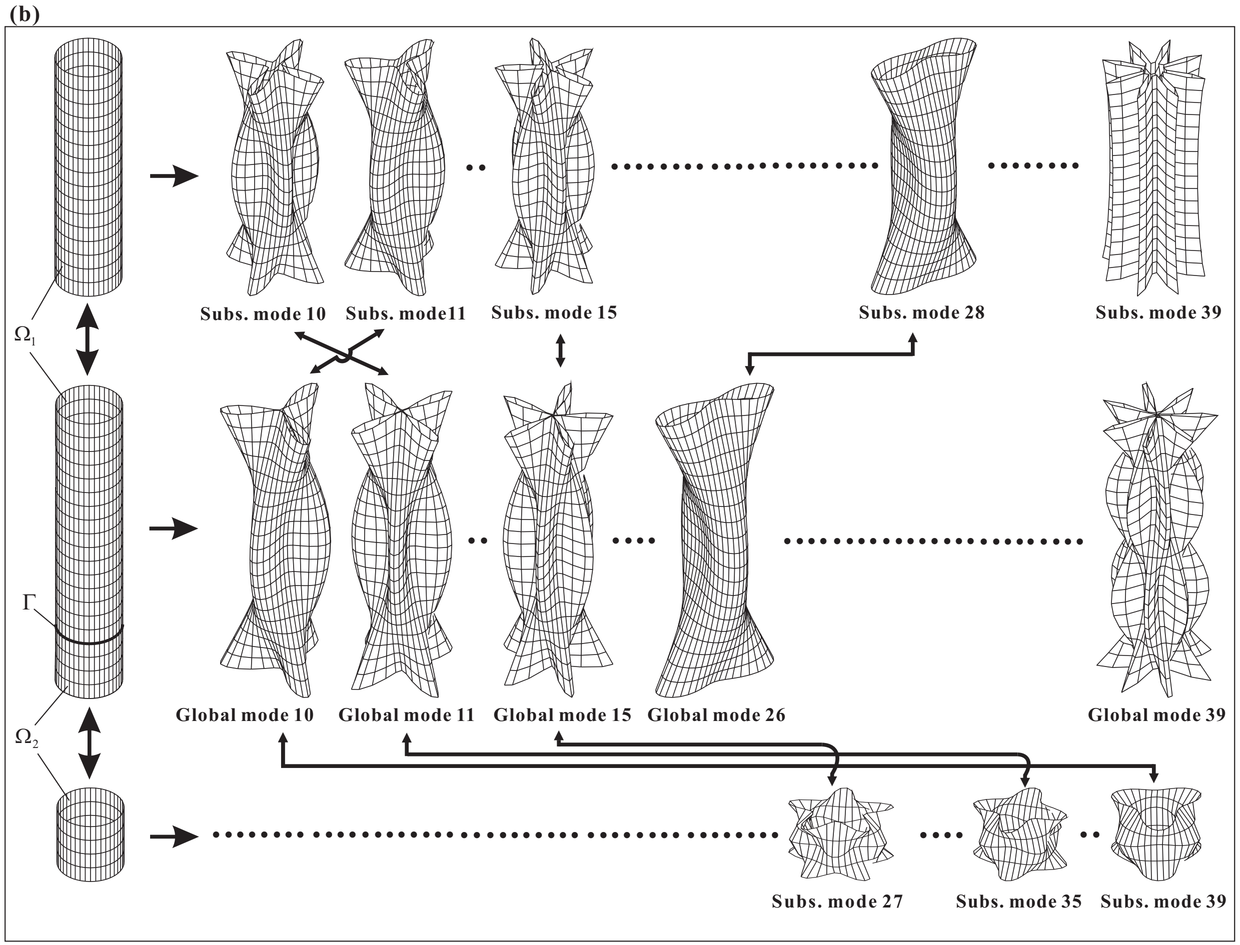


Freq. cut-off

(○)

❖ Limitation of frequency cut-off

Sometimes, higher substructural modes are more significant.



Freq. cut-off

(X)

Key idea: using eigenvector relation between substructural and global structure!

❖ Eigenvector relation in the CB method

➢ Approximated global eigenvector matrix in the CB method

$$\overline{\mathbf{\Phi}}_g = \overline{\mathbf{T}}_0 \overline{\mathbf{\Phi}}_p, \quad \overline{\mathbf{T}}_0 = \begin{bmatrix} \mathbf{\Phi}_d & -\mathbf{K}_s^{-1}\mathbf{K}_c \\ \mathbf{0} & \mathbf{I}_b \end{bmatrix}, \quad \overline{\mathbf{\Phi}}_p = \begin{bmatrix} \overline{\mathbf{\Phi}}_{q_d} \\ \overline{\mathbf{\Phi}}_{u_b} \end{bmatrix}$$

Substructural eigenvector matrix (dominant mode only)

Interface constraint mode

$$\Rightarrow \quad \mathbf{\Phi}_g \approx \underline{\bar{\mathbf{\Phi}}_g} = \begin{bmatrix} \underline{\mathbf{\Phi}_d \bar{\mathbf{\Phi}}_{q_d}} - \mathbf{K}_s^{-1}\mathbf{K}_c \bar{\mathbf{\Phi}}_{u_b} \\ \bar{\mathbf{\Phi}}_{u_b} \end{bmatrix}$$

$$\boxed{\bar{\mathbf{\Phi}}_g \xleftrightarrow{\bar{\mathbf{\Phi}}_{q_d}} \mathbf{\Phi}_d}$$

: $\overline{\mathbf{\Phi}}_{q_d}$ shows the relation between the global and substructural modes.

❖ Calculating modal contributions

$$\bar{\boldsymbol{\Phi}}_{qd} = \begin{bmatrix} \bar{\boldsymbol{\Phi}}_{qd}^{(1)} \\ \bar{\boldsymbol{\Phi}}_{qd}^{(2)} \\ \vdots \\ \bar{\boldsymbol{\Phi}}_{qd}^{(k)} \\ \vdots \\ \bar{\boldsymbol{\Phi}}_{qd}^{(N_s)} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} (\bar{\phi}_{qd}^{(1)})_{11} & (\bar{\phi}_{qd}^{(1)})_{12} & \cdots & (\bar{\phi}_{qd}^{(1)})_{1\bar{N}_p} \\ (\bar{\phi}_{qd}^{(1)})_{21} & (\bar{\phi}_{qd}^{(1)})_{22} & \cdots & (\bar{\phi}_{qd}^{(1)})_{2\bar{N}_p} \\ \vdots & \vdots & \ddots & \vdots \\ (\bar{\phi}_{qd}^{(1)})_{N_d^{(1)}1} & (\bar{\phi}_{qd}^{(1)})_{N_d^{(1)}2} & \cdots & (\bar{\phi}_{qd}^{(1)})_{N_d^{(1)}\bar{N}_p} \end{bmatrix} \\ (\bar{\phi}_{qd}^{(2)})_{ij} \\ \vdots \\ \boxed{(\bar{\phi}_{qd}^{(k)})_{ij}} \\ \vdots \\ (\bar{\phi}_{qd}^{(N_s)})_{ij} \end{bmatrix}$$

$$i = 1, 2, ..., N_d^{(k)},\ j = 1, 2, ..., \bar{N}_p \ \text{and} \ k = 1, 2, ..., N_s,$$

Contribution of the i^th substructural mode in the k^th substructure to the j^th global mode

$$C_i^{(k)} = \sqrt{\sum_{j=1}^{\bar{N}_p} \left[(\bar{\phi}_{qd}^{(k)})_{ij} \right]^2},$$

$$i = 1, 2, ..., N_d^{(k)},\ j = 1, 2, ..., \bar{N}_p \ \text{and} \ k = 1, 2, ..., N_s$$

It is a solution of the reduced eigenvalue problem using CMS. Therefore, the proposed error estimation method requires the intermediate model.

❖ New mode selection method

➢ Two-step mode selection procedure

$$\boxed{\bar{N}_p < \tilde{N}_p \ll N_g}$$

$$\text{(a)}\ \boxed{\mathbf{K}_g\mathbf{\Phi}_g = \lambda\,\mathbf{M}_g\mathbf{\Phi}_g} \xrightarrow[\tilde{N}_d]{} \text{(b)}\ \boxed{\tilde{\mathbf{K}}_p\tilde{\mathbf{\Phi}}_p = \tilde{\lambda}\,\tilde{\mathbf{M}}_p\tilde{\mathbf{\Phi}}_p} \xrightarrow[\bar{N}_d]{} \text{(c)}\ \boxed{\bar{\mathbf{K}}_p\bar{\mathbf{\Phi}}_p = \bar{\lambda}\,\bar{\mathbf{M}}_g\bar{\mathbf{\Phi}}_p}$$

Global EVP | Intermediate EVP | Reduced EVP

➢ The proposed mode selection method

$$C_i^{(k)} = \sqrt{\sum_{j=1}^{\tilde{N}_p}\left[\kappa_j(\tilde{\phi}_{qd}^{(k)})_{ij}\right]^2},$$

$$i = 1, 2, ..., \tilde{N}_d^{(k)},\ j = 1, 2, ..., \tilde{N}_p \text{ and } k = 1, 2, ..., N_s.$$

$$\boxed{\kappa_j = 1 \text{ for } N_t^{\mathrm{L}} \le j \le N_t^{\mathrm{U}},\ \text{otherwise } \kappa_j = 0}$$

κ_j : weighting factor, N_t^L : lower limit of number of the target global mode,

N_t^U : Upper limit of number of the target global mode

❖ Mode selection procedure

Step 1. Initial preparation

(a) $\mathbf{M}_g$ and $\mathbf{K}_g$ are given.

(b) The range of the target global modes is determined by $\omega_t^{\mathrm{L}} \sim \omega_t^{\mathrm{U}}$: $N_t^{\mathrm{L}} \sim N_t^{\mathrm{U}}$.

(c) $\tilde{N}_d$ and $\bar{N}_d$ are determined by $\tilde{\omega}_d$ and $\bar{\omega}_d$, respectively:

$\tilde{\omega}_d = \gamma\beta\omega_t^{\mathrm{U}}, \bar{\omega}_d = \beta\omega_t^{\mathrm{U}}$,

Step 2. Construction of the intermediate reduced model

(a) The substructural eigenvalue problems are solved

$[\mathbf{K}_s^{(k)} - \lambda_j^{(k)}\mathbf{M}_s^{(k)}](\boldsymbol{\phi}^{(k)})_j = \mathbf{0}, \quad$ for $k = 1, 2, \ldots, N_s$.

(b) The dominant substructural modes are selected by the frequency cut-off mode selection method.

(c) The intermediate reduced model is constructed with $\tilde{\mathbf{M}}_p$ and $\tilde{\mathbf{K}}_p$.

Step 3. Construction of the final reduced model

(a) The intermediate eigenvalue problem is solved and $\tilde{\mathbf{\Phi}}_{q_d}$ is found:

$\tilde{\mathbf{K}}_p(\tilde{\boldsymbol{\phi}})_j = \tilde{\lambda}_j\tilde{\mathbf{M}}_p(\tilde{\boldsymbol{\phi}})_j, \quad j = 1, 2, \ldots, \tilde{N}_p$.

(b) The contributions of the substructural modes to the target global modes are calculated: $C_j^{(k)}$ in Equation 21.

(c) The dominant substructural modes are selected in order of $C_j^{(k)}$.

(d) The final reduced model is constructed with $\bar{\mathbf{M}}_p$ and $\bar{\mathbf{K}}_p$.

❖ Eigenvector relation in the F-CMS method

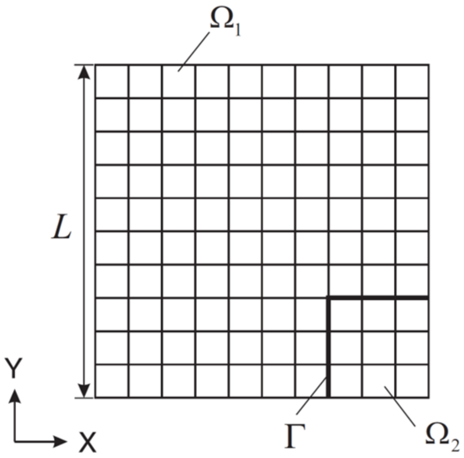
Note that, when we define the eigenvector relation between the global and reduced models, the proposed mode selection method can be also employed for other CMS methods.

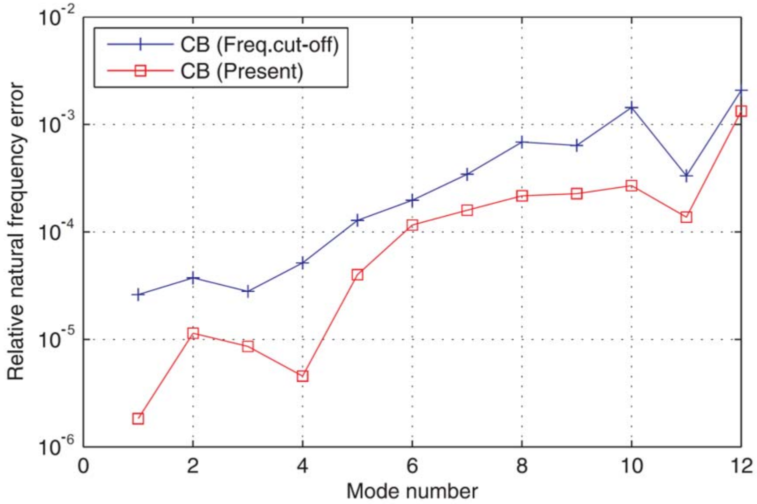
- Approximated global eigenvector matrix in the F-CMS method

$$\mathbf{\Phi}_g \approx \underline{\bar{\mathbf{\Phi}}_g} = \mathbf{L}\bar{\mathbf{\Phi}}_{u_b} = \underline{\mathbf{\Phi}_d \bar{\mathbf{\Phi}}_{q_d}} + \mathbf{R}_s \bar{\mathbf{\Phi}}_{\alpha_s} - \hat{\mathbf{F}}_r \mathbf{B} \bar{\mathbf{\Phi}}_{\lambda_\ell}$$

$$\boxed{\bar{\mathbf{\Phi}}_g \xleftrightarrow{\bar{\mathbf{\Phi}}_{q_d}} \mathbf{\Phi}_d}$$

❖ Numerical examples

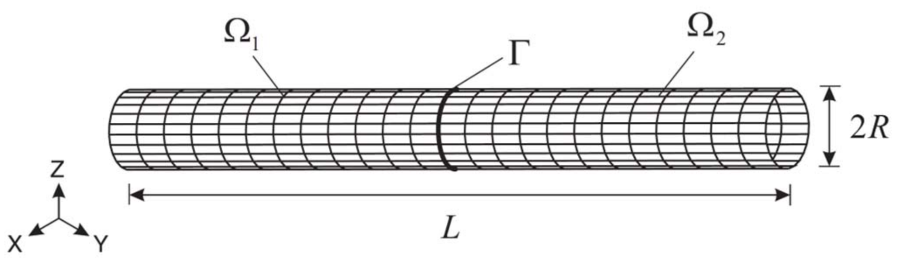




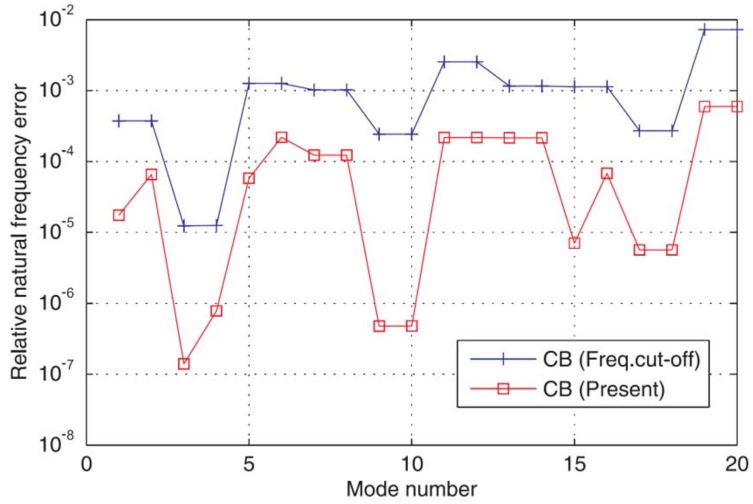
Substructural mode number	Freq.cut-off (rad/sec) Ω_1	Freq.cut-off (rad/sec) Ω_2	Present, $C_j^{(k)}$ Ω_1	Present, $C_j^{(k)}$ Ω_2
1	2.268E+00 (○)	3.860E+01 (○)	8.589E+00 (○)	5.871E-01 (○)
2	3.583E+00 (○)	1.598E+02 (○)	2.988E+00 (○)	2.780E-02 (○)
3	6.685E+00 (○)	1.899E+02 (×)	1.746E+00 (○)	1.189E-02 (○)
4	1.358E+01 (○)	3.386E+02 (×)	1.173E+00 (○)	0.000E+00 (×)
5	1.570E+01 (○)	8.477E+02 (×)	1.307E+00 (○)	0.000E+00 (×)
⋮	⋮	⋮	⋮	⋮
⋮	(○)	(×)	(○)	(×)
⋮	⋮	⋮	⋮	⋮
18	9.199E+01 (○)	7.347E+05 (×)	7.222E-02 (○)	0.000E+00 (×)
19	1.060E+02 (○)	7.348E+05 (×)	5.794E-02 (○)	0.000E+00 (×)
20	1.190E+02 (○)	7.348E+05 (×)	4.072E-03 (×)	0.000E+00 (×)
21	1.256E+02 (○)	7.350E+05 (×)	6.004E-03 (×)	0.000E+00 (×)
22	1.261E+02 (○)	8.382E+05 (×)	4.851E-03 (×)	0.000E+00 (×)
23	1.456E+02 (○)	8.382E+05 (×)	4.487E-03 (×)	0.000E+00 (×)
24	1.475E+02 (○)	8.382E+05 (×)	7.815E-03 (○)	0.000E+00 (×)
25	1.531E+02 (○)	8.382E+05 (×)	1.520E-02 (○)	0.000E+00 (×)
26	1.657E+02 (×)	8.383E+05 (×)	1.363E-02 (○)	0.000E+00 (×)
27	1.706E+02 (×)	8.384E+05 (×)	1.070E-02 (○)	0.000E+00 (×)
28	1.708E+02 (×)	8.384E+05 (×)	1.130E-02 (○)	0.000E+00 (×)

○ : selected, × : unselected

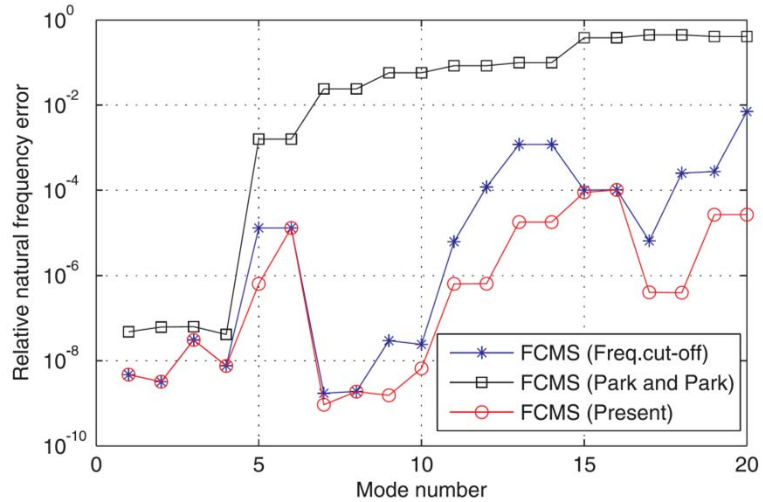
❖ Numerical examples



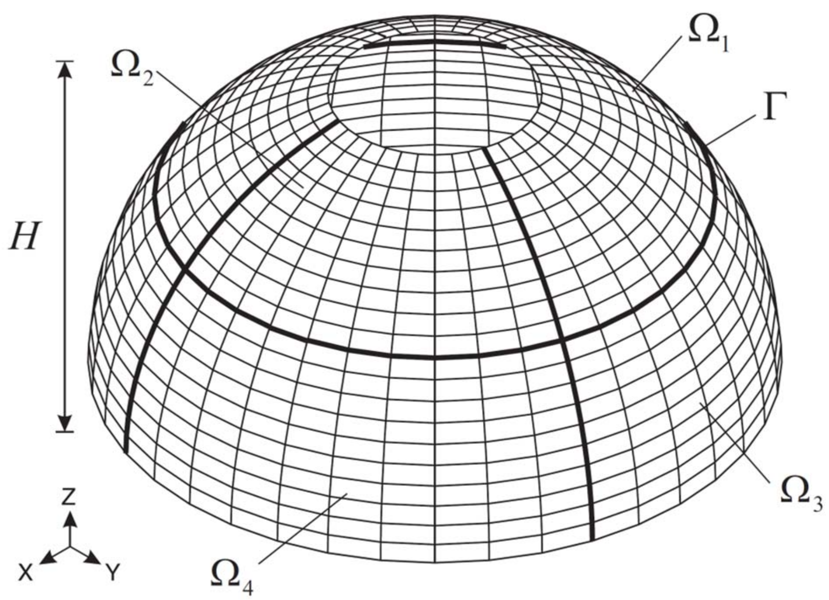
(a)



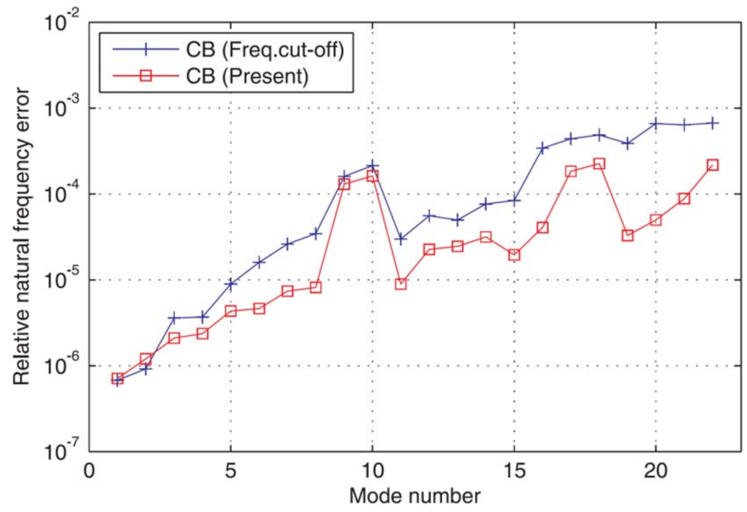
(b)



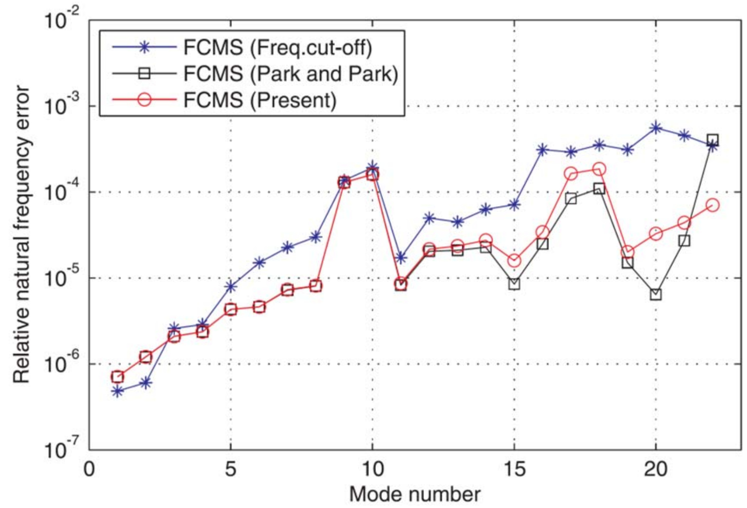
❖ Numerical examples



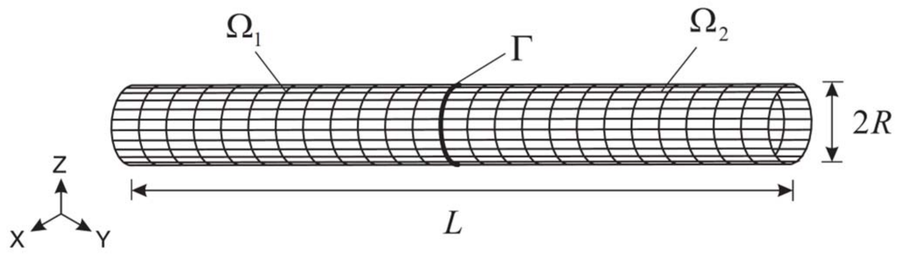
(a)

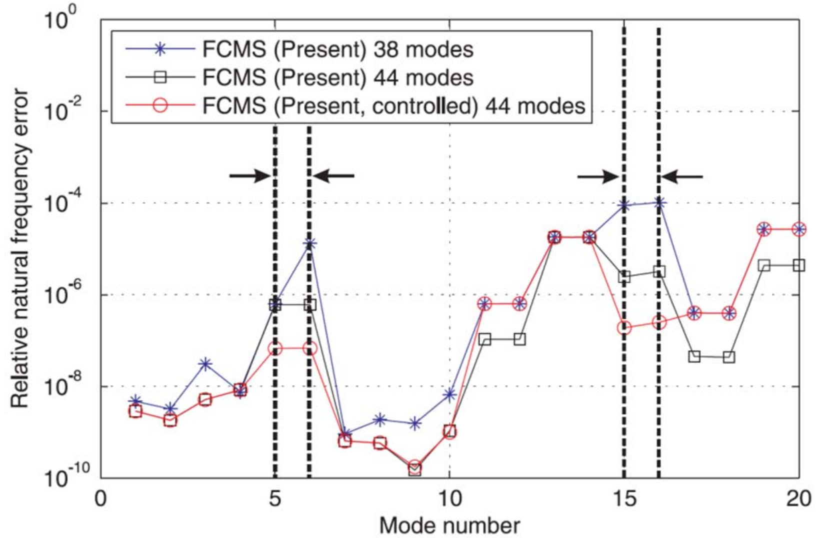


(b)



❖ Accuracy control strategy





$$C_i^{(k)} = \sqrt{\sum_{j=1}^{\tilde{N}_p} \left[\kappa_j (\tilde{\phi}_{qd}^{(k)})_{ij} \right]^2},$$

$$i = 1, 2, ..., \tilde{N}_d^{(k)},\ j = 1, 2, ..., \tilde{N}_p \text{ and } k = 1, 2, ..., N_s.$$

$$\kappa_j = 1 \text{ for } N_t^{\mathrm{L}} \le j \le N_t^{\mathrm{U}}, \text{ otherwise } \kappa_j = 0$$

❖ Closure

1. **A general mode selection method for CMS methods was proposed using eigenvector relation between global and substructural modes.**

2. **Error control strategy for locally fluctuated eigenvalue error was also proposed.**

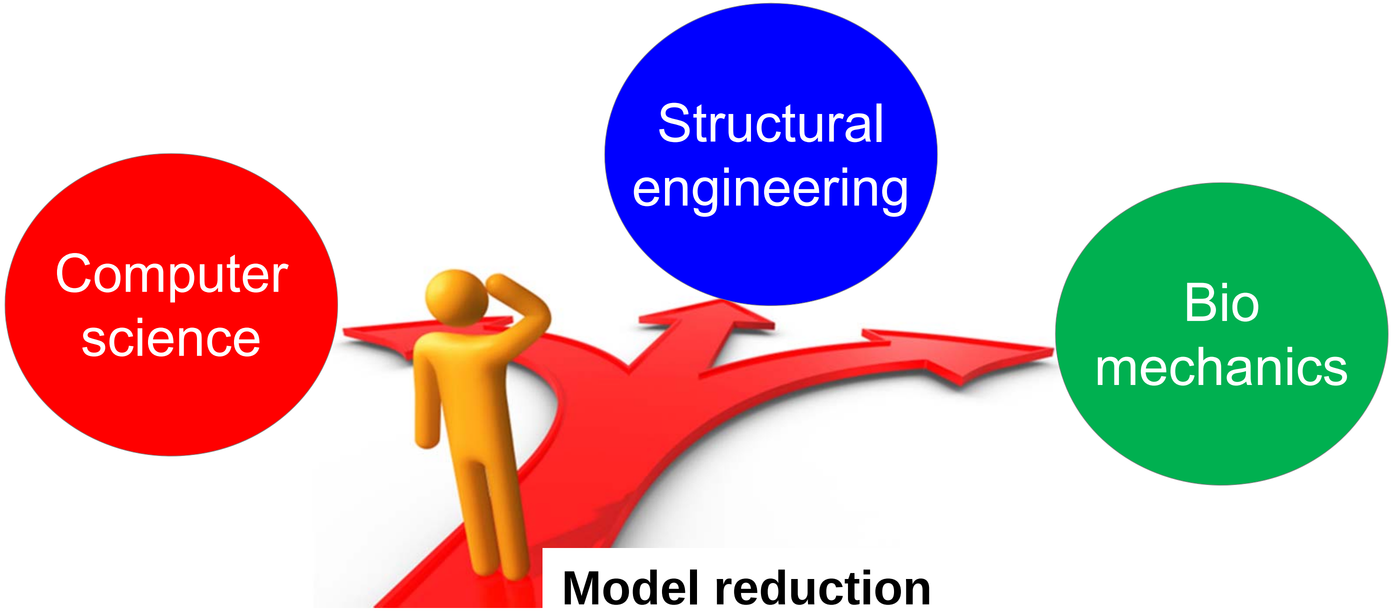
3. **Efficient model reduction algorithm could be developed by combinations of the proposed error estimator and the existing mode/DOFs selection methods.**

KC Park, JG Kim, PS Lee. A mode selection criterion based on flexibility approach in component mode synthesis, Proceedings of 53rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Material Conference, 2012.

JG Kim, PS Lee, KC Park. A mode selection method for structural component mode synthesis, International Journal for Numerical Methods in Engineering, submitted.

Conclusions

1. We proposed two enhanced CMS method: the enhanced CB and AMLS method.
2. We developed the error estimator, and it employed in CB, AMLS, F-CMS methods and Guyan reduction.
3. A mode selection method for CMS methods was proposed using eigenvector relation between global and substructural modes.
4. Interface reduction method for the F-CMS method was also developed (not presented here).



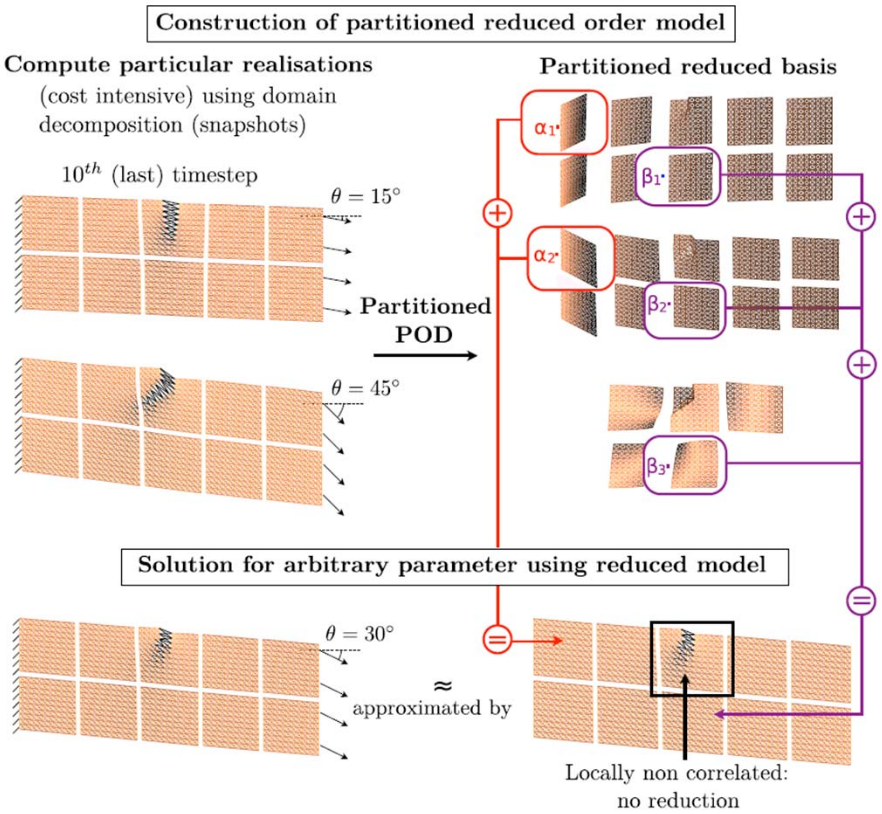
Thank you

Extension of model reduction

❖ 1. Reduced order modeling

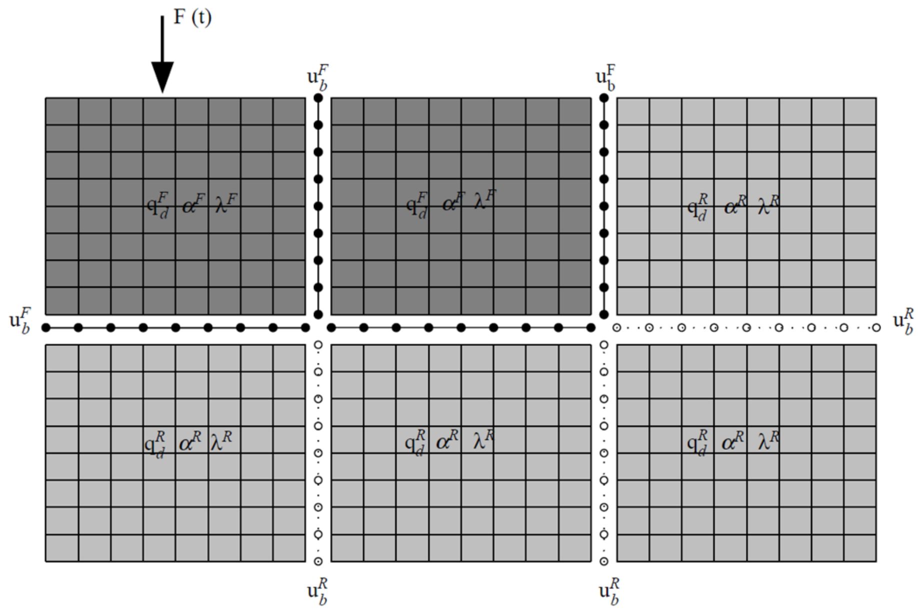
➢ Research trend

- Crack, multi-scale, multi-physics (FSI), transient analysis….etc.



< Nonlinear crack problem >

Kerfrien et al. (2012)



< Transient analysis>

Markovic et al. (2009)

Interface handling is also important topic!

❖ 2. Eigenvalue problem solver

➢ Representative solution method of the eigenvalue problem

- QR algorithm
- Householder transformation
- Subspace iteration
- Lanczos algorithm

➢ Recently research

- CMS + subspace iteration : Yin, Voss and Chen (2013), ADINA letter (2013)

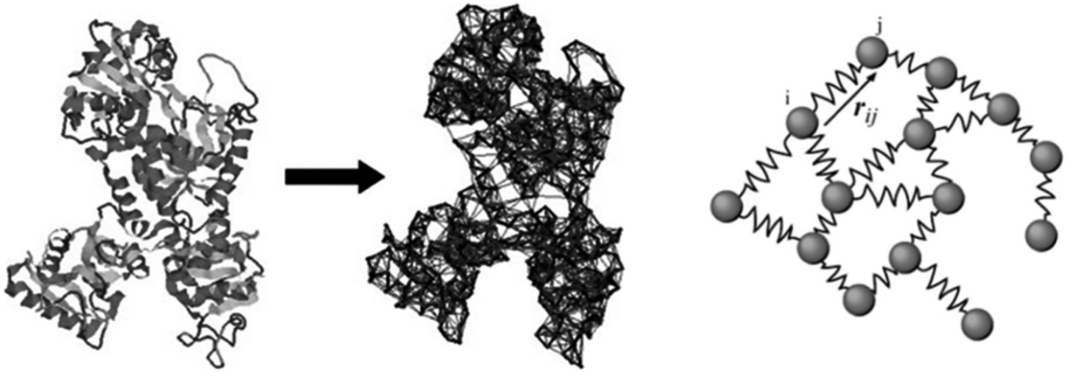
➢ Idea

- CMS + error estimation method = Iterative CMS method

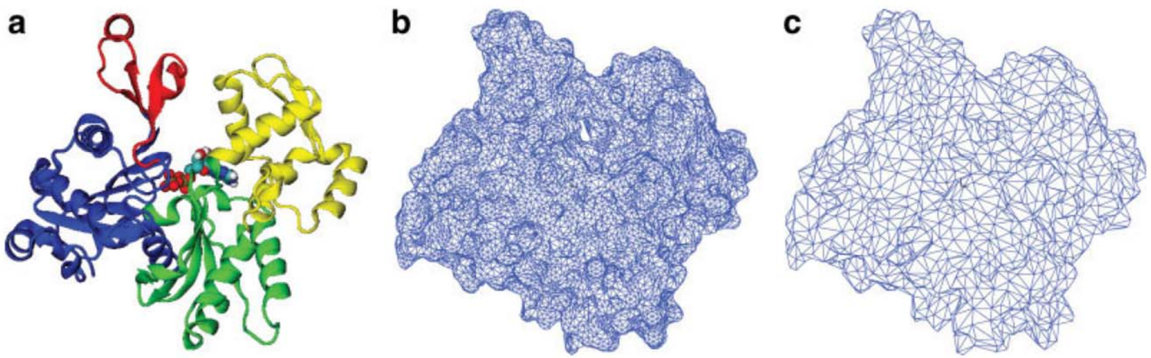
Let us develop more efficient method of eigenvalue problem than previous methods.

❖ 3. Protein dynamics

1. Elastic network model (*Jeong et al. 2006*)



2. Finite element model (*M. Bathe 2007*)



✓ **Objective: reducing the computational cost**

Molecular dynamics

Coarse grained modeling

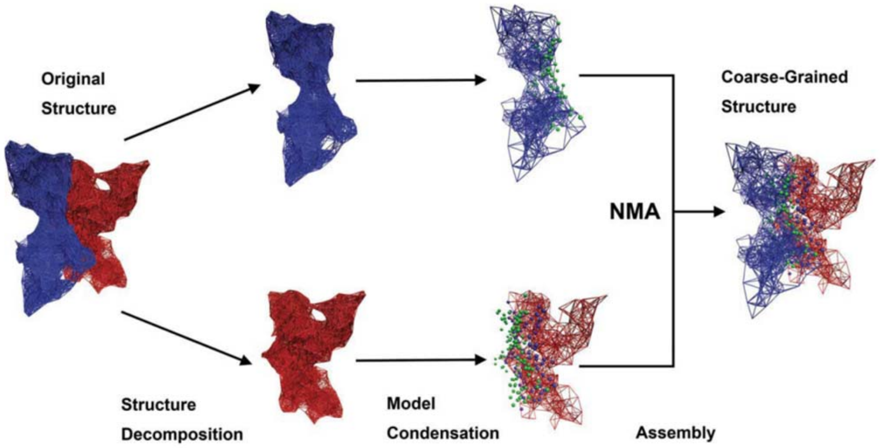
Normal mode analysis
(= Eigenvalue problem)

Next step?

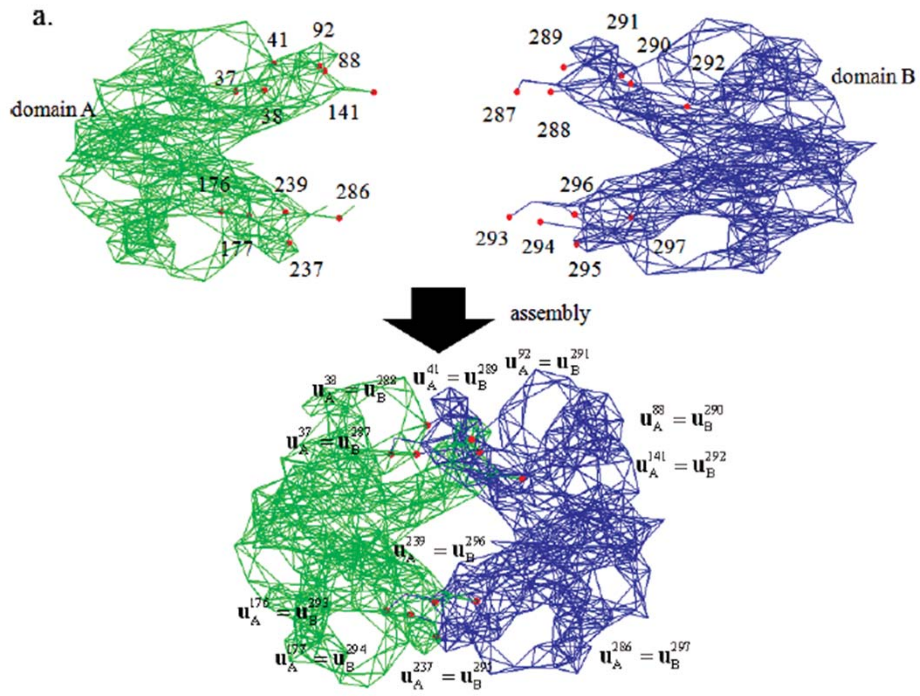
❖ 3. Protein dynamics: related researches

나성수 교수, 고려대학교 기계공학과

엄길호 교수, 성균관대학교 체육학과 (전 **KIST, Microsystem Research Center**)



Kim et al. 2011



Kim et al. 2009

❖ 3. Protein dynamics: error estimation

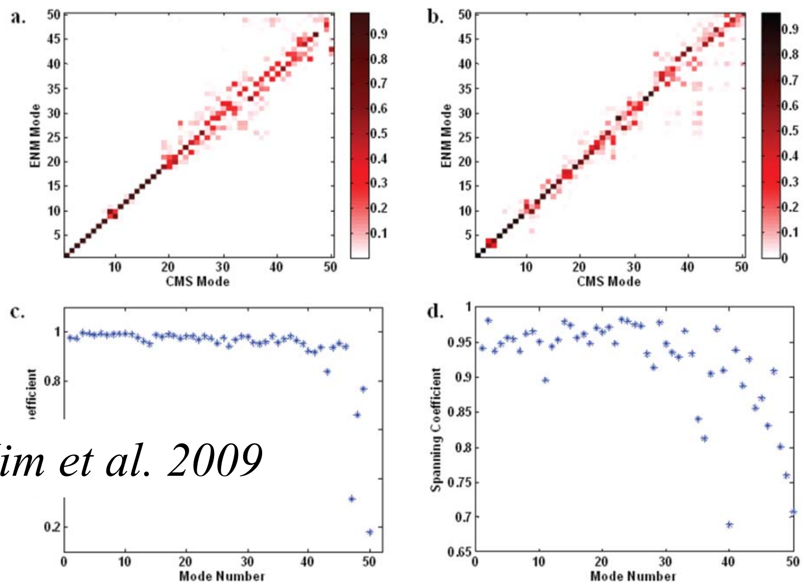


Figure 4. Overlap between normal modes obtained from ENM and CMS for hemoglobin in (a) close form (pdb: 1a3n) and (b) open form (pdb: 1bbb). Spanning coefficient between normal modes computed from ENM and CMS for hemoglobin in (c) close form and (d) open form. It is shown that low-frequency normal modes estimated from CMS are highly correlated with those from ENM.

$$\mathbf{v}_i^{ENM} \bullet \mathbf{v}_j^{CMS}$$

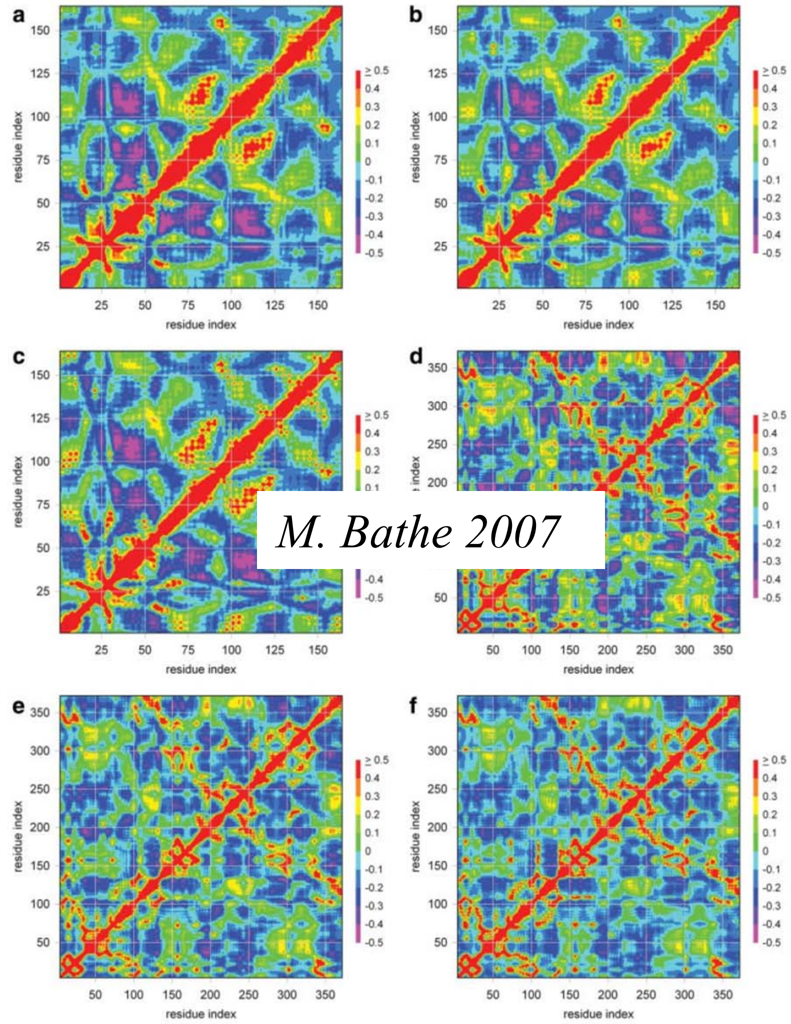


Figure 4

Correlated fluctuations of α-carbon atoms computed for T4 lysozyme using (a) ATM-NMA; (b) the RTB procedure; (c) the FEM; and for G-actin using (d) ATM-NMA, (e) the RTB procedure, and (f) the FEM. Figures are rendered using DPLOT version 2.1.1.9. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]