

Ph. D. Thesis Presentation

Partition of Unity Based Shell Finite Elements

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Ph. D. Thesis Presentation

Committee

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- Prof. YeunWoo Cho (Ocean Systems Engineering, KAIST)
- Prof. Seunghwa Ryu (Mechanical Engineering, KAIST)

Goal

: To develop the efficient and robust shell finite elements based on partition of unity concept.

Limitations :

- Linear and nonlinear static analyses
- Shell elements with the effects shear deformation (Reissner-Mindlin theory)

Contents : Partition of unity based shell finite elements

1. Part I (Introduction to Partition of Unity (PU) Based FEM)

- Brief History of the Finite Element Method
- Application of the Finite Element Method
- Finite Element Analysis Procedures
- Ultimate Goal of the Finite Element Method
- Partition of Unity Based Shell Finite Element

Enriched MITC3

2. Part II (PU Based Shell Element with Improved Membrane Behaviors)

- Introduction to Nonlinear Finite Element Analysis
- Introduction and Scope of Research
- **MITC3+ in the Nonlinear Analysis**
 - ✓ Key Concepts / Nonlinear Formulation
 - ✓ Benchmark Problems
- **The Method with Improved Membrane Behaviors**
 - Comparison with Other Methods
 - Key Concepts / Nonlinear Formulation
 - Benchmark Problems

MITC3+

Enriched MITC3+

3. Future Works

4. Conclusions

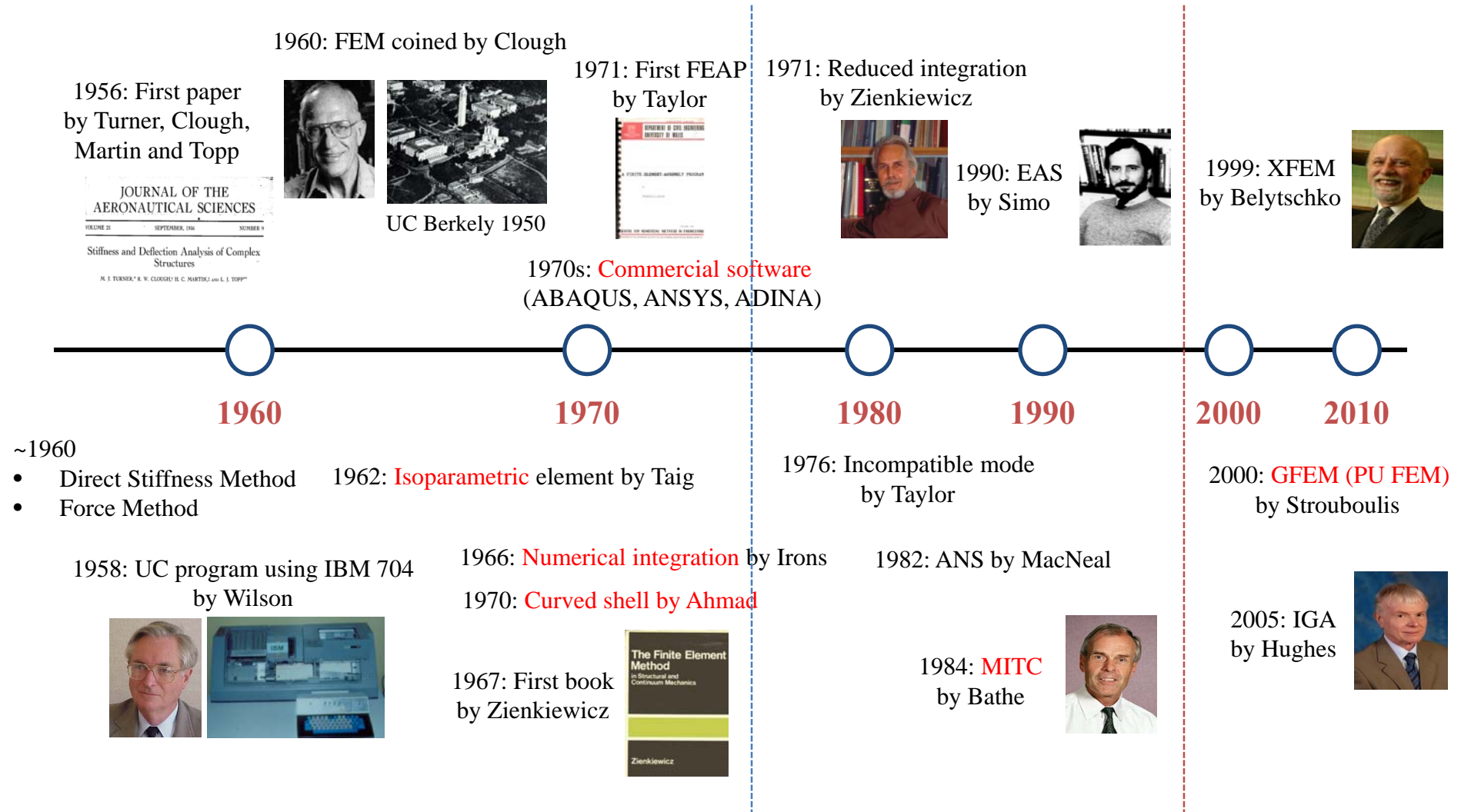
Part I:

Introduction to Partition of Unity (PU) Based FEM

- Brief History of the Finite Element Method
- Application of the Finite Element Method
- Finite Element Analysis Procedures
- Ultimate Goal of the Finite Element Method
- Partition of Unity Based Shell Finite Elements

Brief history – Events in the Finite Element Methods

Events in the Finite Element Methods



List of finite element software packages – Commercial FEA software

FEA software list

Proprietary / Commercial Finite Element Analysis (FEA) Software

Commercial (48)

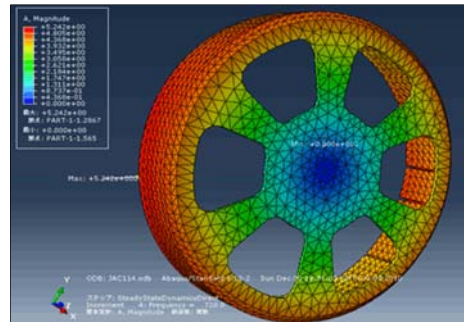
ABAQUS
ANSYS
ADINA
NASTRAN
ALGOR
LS-DYNA
PAM-CRASH
MARC
LUSAS
COMSOL
VISUALFEA
FEMAP

.....

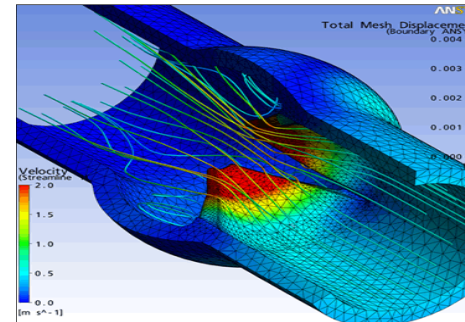
Open source (19)

FreeFem++
OOFEM
jFEM
GetFEM++
Code Aster

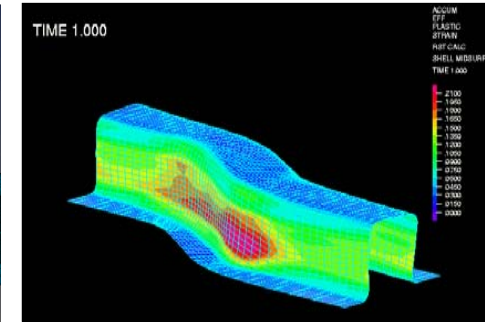
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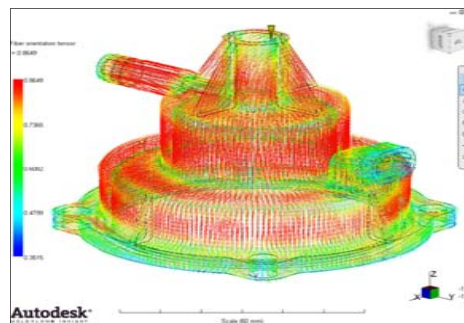
ABAQUS



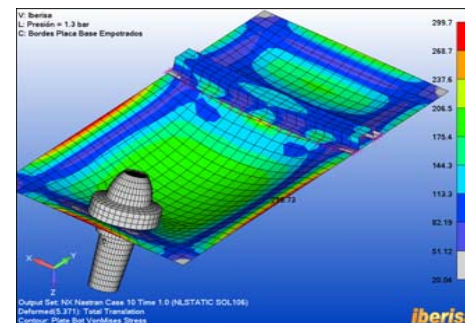
ANSYS



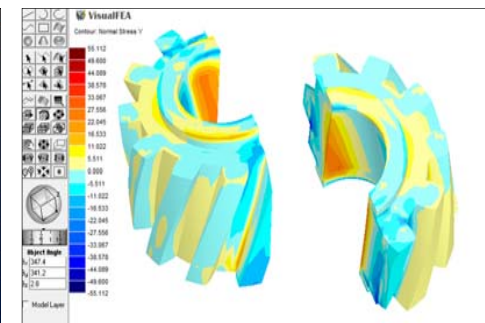
ADINA



ALGOR



NASTRAN

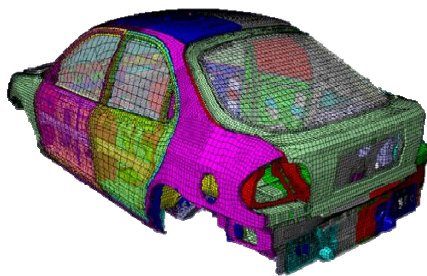


VISUALFEA

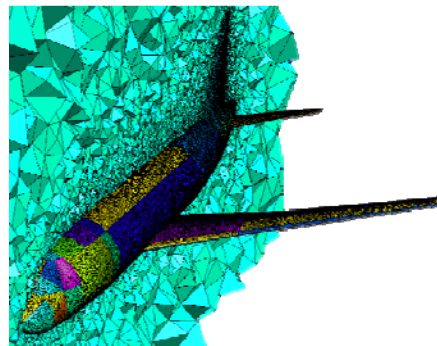
Commercial FEA software prices start from around \$1,500 to \$60,000.

Figure source : google image

Applications of FEM #1 – Industry fields



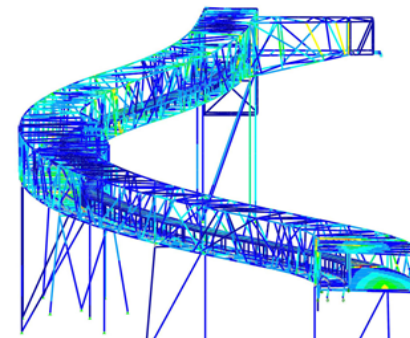
Automobile



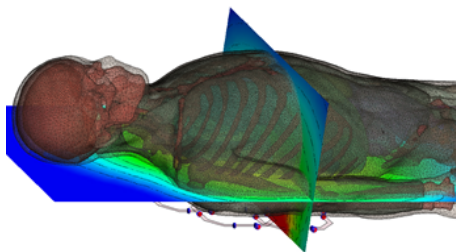
Aerospace



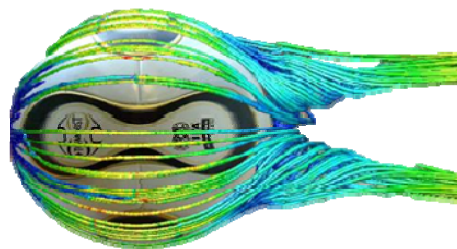
Electronics



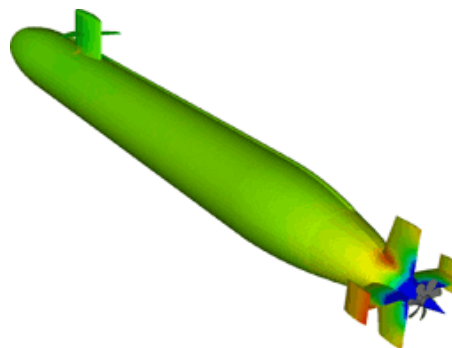
Civil engineering



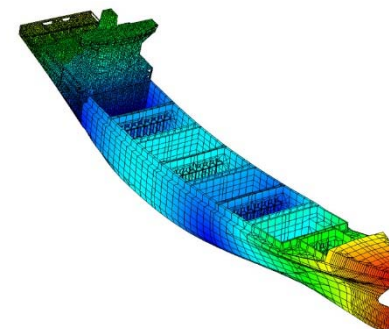
Healthcare



Sports



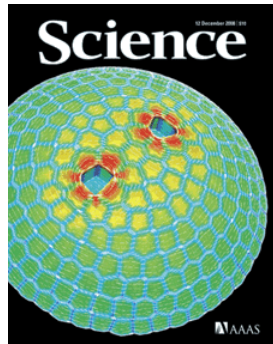
Defense



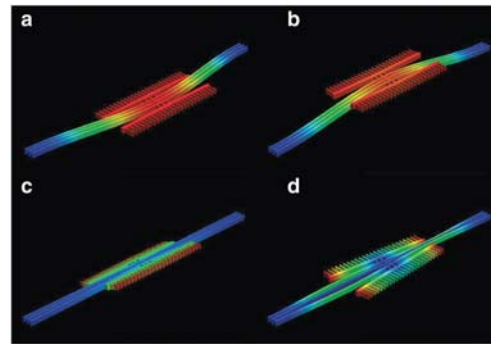
Ship building

Figure source : google image

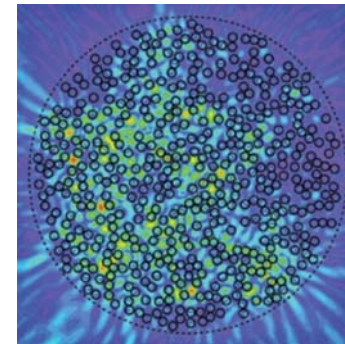
Applications of FEM #2 – Research fields (High profile journal)



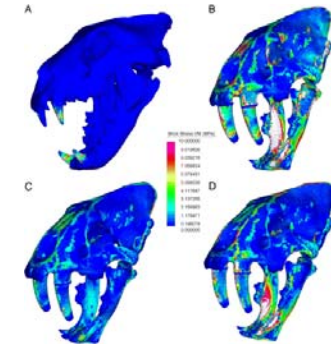
Olivier et al., Science 2008
(Developmental patterning)



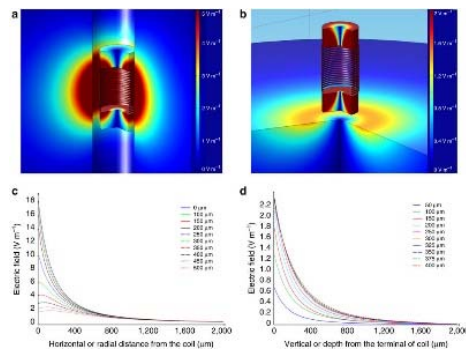
Dario et al., Nature Communications 2011
(Frequency stabilization)



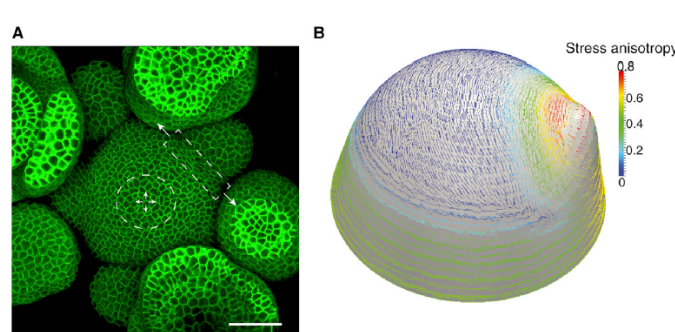
Diederik et al. Nature Photonics 2013
(Disordered photonics)



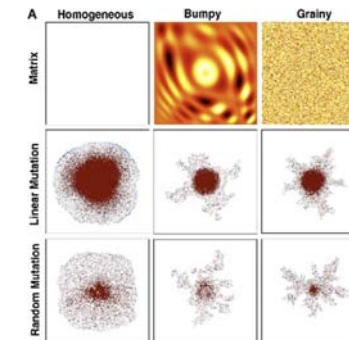
McHenry et al. PNAS 2007
(Predatory behavior in *Smilodon*)



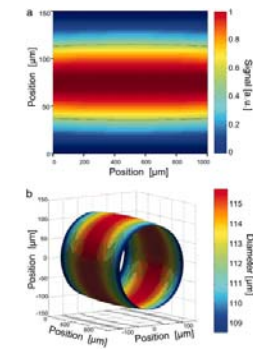
Giorgio et al., Nature Communications 2012
(Microscopic magnetic simulation)



Sampathkumar et al. Cell Current Biology 2014
(Plant development)



Anderson et al. Cell Theory 2006
(Tumor morphology)



Masson et al. PNAS 2006
(Ionic contrast terahertz)

Figure source : journals

Is there nothing left to improve FEMs?

- CLUE**
- Many researchers and their contributions over 50 years
 - More than 10,000,000 papers to improve performance of the FEMs
 - Most widely used numerical methods in engineering
 - Stabilized commercial software products (e.g. ABQUS, ANSYS, ADINA,...)

ANSWER : Of course, more research is needed despite the remarkable developments and improvements.

Step 1

: Observation of the system

Column under an eccentric compressive load



This eccentric load is perpendicular to the column, but it doesn't pass through the column's centroid.

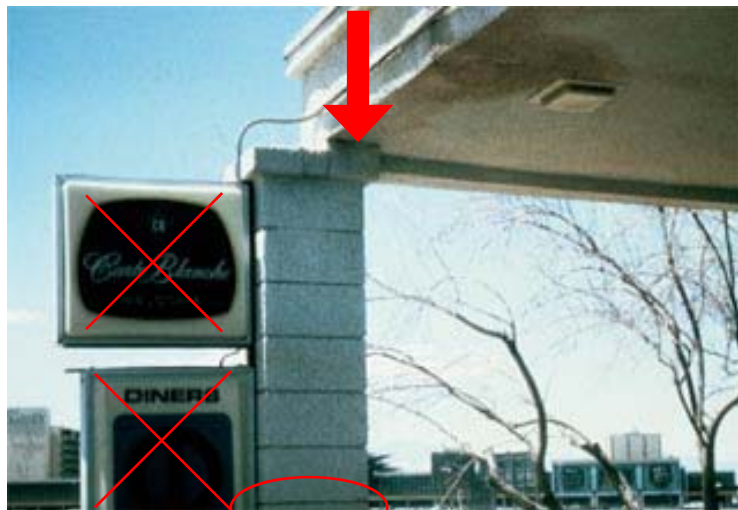
Figure source : google image

Step 2

: Idealization of the system

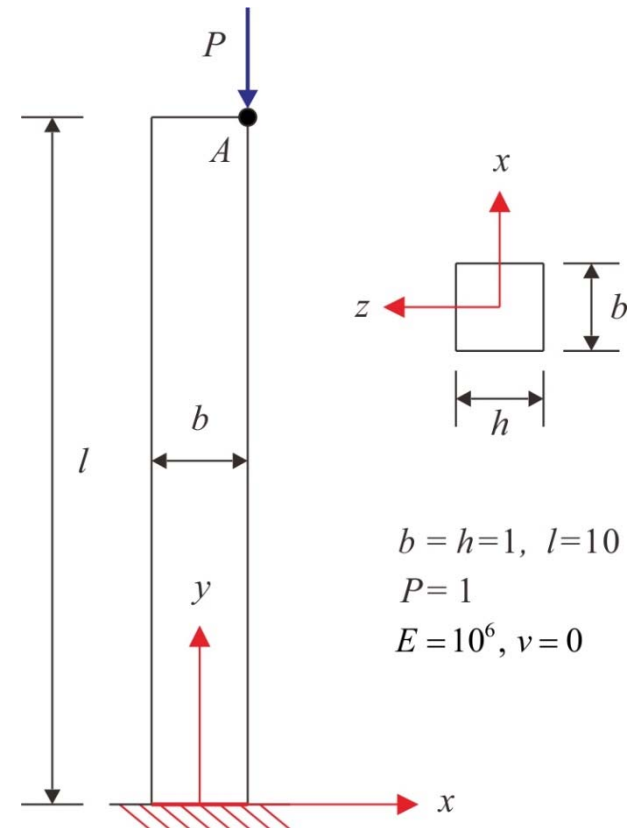
Major assumption : Linear elastic material

Applying an eccentric load



Imposing the boundary condition

Idealization



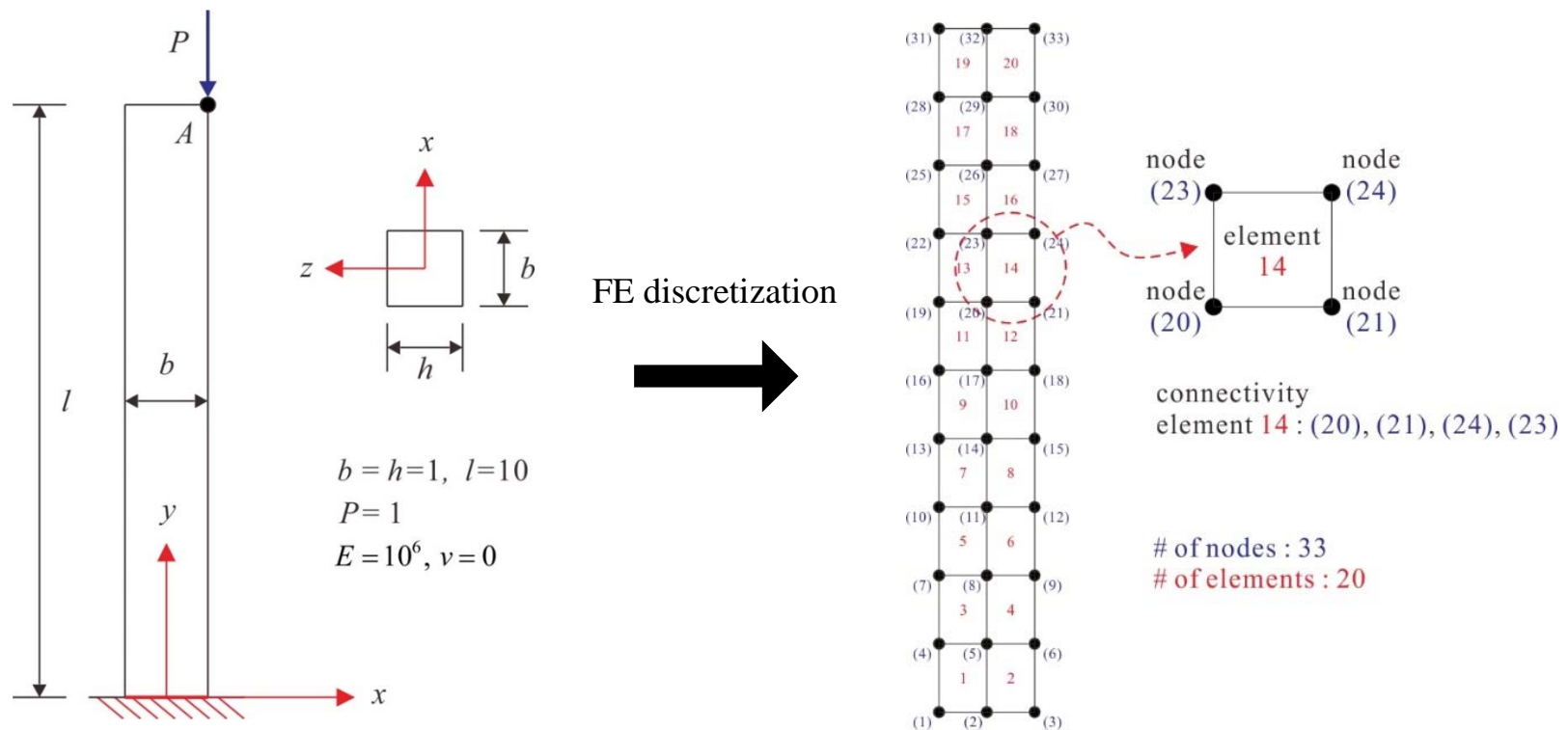
FE analysis procedures – Step 3: Discretization of the domain

Step 3

: Discretization of the domain

(using ADINA founded in 1974, ADINA 9.0.5 latest released version (2014.10))

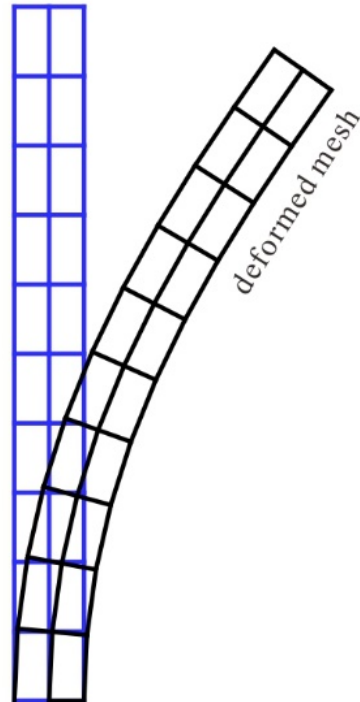
(setting the material properties, imposing the boundary condition, and applying loads)



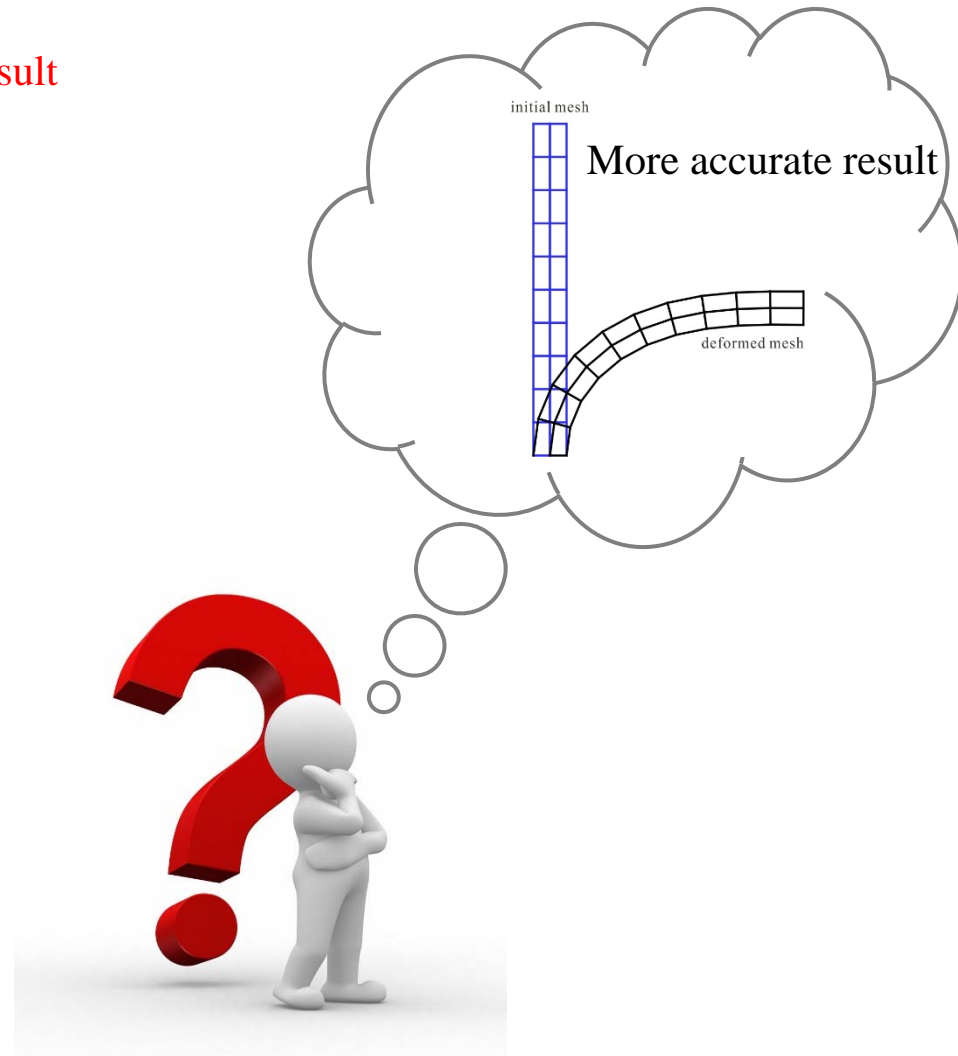
Step 4

: Solving the problem and interpreting the result

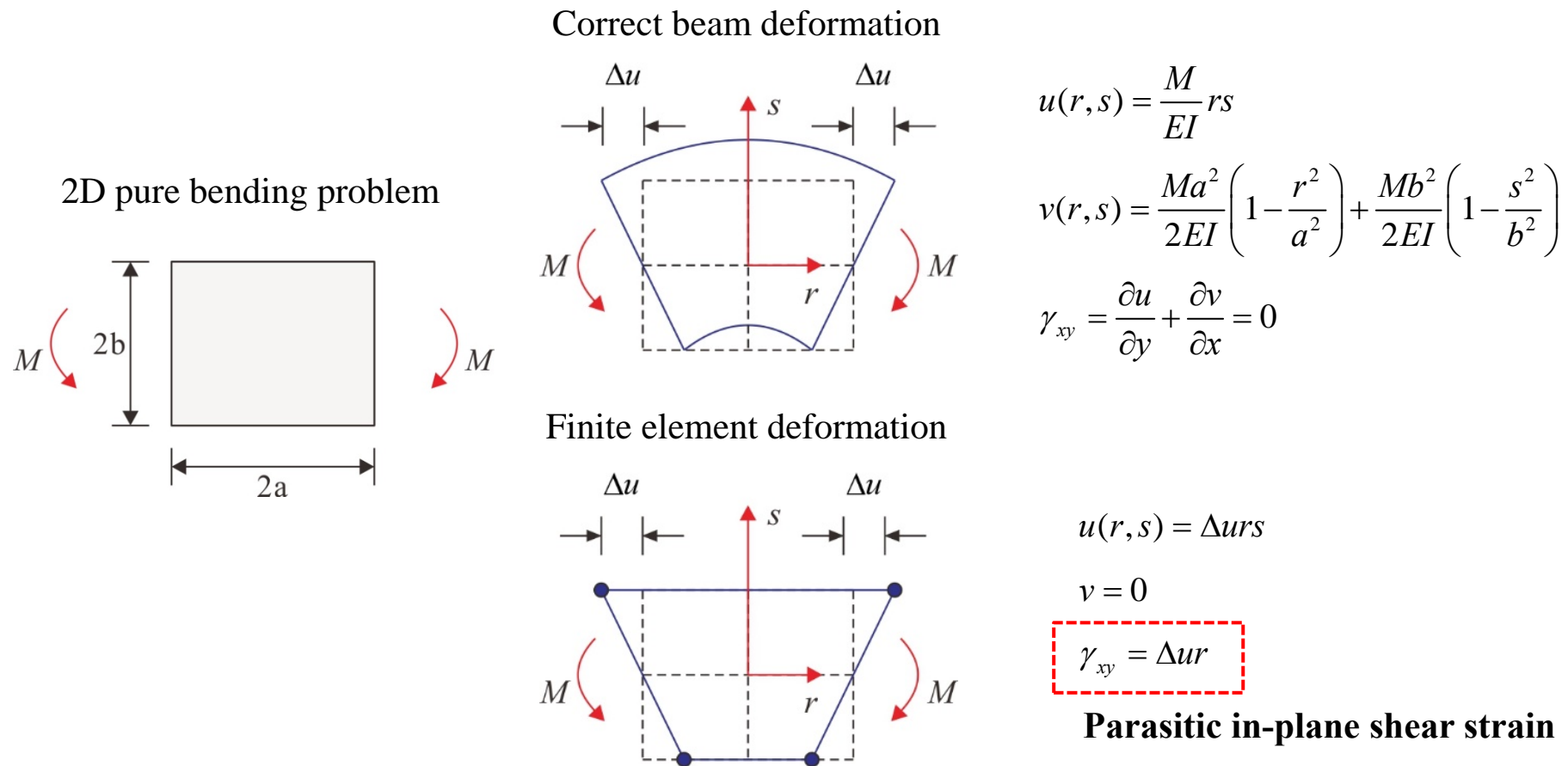
initial mesh



Result : deformed shape of the column



The cause of this problem – In-plane shear locking



In-plane shear locking : The element has an excess of shear strain which contributes to the poor ability of the element to reproduce bending modes.

Remedies – Books and manual books

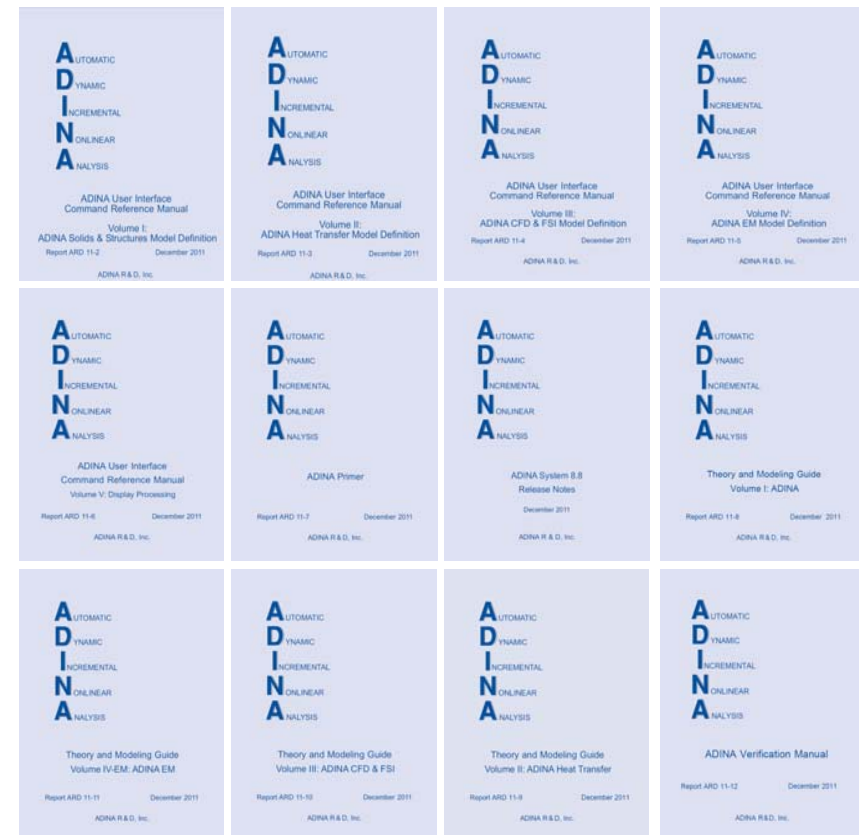
Amazon search

450 results in science & math, keyword : “finite element method”



Commercial software : ADINA

Manual book : 12 books



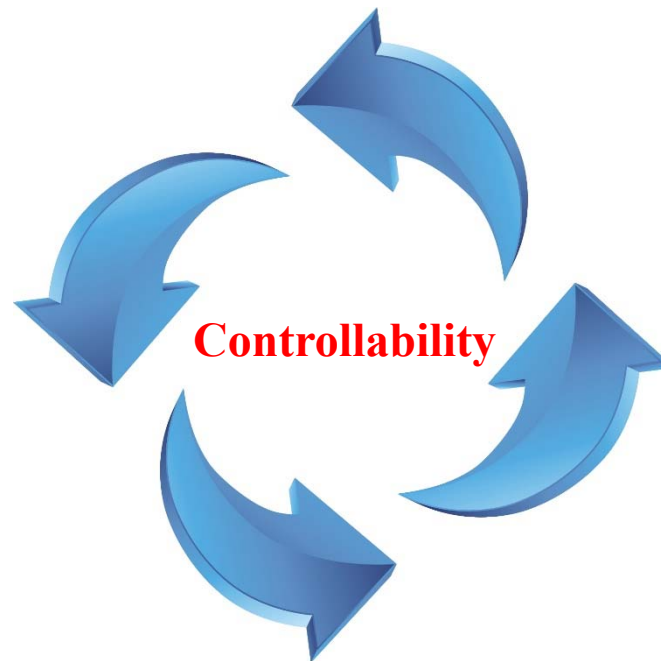
The ultimate goal of the finite element method #1 – User conveniences & Accuracy

Accurate solution & **User conveniences**

What does “user conveniences” mean?

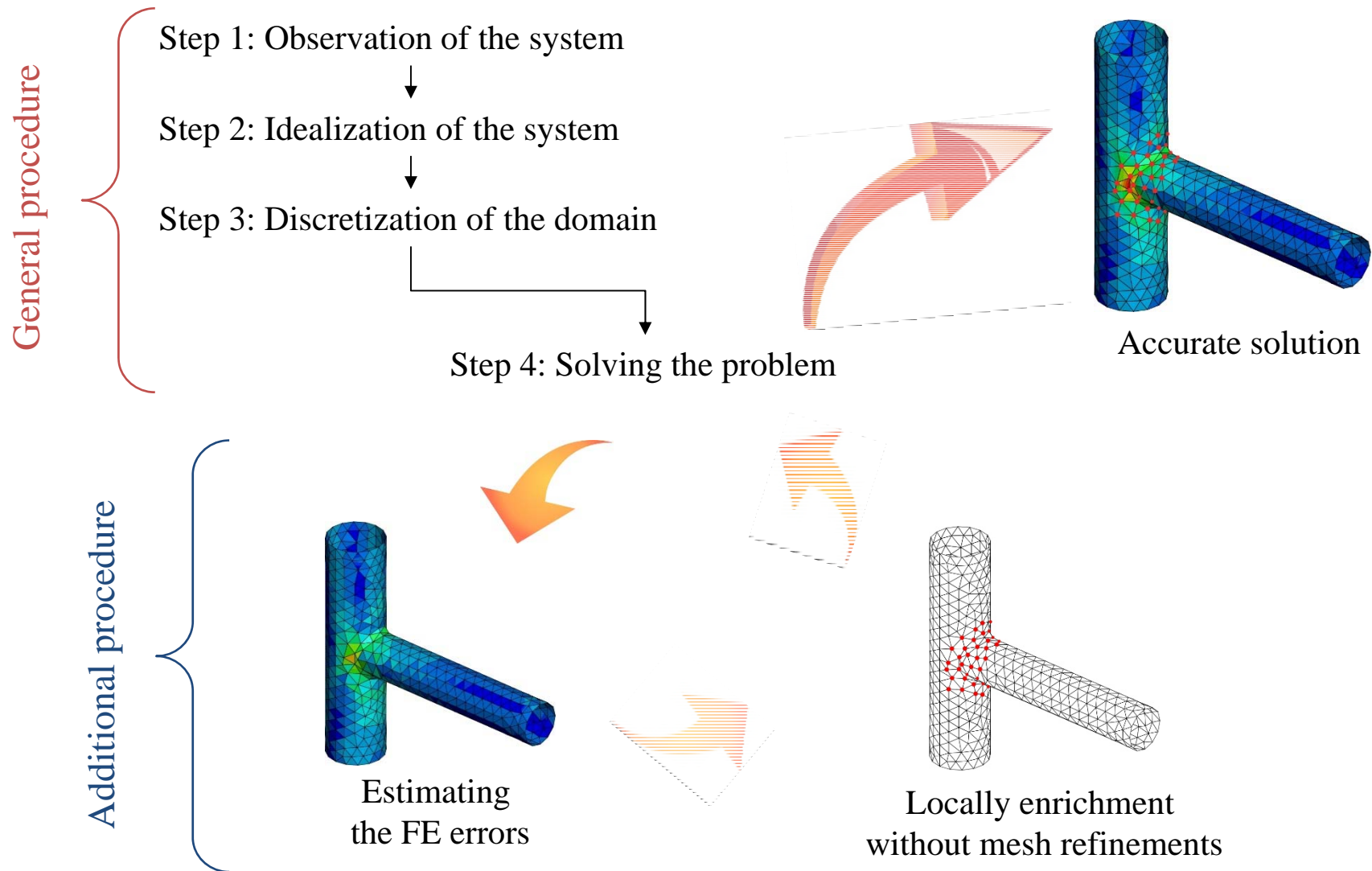


One click



Reliable results

The ultimate goal of the finite element method #2 – User conveniences & Accuracy



The ultimate goal of the finite element method #3 – Error estimates

“A Finite element solution contains enough information to estimate its own error”

Cook RD et al., Concepts and applications of finite element analysis, 4th ed., Wiley.

“Nowadays, a booming activity in error estimation is fostered by better understanding of mathematical foundations”

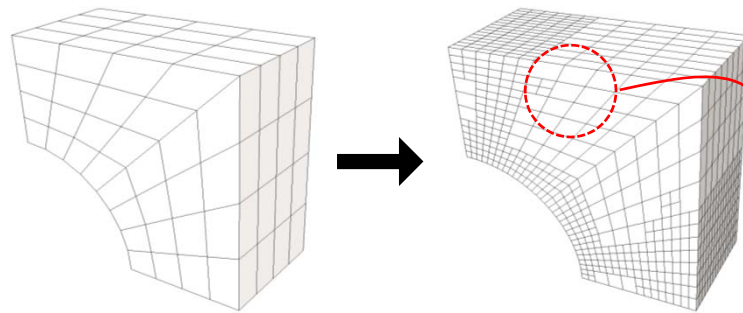
Aninsworth M, Oden JT, A posteriori error estimation in finite element analysis, Wiley

Two types of error estimates serve very different purpose

- A priori error estimates
 - To check the order of convergence of a given FE method
 - **A posteriori error estimates**
 - To indicate where the error is particularly high
- I. Residuals – based method
Babuska I and Rheinboldt WC, Int. J. Numer. Meth. Eng.;12:1597-615, 1978 (1371 cited)
 - II. Recovery – based method
Zienkiewicz OC and Zhu JZ, Int. J. Numer. Meth. Eng.;24:337-57,1987 – ZZ method (2147 cited)

The ultimate goal of the finite element method #4 – Error estimates

Error estimate / Adaptive mesh refinement

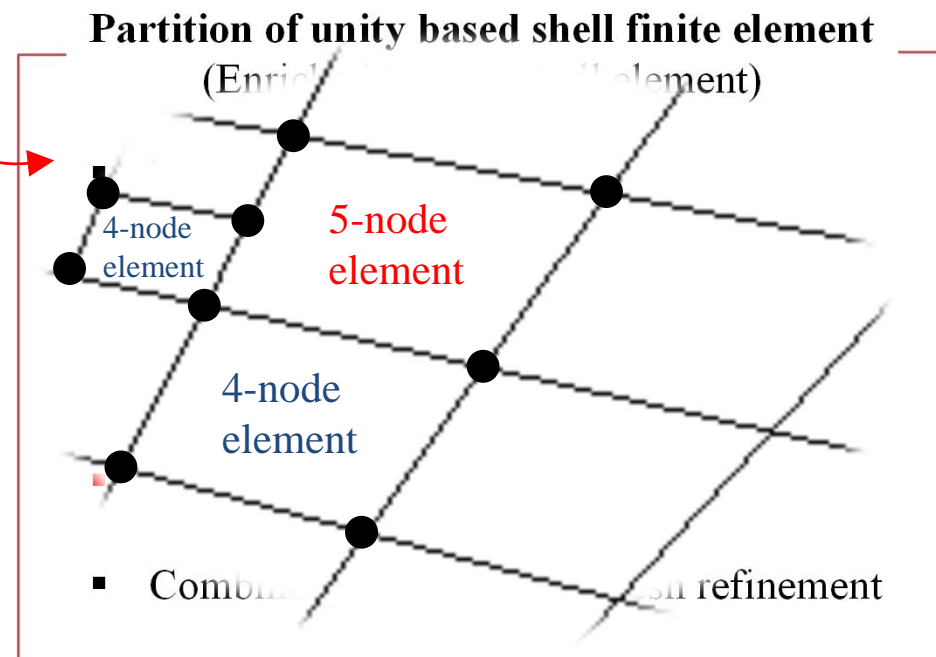


Hyang et al., DDMSE;78:53-74,2011

Disadvantages

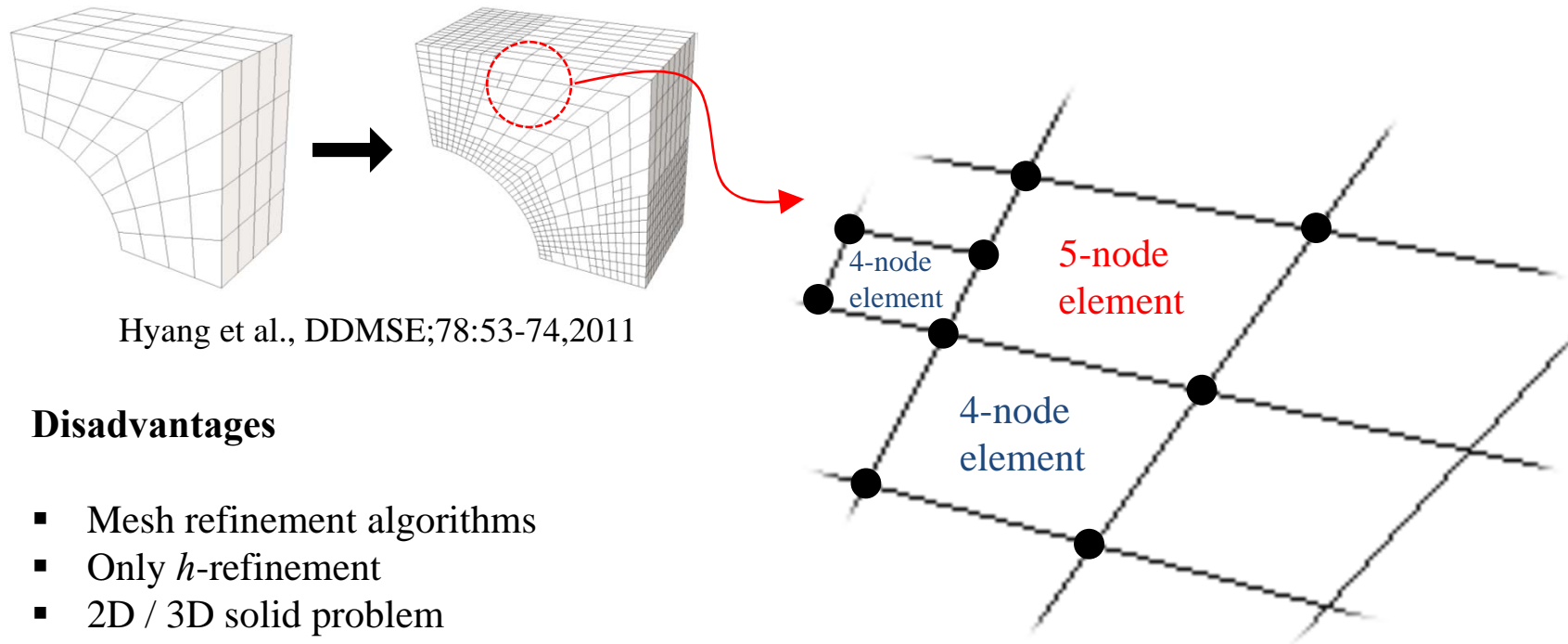
- Mesh refinement algorithms
- Only h -refinement
- 2D / 3D solid problem
- Transition elements

If shell problem,
how can treat the locking phenomenon?



The ultimate goal of the finite element method #4 – Error estimates

Error estimate / Adaptive mesh refinement



Disadvantages

- Mesh refinement algorithms
- Only h -refinement
- 2D / 3D solid problem
- Transition elements

If shell problem,
how can treat the locking phenomenon?

FEM

$$u(\mathbf{x}) = \sum_{i=1}^3 h_i u_i$$

h_i : shape function
 u_i : nodal displacement **variable**
 $u_i(\mathbf{x})$: local approximation **function**

PU based FEM

$$\bar{u}(\mathbf{x}) = \sum_{i=1}^3 h_i u_i(\mathbf{x})$$

Local approximation , $u_i(\mathbf{x})$

$$u_i(\mathbf{x}) = \mathbf{p}^T \mathbf{a}(\mathbf{x})$$

$$\mathbf{p}^T(\mathbf{x}) = [1, x, y, xy, \dots]$$

$$\mathbf{a}(\mathbf{x}) = [a_{1i}, a_{2i}, a_{3i}, a_{4i}, \dots]$$

$$u_i(\mathbf{x}) = a_{1i} + xa_{2i} + ya_{3i}$$

$$u_i(x_i, y_i) = a_{1i} + x_i a_{2i} + y_i a_{3i} = u_i$$

$$a_{1i} = u_i - x_i a_{2i} - y_i a_{3i}$$

$$u_i(\mathbf{x}) = u_i + (x - x_i) a_{2i} + (y - y_i) a_{3i}$$

$$\begin{aligned}
 \bar{u}(\mathbf{x}) &= \sum_{i=1}^3 h_i u_i(\mathbf{x}) \\
 &= \sum_{i=1}^3 h_i [u_i + (x - x_i) a_{2i} + (y - y_i) a_{3i}] \\
 &= \sum_{i=1}^3 h_i u_i + \sum_{i=1}^3 h_i (x - x_i) a_{2i} + \sum_{i=1}^3 h_i (y - y_i) a_{3i} \\
 &= \mathbf{u} + \hat{\mathbf{u}}
 \end{aligned}$$

- Use of the high order interpolation
- Local use

PU based MITC3 shell element – Locking treatment

The linear terms of the enriched covariant strain components

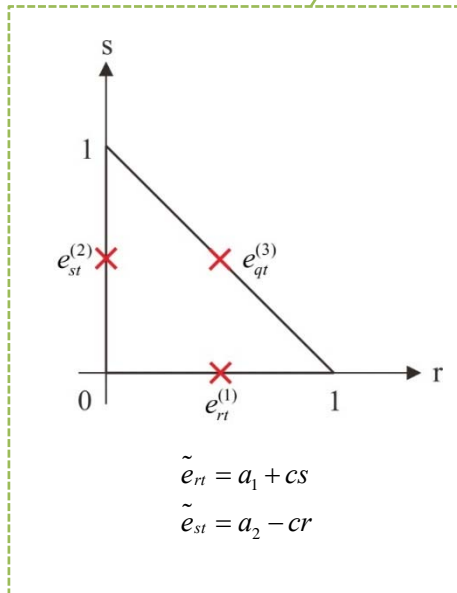
$$\bar{\mathbf{g}}_i = \frac{\partial(\mathbf{x} + \hat{\mathbf{x}})}{\partial r_i} = \frac{\partial \mathbf{x}}{\partial r_i} = \mathbf{g}_i$$

$$\bar{e}_{ij} = \frac{1}{2}(\bar{\mathbf{g}}_i \cdot \bar{\mathbf{u}}_{,j} + \bar{\mathbf{g}}_j \cdot \bar{\mathbf{u}}_{,i})$$

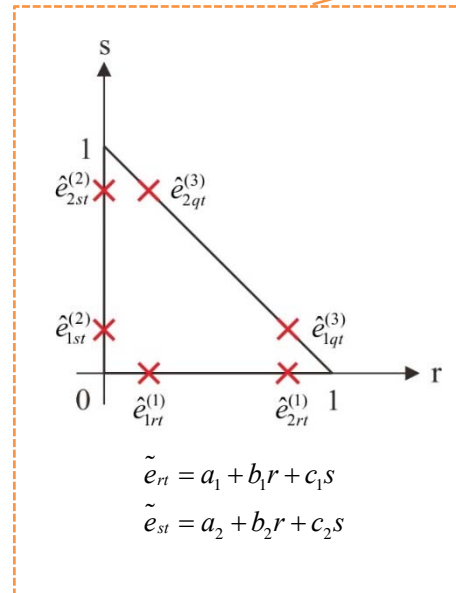
$$\bar{e}_{ij} = \frac{1}{2}(\mathbf{g}_i \cdot \mathbf{u}_{,j} + \mathbf{g}_j \cdot \mathbf{u}_{,i}) + \frac{1}{2}(\mathbf{g}_i \cdot \hat{\mathbf{u}}_{,j} + \mathbf{g}_j \cdot \hat{\mathbf{u}}_{,i})$$

PU based MITC3 shell element
(also called **enriched MITC3**)

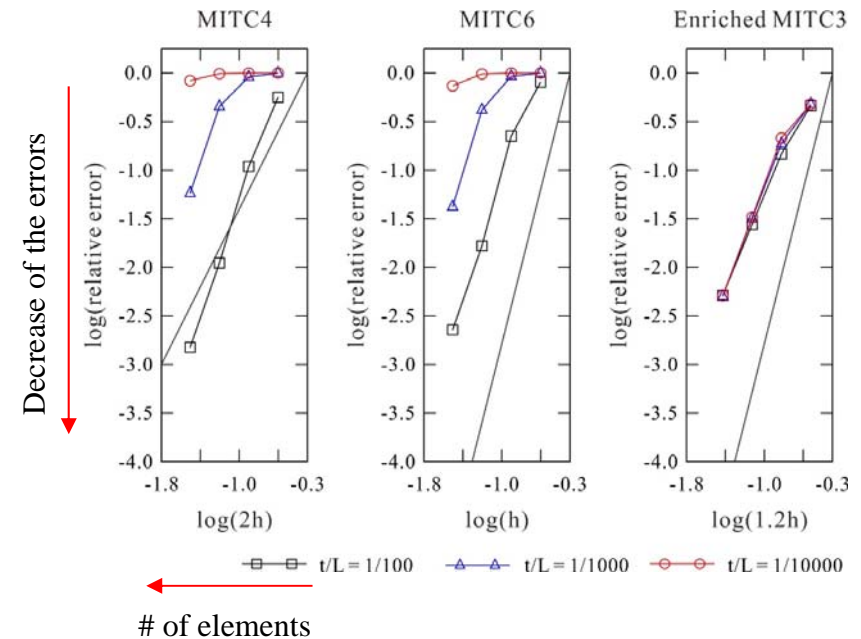
MITC3



MITC6



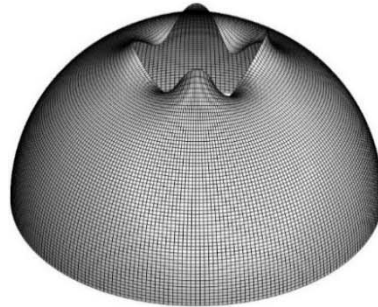
Convergence study



PU based MITC3 shell element – “Highly sensitive” shell problem

Fine mesh

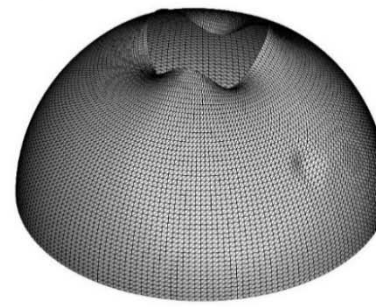
(a)



MITC4

(DOFs = 46,080, Ref. strain energy)

(b)

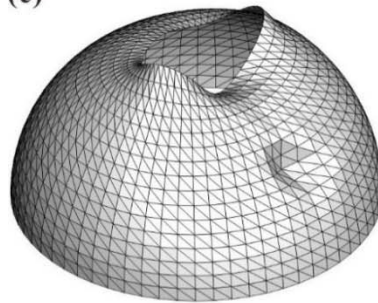


MITC3

(DOFs = 46,080, Error = 45.66%)

Coarse mesh

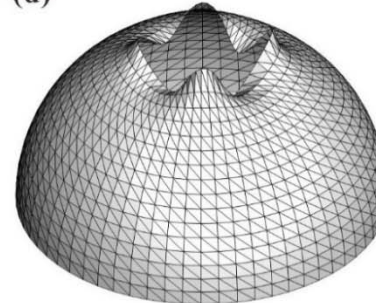
(c)



MITC3

(DOFs = 5,120, Error = 75.81%)

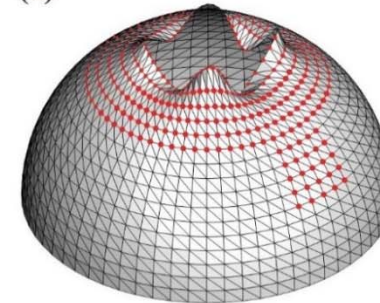
(d)



Fully enriched MITC3

(DOFs = 15,360, Error = 3.11%)

(e)

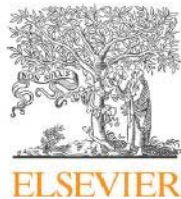


Locally enriched MITC3

(DOFs = 9,120, Error = 6.49%)

Jeon HM, Lee PS, Bathe KJ. *The MITC3 shell finite element enriched by interpolation covers. Comput Struct* 2014;134:128-42.

Computers and Structures 134 (2014) 128–142



Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc



The MITC3 shell finite element enriched by interpolation covers



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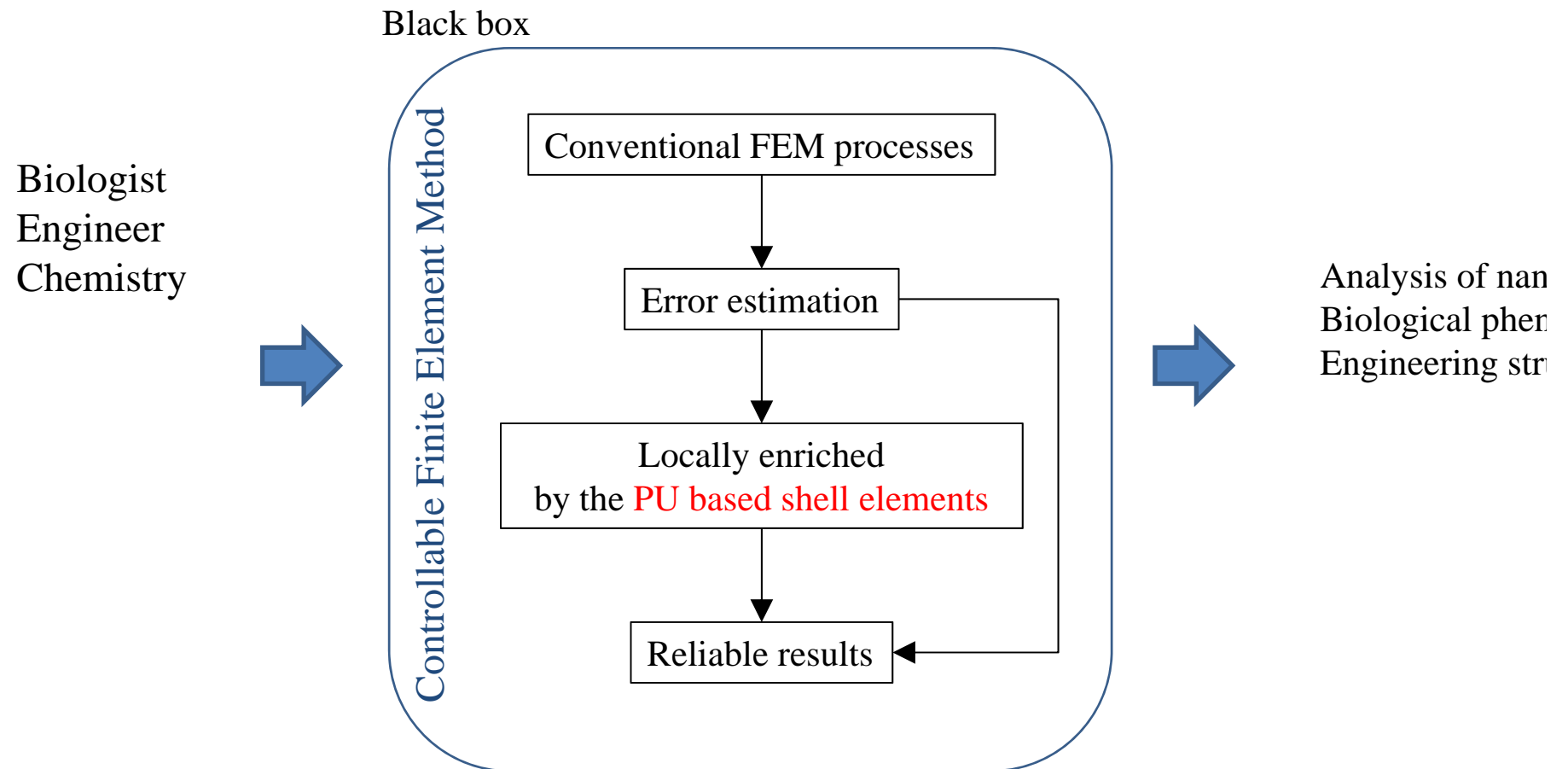
Triangular elements

ABSTRACT

In this paper, we develop a scheme to enrich the 3-node triangular MITC shell finite element by interpolation cover functions. The MITC method is used for the standard and enriched displacement interpolations. The enriched 3-node shell finite element not only captures higher gradients but also decreases inter-elemental stress jumps. In particular, the enrichment scheme increases the solution accuracy without any traditional local mesh refinement. Convergence studies considering a fully clamped square plate problem, cylindrical shell problems, and hyperboloid shell problems demonstrate the good predictive capability of the enriched MITC3 shell finite element, even when distorted meshes are used. We evaluate the effectiveness of the method, and also illustrate the use of the enrichment scheme applied only locally through the solution of two additional shell problems: a shaft–shaft interaction problem and a monster shell problem.

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The ultimate goal of the FEMs – PU based shell finite elements



Part II:

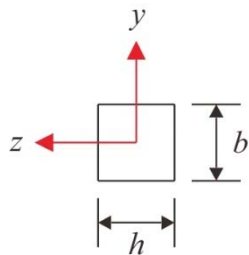
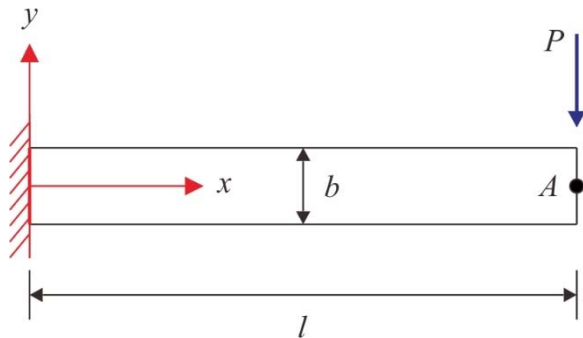
PU based Shell Element with Improved Membrane Response)

- Introduction to Nonlinear Finite Element Analysis
- Introduction and Scope of Research
- **MITC3+ Shell Element in the Nonlinear Analysis**
 - Key Concepts / Nonlinear Formulation
 - Benchmark Problems
- **The Method with Improved Membrane Behaviors**
 - Comparison with Other Methods
 - Key Concepts / Nonlinear Formulation
 - Benchmark Problems

Introduction to nonlinear FEA #1 – Linear analysis vs Nonlinear analysis

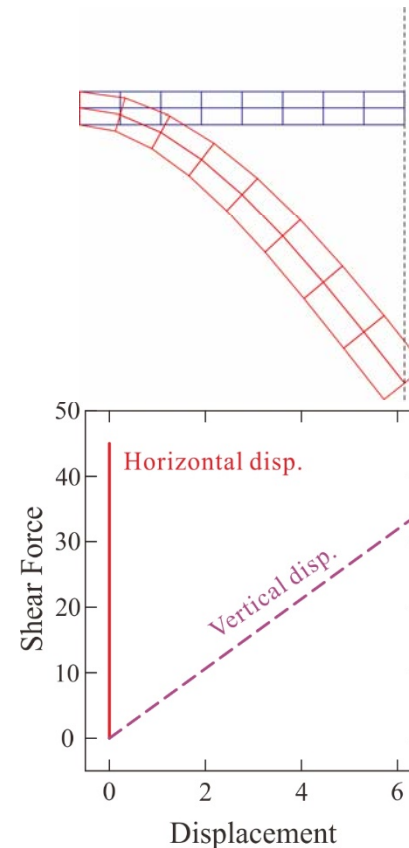
- Classification
- Geometric nonlinear analysis
 - Material nonlinear analysis
 - Changing in boundary condition

Cantilever beam subjected to end shear force

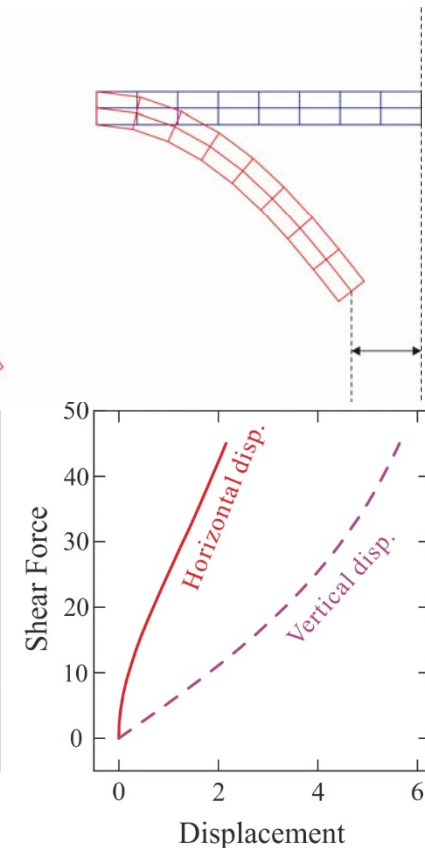


$$\begin{aligned} b &= h = 1, \quad l = 10, \\ E &= 1.2 \times 10^4, \quad \nu = 0.3 \\ P &= 4.5 \end{aligned}$$

Linear analysis



Nonlinear analysis



Introduction to nonlinear FEA #2 – Linear analysis vs Nonlinear analysis

	LINEAR ANALYSIS	NONLINEAR ANALYSIS
1. Strain	Infinitesimal strain $e_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$	Green-Lagrange strain $e_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial u_k}{\partial x_i} \cdot \frac{\partial u_k}{\partial x_j} \right)$
2. Rotation (curved beam/shell)	Infinitesimal rotation ${}^{t+\Delta t} \mathbf{V}_n^i = {}^{t+\Delta t} \mathbf{Q}^i {}^t \mathbf{V}_n^i$ ${}^{t+\Delta t} \mathbf{Q}^i = \mathbf{I}_3 + {}^{t+\Delta t} \mathbf{\Theta}^i$ ${}^{t+\Delta t} \mathbf{\Theta}^i = \begin{bmatrix} 0 & -{}^{t+\Delta t} \theta_3^i & {}^{t+\Delta t} \theta_2^i \\ {}^{t+\Delta t} \theta_3^i & 0 & -{}^{t+\Delta t} \theta_1^i \\ -{}^{t+\Delta t} \theta_2^i & {}^{t+\Delta t} \theta_1^i & 0 \end{bmatrix}$ $\blacktriangleright -\mathbf{V}_2^i \alpha_i + \mathbf{V}_1^i \beta_i$	Finite rotation ${}^{t+\Delta t} \mathbf{V}_n^i = {}^{t+\Delta t} \mathbf{Q}^i {}^t \mathbf{V}_n^i$ ${}^{t+\Delta t} \mathbf{Q}^i = \mathbf{I}_3 + {}^{t+\Delta t} \mathbf{\Theta}^i + \frac{1}{2!} ({}^{t+\Delta t} \mathbf{\Theta}^i)^2$ ${}^{t+\Delta t} \mathbf{\Theta}^i = \begin{bmatrix} 0 & -{}^{t+\Delta t} \theta_3^i & {}^{t+\Delta t} \theta_2^i \\ {}^{t+\Delta t} \theta_3^i & 0 & -{}^{t+\Delta t} \theta_1^i \\ -{}^{t+\Delta t} \theta_2^i & {}^{t+\Delta t} \theta_1^i & 0 \end{bmatrix}$ $\blacktriangleright -\alpha_i {}^t \mathbf{V}_2^i + \beta_i {}^t \mathbf{V}_1^i - \frac{1}{2} (\alpha_i^2 + \beta_i^2) {}^t \mathbf{V}_n^i$
3. Framework	-	<ul style="list-style-type: none"> ◇ Total Lagrangian formulation ◇ Update Lagrangian formulation ◇ Corotational formulation
4. Iterative scheme	-	<ul style="list-style-type: none"> ○ Full Newton-Raphson method ○ Modified Newton-Raphson method ○ BFGS matrix update method

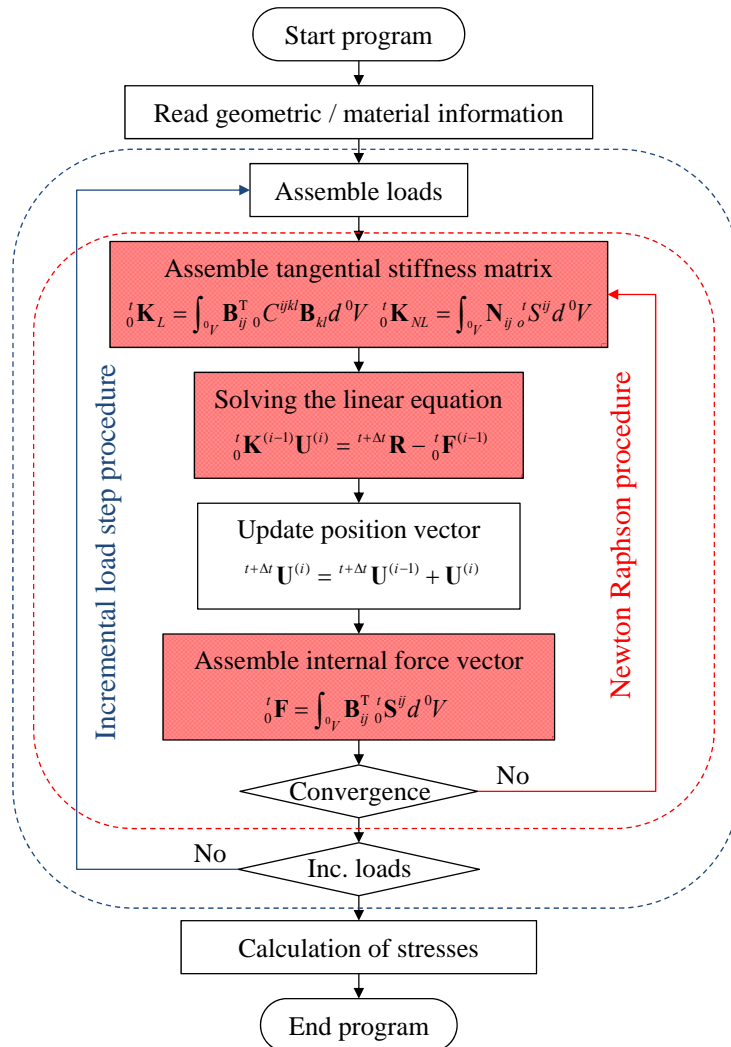
Three types of approaches

: to improve the nonlinear finite element solution

1. Using the **high performance computing**
2. Obtaining the **Equilibrium path**
3. Development and improvement of the **finite elements** (present research)

Three types of approaches #1 – High performance computing

Geometric nonlinear FE procedure



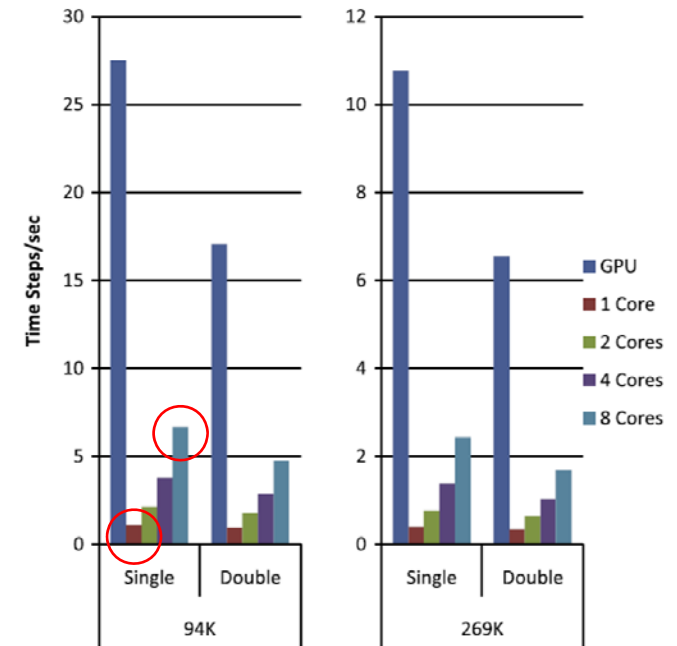
High performance computing



Two quad core Intel Xeon X5560
Core : 8 running at 3.2 GHz
RAM : 48 GB of DDR31333MHz
Parallel library : OpenMP



NVIDIA GeForce GTX480
Core : 480 CUDA, 1401 MHz
RAM : 1.5 GB of video memory
Parallel library : CUDA

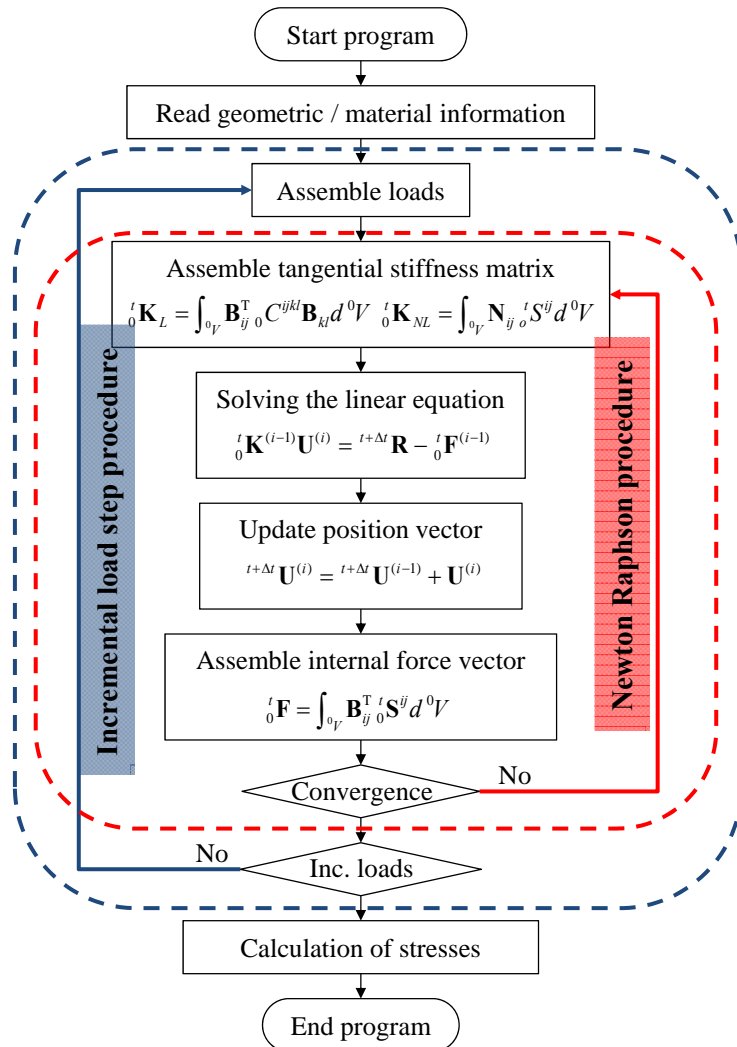


Dick et al., Simul Model Pract Th 2011;19:801-16.

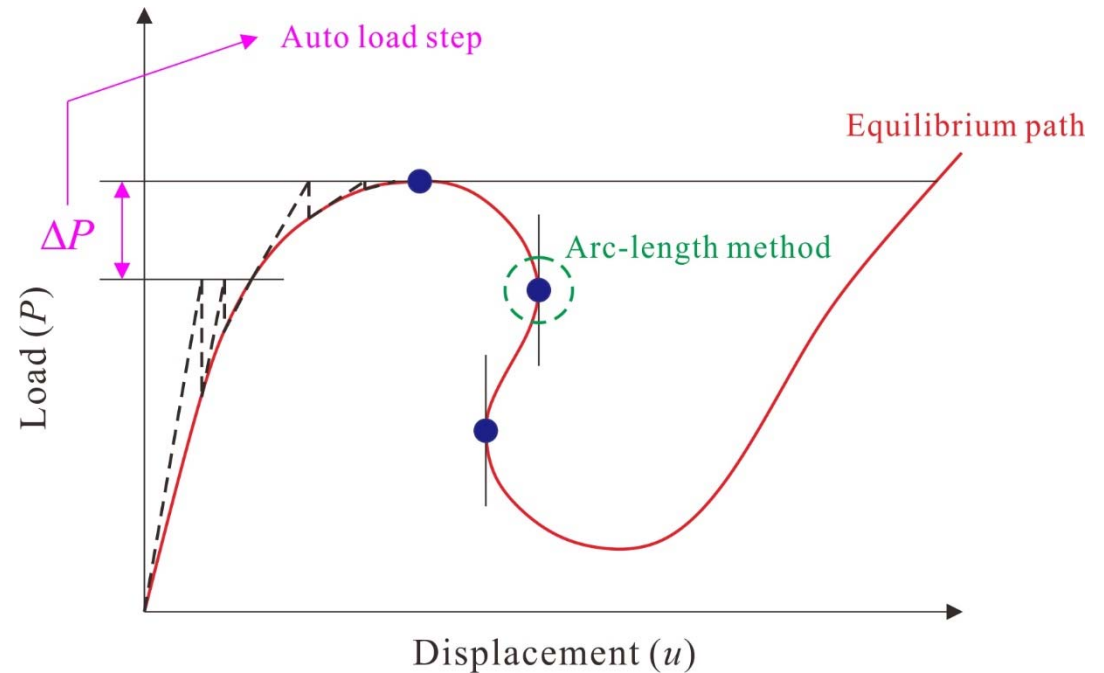
Karatarakis et al., Comput Methods Appl Mech Engrg 2014;269:334-55.

Three types of approaches #2 – Obtaining the equilibrium path

Geometric nonlinear FE procedure



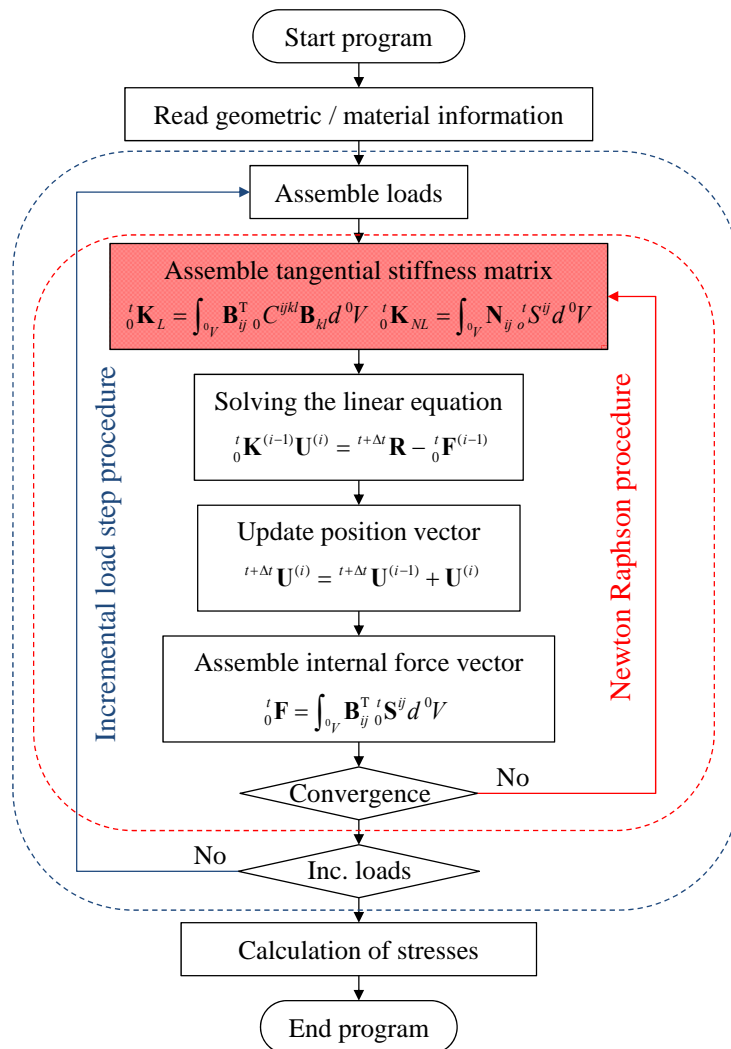
Obtaining the equilibrium path



- Load control / Displacement control (Batoz and Dhatt, 1979)
- Limit point analysis – Arc length method (Riks(1972), Crisfield(1980))
- Others – Automatic load step(Ramm, 1982), Line search(Box 1969)

Three types of approaches #3 – Development and improvement of the finite elements

Geometric nonlinear FE procedure



Different types of shell elements in commercial software

ABAQUS

S3, S3R, S4, S4R, SAX1, SAX2, SAX2T, SAXA1n, SAXA2n, STRI3, S4R5, STRI65, S8R, S8RT, S8R5, S9R5

ANSYS

SHELL28, SHELL41, SHELL43, SHELL51, SHELL57, SHELL61, SHELL63, SHELL91, SHELL93, SHELL99, SHELL143, SHELL150, SHELL157, SHELL163, SHELL181, SHELL185

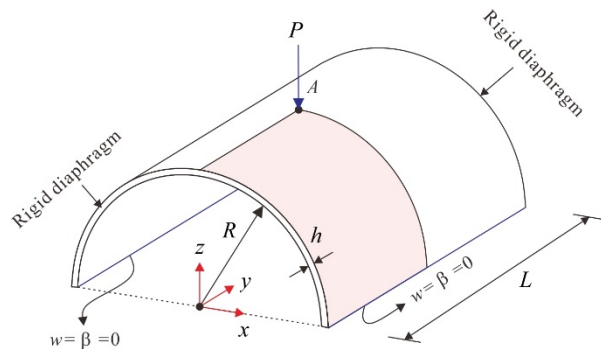
ADINA

Axisymmetric shell, DISP10 Collapsed, MITC3, MITC4, MITC4i, MITC6, MITC8, MITC9, MITC16

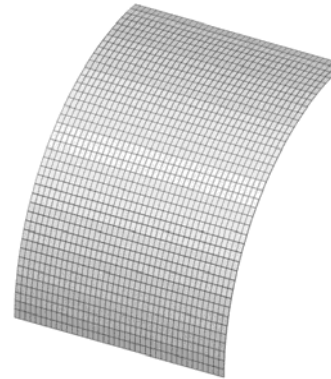
MSC Nastran™

QUAD4, QUAD8, QUADR, TRIA3, TRIA6, TRIAR, TRIA, CONEAX, RTRPLT

Validation of the nonlinear FE code – CODE vs ADINA



- MITC4 shell element
- Full Newton-Raphson method
- 100 load step



Deformed shapes

- CODE (C++)
- TECPLOT / PARAVIEW

Commercial software (ADINA)

```

I PROGRAM ADINA - VERSION 8.8.0 (build 01.26.2012) *** NO HEADING DEFINED ***

PRINT OUT FOR TIME (LOAD) STEP 100 (TIME STEP= 0.100
DIAG ELEMENT (WITH MAX ABS VALUE) OF THE FACTORIZED MATRIX = 0.77405E+05 NODE = 4
DIAG ELEMENT (WITH MIN ABS VALUE) OF THE FACTORIZED MATRIX = 0.21405E+03 NODE = 4

INTERMEDIATE PRINTOUT DURING EQUILIBRIUM ITERAT
    
```

	OUT-OF-BALANCE ENERGY	OUT-OF-BALANCE FORCE NODE-DOF MAX VALUE	NORM OF MOMENT NODE-DOF MAX VALUE	NORM OF INCREMENTAL DISP. NODE-DOF MAX VALUE	CONVERGENCE RATIOS FOR OUT-OF-BALANCE ENERGY	CON F
ITE= 0	5.35E+00	3.00E+01 1-Z -3.00E+01	8.18E-06 77-Y -4.76E-06	6.31E+00 34-Z -2.58E-01 -7.38E-02	2.40E-01 36-Y 0.00E+00 0.00E+00	1.00E+00 2.73E-05
ITE= 1	3.23E+00	2.80E+02 77-X -1.02E+02	3.02E+01 117-Y 1.18E+01	2.20E-01 36-Z -1.73E-02 -5.49E-03	1.23E-02 36-Y 0.00E+00 0.00E+00	6.04E-01 5.28E+00
ITE= 2	2.94E-02	1.28E+00 76-Z -5.76E-01	4.84E-01 77-Y -2.70E-01	5.37E-01 34-Z -2.66E-02 -8.00E-03	2.29E-02 36-Y 0.00E+00 0.00E+00	5.50E-03 4.56E-03 8.47E-02
ITE= 3	3.47E-04	2.84E+00 76-X 1.12E+00	3.04E-01 77-Y -1.19E-01	1.39E-02 34-Z -7.24E-04 -2.34E-04	6.10E-04 36-Y 0.00E+00 0.00E+00	6.48E-05 1.01E-02 5.31E-02

CODE (C / C++ / Fortran)

```

[Problem_Pinched_Cylinder_Octant1]

[INFEM_shell_MITC4]

-----
Young's modulus : 3.000E+04, Poisson ratio : 0.300E+00
thickness : 1.00E+00
# total elements : 1600, # total nodes : 1681
# total DOFs : 8405, # free DOFs : 8079, # fix DOFs : 326
# of load step : 100
Newton-Rhapson iteration tolerance : 1.00E-03
required memory = 1.258E+01 MB
-----

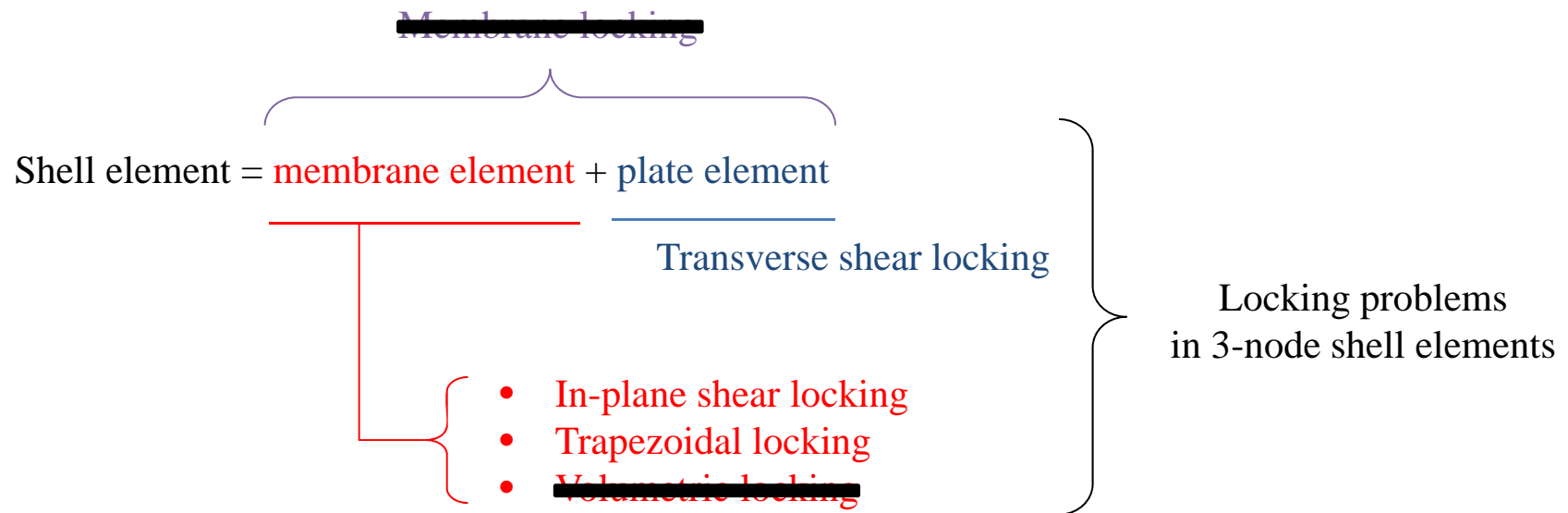
1/ Newton-Rhapson iteration

step  iter  OOB<energy>  norm(OOB<R-F>)  norm<disp>  norm<rot>  criteria
99      0    4.76E+00    9.76E+03    5.43E+00    2.02E-01    1.00E+00
99      1    1.54E+00    9.81E+03    1.82E-01    9.35E-03    3.24E-01
99      2    6.58E-03    9.81E+03    2.23E-01    9.39E-03    1.38E-03
99      3    9.70E-06    9.81E+03    2.04E-03    8.03E-05    2.04E-06

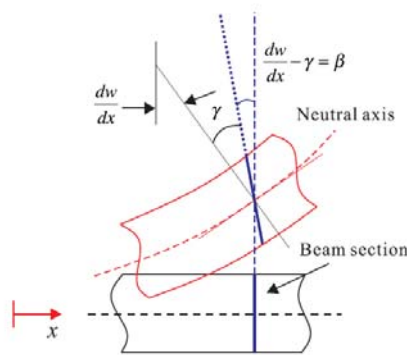
100     0    5.35E+00    9.81E+03    6.31E+00    2.40E-01    1.00E+00
100     1    3.23E+00    9.88E+03    2.20E-01    1.23E-02    6.04E-01
100     2    2.94E-02    9.88E+03    5.37E-01    2.29E-02    5.50E-03
100     3    3.47E-04    9.88E+03    1.39E-02    6.10E-04    6.48E-05

2/ Printing Output Files
3/ Completed !
    
```

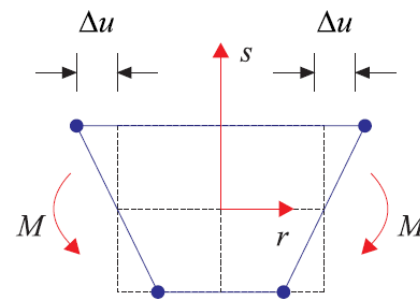
Introduction to locking phenomenon – 3-node triangular shell element



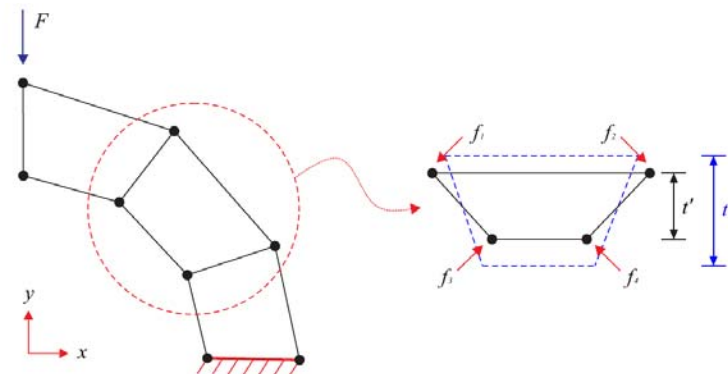
Transverse shear locking



In-plane shear locking

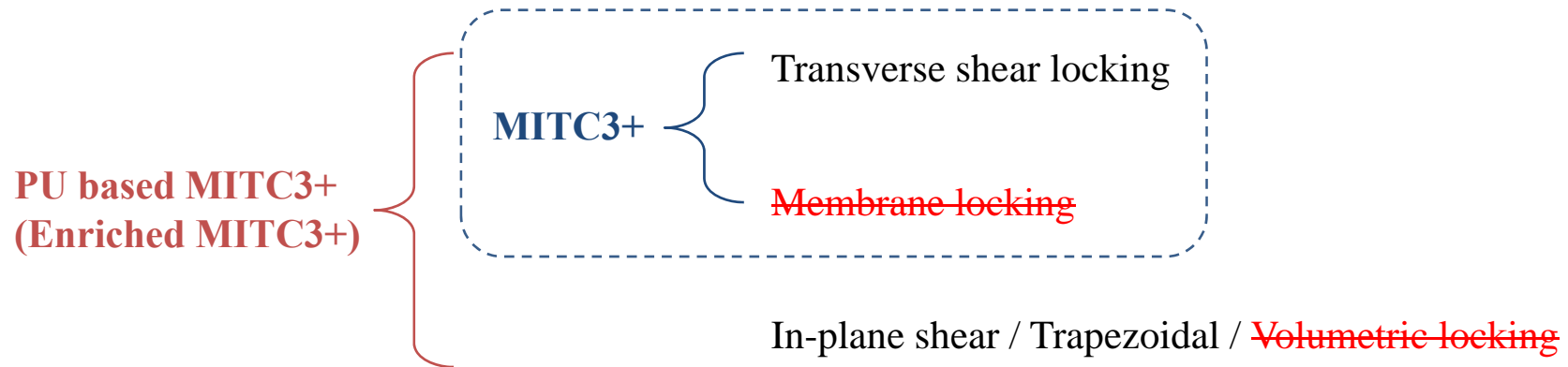


Trapezoidal locking



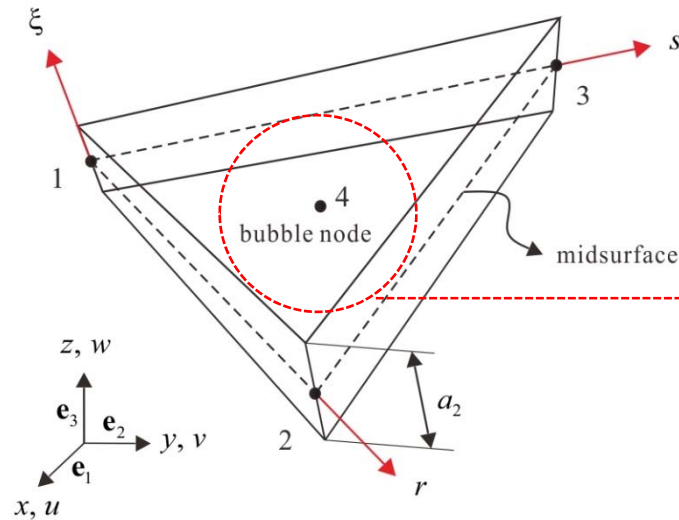
Newly developed shell elements – MITC3+ and enriched MITC3+ shell elements

Two types of newly developed triangular shell elements



	MITC3+	Enriched MITC3+
1. BASIS	Original MITC3 shell element	MITC3+ shell element
2. METHODS	Cubic bubble function	Cubic bubble function Partition of unity approximation
4. FEATURE	Improved bending behaviors	Improved bending / membrane behaviors

MITC3+ shell element – Key concepts



Original 3-node shell element

$$\mathbf{u}(r, s, \xi) = \sum_{i=1}^3 h_i(r, s) \mathbf{u}_i + \frac{\xi}{2} \sum_{i=1}^3 a_i h_i(r, s) (-\mathbf{V}_2^i \alpha_i + \mathbf{V}_1^i \beta_i)$$

3-node shell element with the bubble function

$$\mathbf{u}(r, s, \xi) = \sum_{i=1}^3 h_i(r, s) \mathbf{u}_i + \frac{\xi}{2} \sum_{i=1}^4 a_i f_i(r, s) (-\mathbf{V}_2^i \alpha_i + \mathbf{V}_1^i \beta_i)$$

➡ Suggested new assumed strain fields

➡ MITC3+ shell element

- Lee Y et al., The MITC3+ shell element and its performance, Comput Struct 2014;138:12-23
- Excellent convergence behavior
- Only linear analysis conditions have been considered

Kinematics

Incremental Green-Lagrange strain tensor components :

$${}_0\varepsilon_{ij} = \frac{1}{2}({}^{t+\Delta t}\mathbf{g}_i \cdot {}^{t+\Delta t}\mathbf{g}_j - {}^t\mathbf{g}_i \cdot {}^t\mathbf{g}_j) = \frac{1}{2}(\mathbf{u}_{,i} \cdot {}^t\mathbf{g}_j + {}^t\mathbf{g}_i \cdot \mathbf{u}_{,j} + \boxed{\mathbf{u}_{,i} \cdot \mathbf{u}_{,j}})$$

Displacement interpolation with finite rotations : nonlinear part in the GL strain

$$\mathbf{u} = \sum_{i=1}^3 h_i(r,s) \mathbf{u}_i + \frac{t}{2} \sum_{i=1}^4 a_i f_i \left[-\alpha_i {}^t\mathbf{V}_2^i + \beta_i {}^t\mathbf{V}_1^i - \frac{1}{2}(\alpha_i^2 + \beta_i^2) {}^t\mathbf{V}_n^i \right]$$

cf. linear analysis $\mathbf{u} = \sum_{i=1}^3 h_i \mathbf{u}_i + \sum_{i=1}^4 \frac{t}{2} a_i f_i (-\mathbf{V}_2^i \alpha_i + \mathbf{V}_1^i \beta_i)$

Iterative solution procedures

With the full Newton-Raphson iteration scheme, the equation for the i -th iteration in a finite element model are

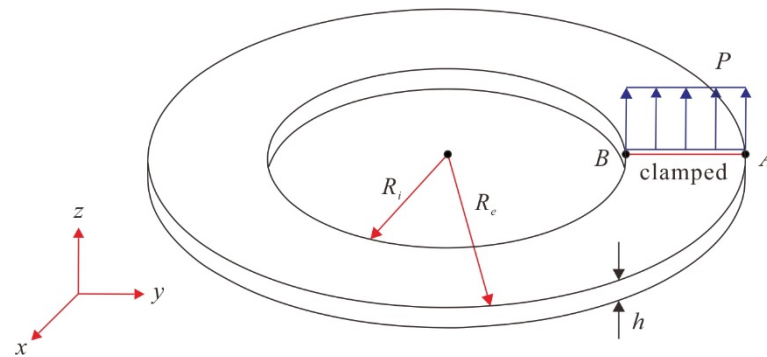
$${}_0\mathbf{K}^{(i-1)} \Delta \mathbf{U}^{(i)} = {}^{t+\Delta t}\mathfrak{R} - {}^t\mathbf{F}^{(i-1)}$$

for the displacement,

$${}^{t+\Delta t}\mathbf{U}^{(i)} = {}^{t+\Delta t}\mathbf{U}^{(i-1)} + \Delta \mathbf{U}^{(i)}$$

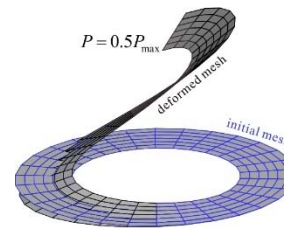
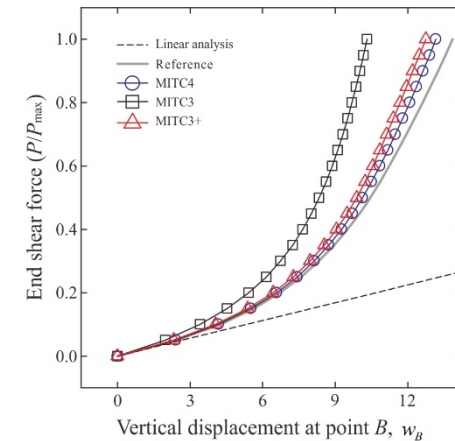
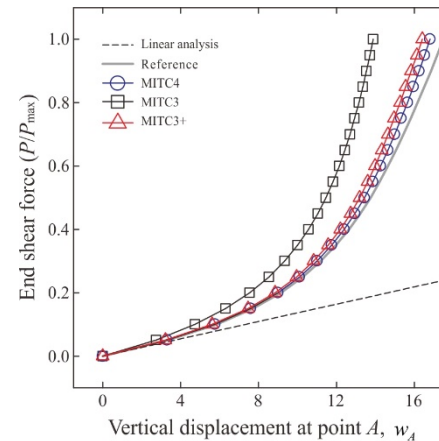
Nonlinear performances of the MITC3+ element #1 – Slit annular plate problem

Slit annular plate problem

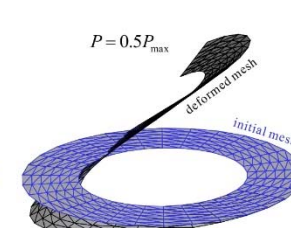


$$R_i = 6, R_e = 10, h = 0.03, P = 3.2$$

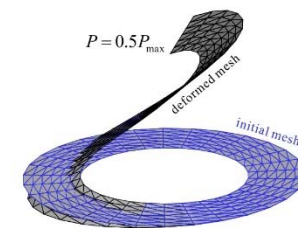
$$E = 21 \times 10^6, \nu = 0$$



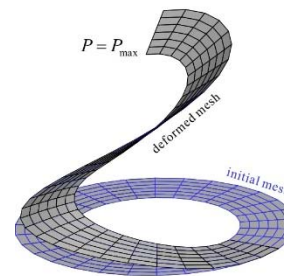
MITC4



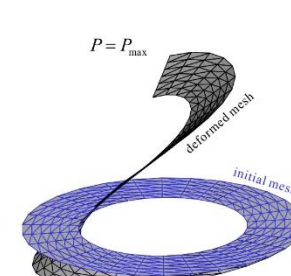
MITC3



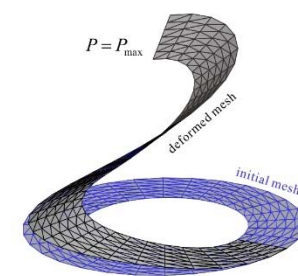
MITC3+



MITC4



MITC3

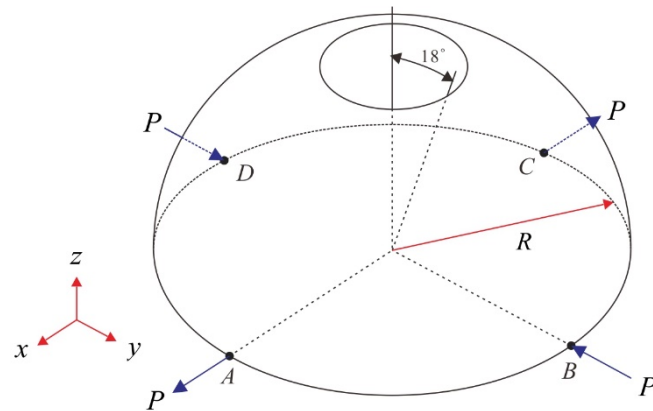


MITC3+

Element	Description
MITC4	4-node quadrilateral shell element (Bathe, 1986)
MITC3	3-node triangular shell element (Lee, 2004)
MITC3+	3-node triangular shell element with a cubic bubble function (present)
Reference	9-node quadrilateral shell element

Nonlinear performances of the MITC3+ element #2 – Hemisphere shell

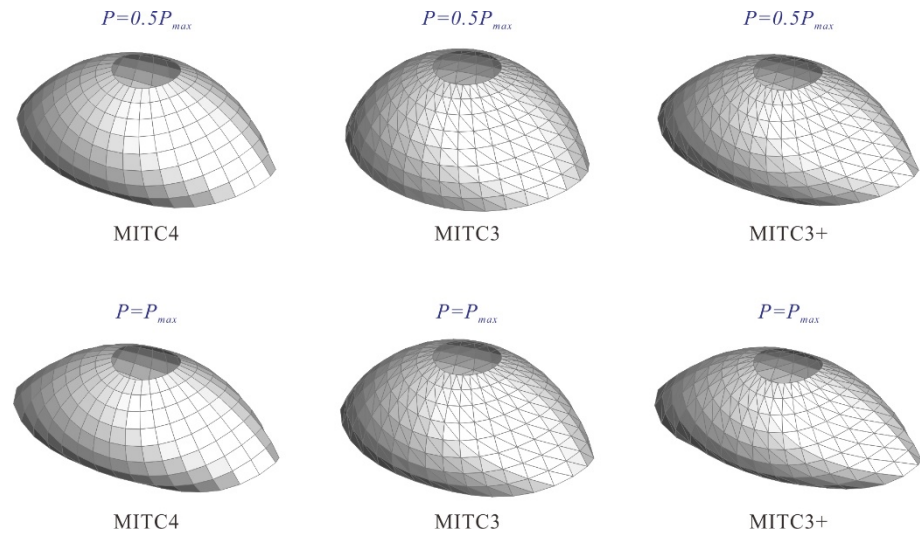
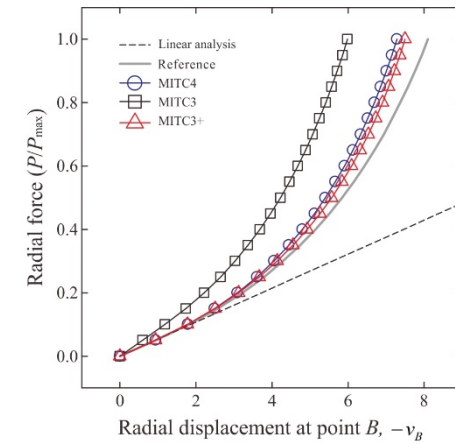
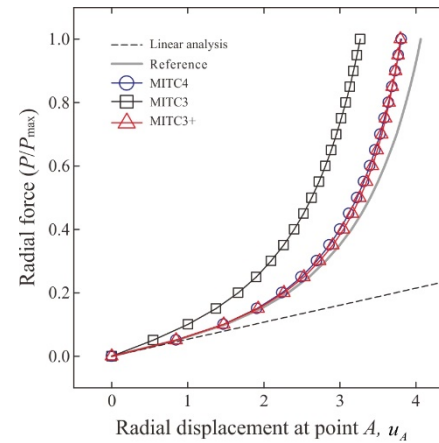
Hemisphere shell problem



$$R = 10, h = 0.04, P = 400$$

$$E = 6.825 \times 10^7, \nu = 0.3$$

Element	Description
MITC4	4-node quadrilateral shell element (Bathe, 1986)
MITC3	3-node triangular shell element (Lee, 2004)
MITC3+	3-node triangular shell element with a cubic bubble function (present)
Reference	9-node quadrilateral shell element



Nonlinear performances of the MITC3+ element #3 – Other problems

Other benchmark problems solved

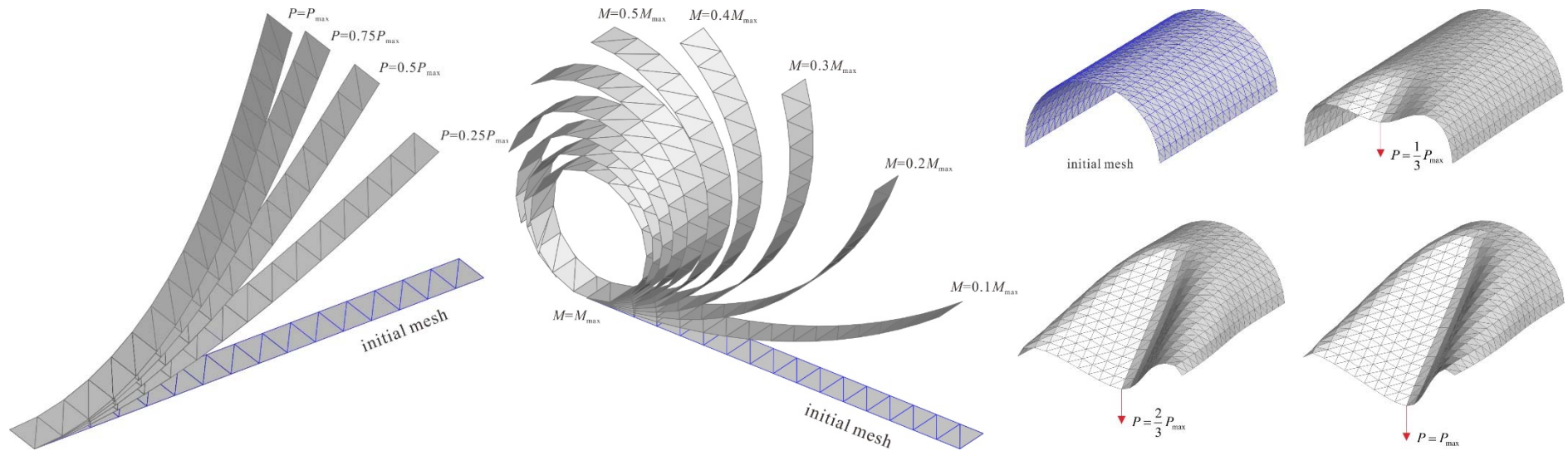
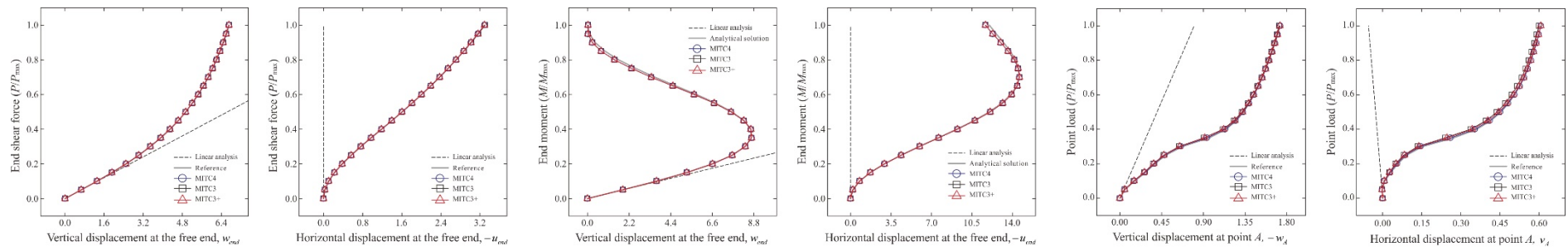


Plate under end shear force

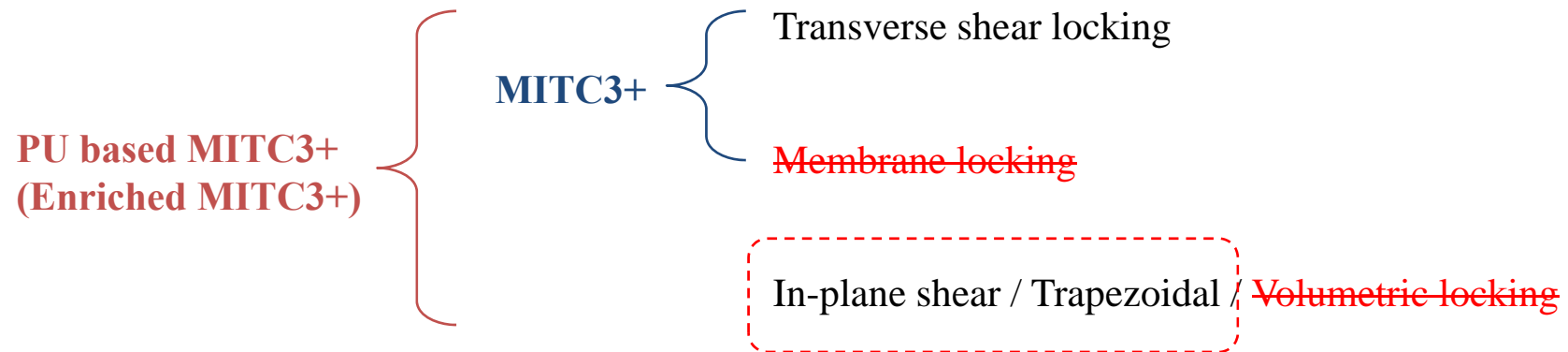
Plate subjected to end moment

Clamped semi-cylindrical shell



Newly developed shell elements – MITC3+ and enriched MITC3+ shell elements

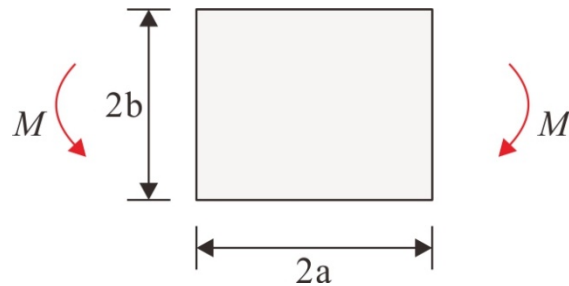
Two types of newly developed triangular shell elements



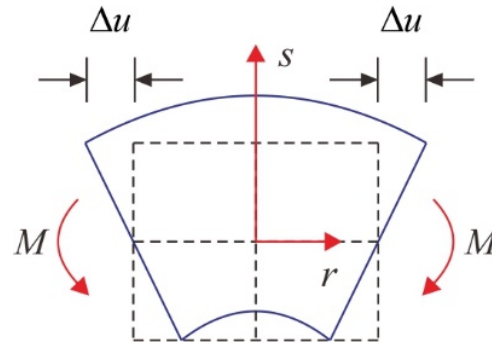
	MITC3+	Enriched MITC3+
1. BASIS	Original MITC3 shell element	MITC3+ shell element
2. METHODS	Cubic bubble function	Cubic bubble function Partition of unity approximation
4. FEATURE	Improved bending behaviors	Improved bending / membrane behaviors

In plane shear locking – Bilinear quadrilateral finite element

Pure bending 2D problem



Correct beam deformation

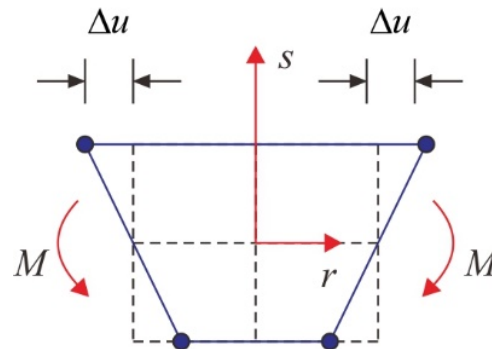


$$u(r, s) = \frac{M}{EI} rs$$

$$v(r, s) = \frac{Ma^2}{2EI} \left(1 - \frac{r^2}{a^2}\right) + \frac{Mb^2}{2EI} \left(1 - \frac{s^2}{b^2}\right)$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0$$

Finite element deformation



$$u(r, s) = \Delta u rs$$

$$v = 0$$

$$\gamma_{xy} = \Delta u r$$

Parasitic in-plane shear strain

In-plane shear locking : The element has an excess of shear strain which contribute to the poor ability of the element to reproduce bending modes.

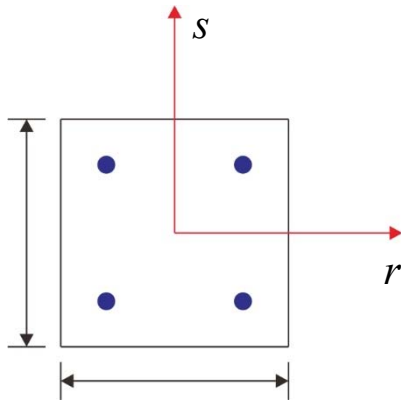
The methods to improve membrane behaviors

- Reduced Integration method
- Incompatible mode
- Additional bubble node
- Assumed Natural Strain(ANS) method
- Drilling degrees of freedom
- Enhanced Assumed Strain(EAS) method
- Discrete Shear Gap (DSG) method
-

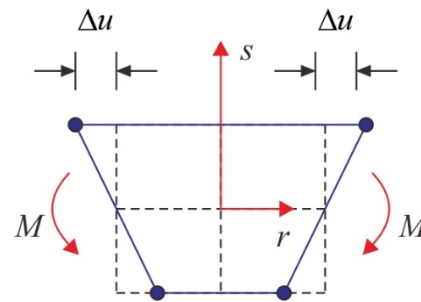
- Partition of unity approximations

Comparison with other methods #2 – Reduced Integration method

Gauss quadrature $I = \int_{-1}^1 \int_{-1}^1 f(r,s) ds dr \approx \sum_{i=1}^M \sum_{j=1}^M W_{ij} f(r_i, s_j)$



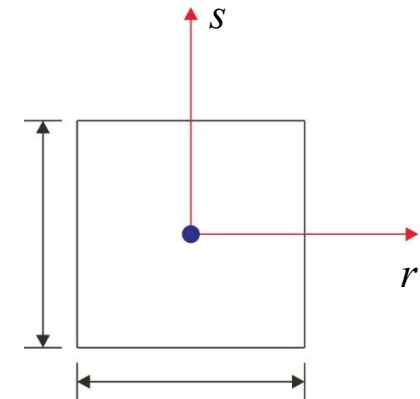
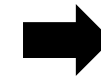
Full integration



$$u(r,s) = \Delta u r s$$

$$v = 0$$

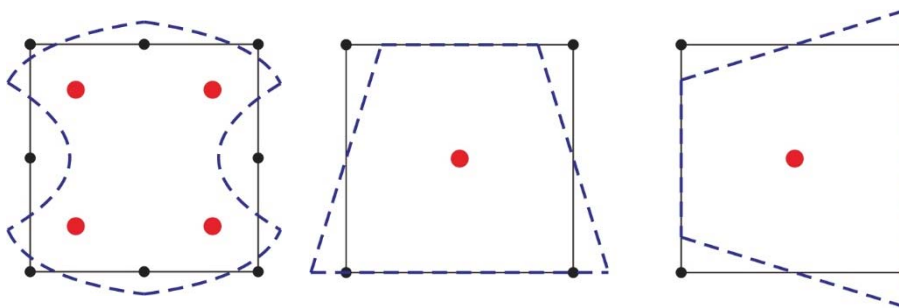
$$\gamma_{xy} = \Delta u r$$



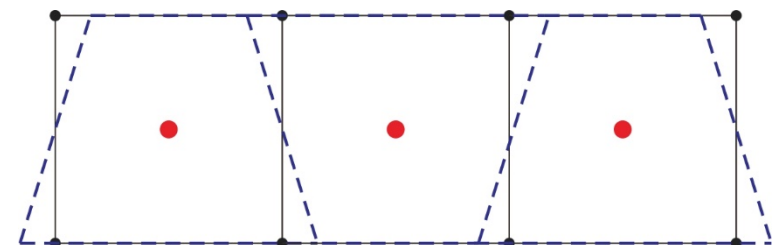
Reduced integration
(Zienkiewicz OC et al., 1971)

Geometric-invariant
Quadrilateral element

Hourglass control (Flanagan & Belytschko, 1981)



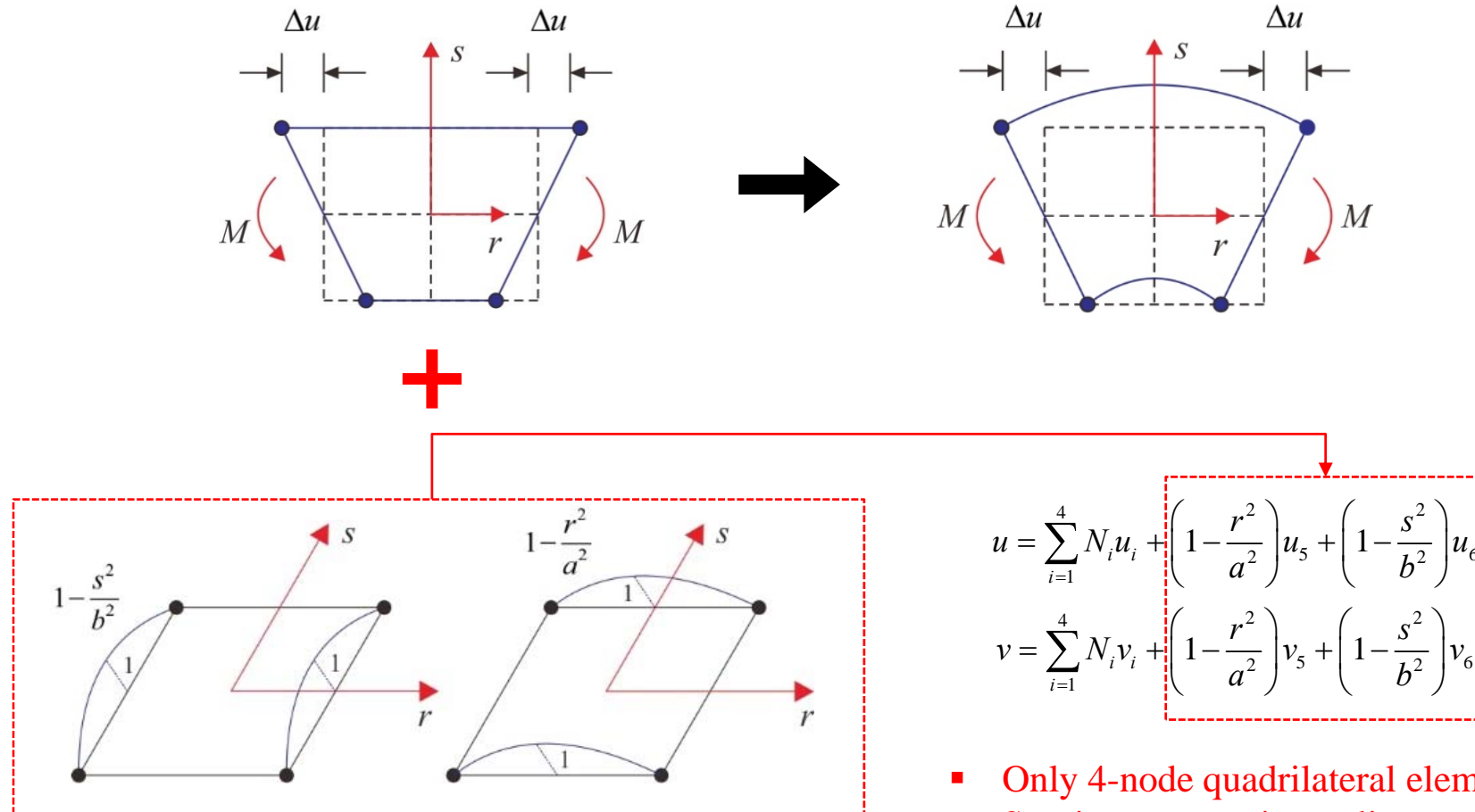
2 hour-glass modes



Propagation of hour-glass modes through a mesh

Comparison with other methods #3 – Incompatible modes

Incompatible modes



Incompatible modes (Wilson(1971), Taylor(1976))

- Only 4-node quadrilateral element
- Spurious energy in nonlinear analysis
(Sussman T and Bathe KJ, Comput Struct, 2014)

Comparison with other methods #4 – Partition of unity approximation

Partition of unity approximation

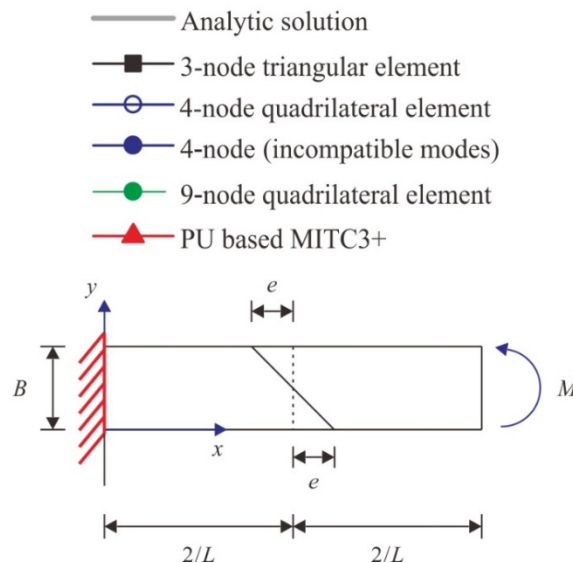
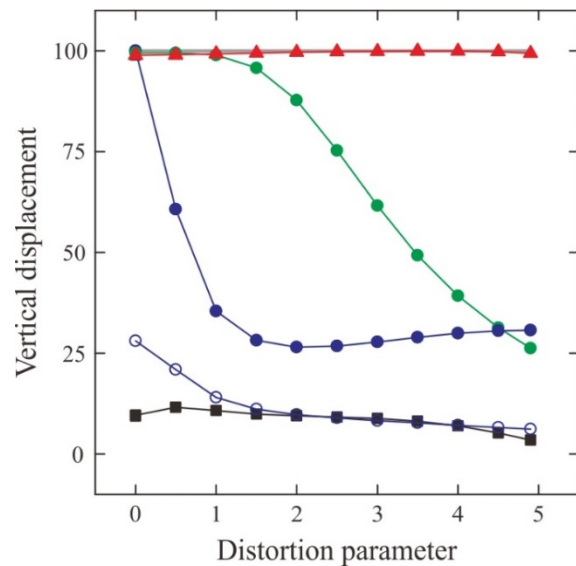
$$\bar{u}(\mathbf{x}) = \underbrace{\sum_{i=1}^3 h_i u_i}_{\text{Standard term}} + \underbrace{\sum_{i=1}^3 h_i (x - x_i) a_{2i} + h_i (y - y_i) a_{3i}}_{\text{Additional term}}$$

$$\bar{u}(\mathbf{x}) = u(x) + \hat{u}(x)$$

$$u(x) = \sum_{i=1}^3 h_i u_i$$

$$\hat{u}(x) = \sum_{i=1}^3 h_i (x - x_i) a_{2i} + h_i (y - y_i) a_{3i}$$

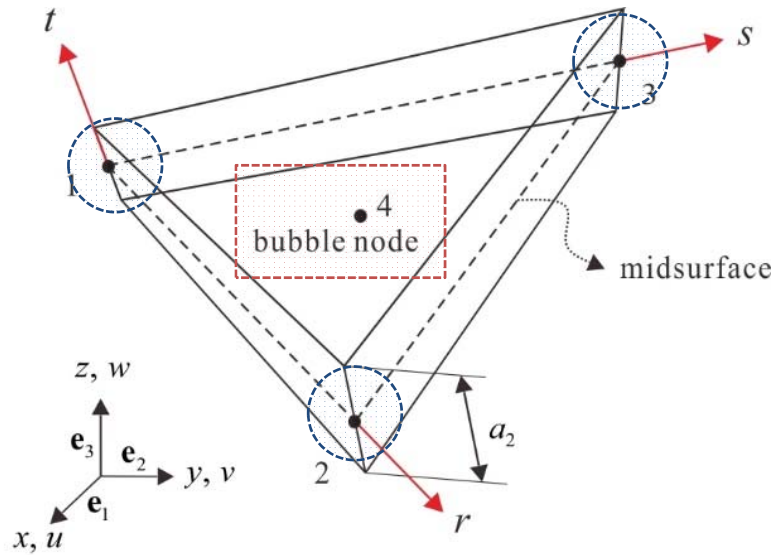
Distortion sensitivity test



Advantages

- 3-node triangular element
- Geometric invariant
- Pass basic tests
- Accurate results than other methods

Proposed remedy to improved membrane response – Partition of unity approximation



$$\begin{bmatrix} e_{xx} \\ e_{yy} \\ e_{xy} \\ e_{yz} \\ e_{zx} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{u}}{\partial x} \\ \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{v}}{\partial y} \\ \frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x} \\ \frac{\partial \bar{v}}{\partial z} + \frac{\partial \bar{w}}{\partial y} \\ \frac{\partial \bar{w}}{\partial x} + \frac{\partial \bar{u}}{\partial z} \end{bmatrix}$$

High order interpolation
(enhanced membrane)

MITC method
(transverse shear locking)

Original 3-node displacement interpolation

$$\mathbf{u} = \sum_{i=1}^3 h_i \mathbf{u}_i + \frac{t}{2} \sum_{i=1}^3 a_i h_i (-\mathbf{V}_2^i \alpha_i + \mathbf{V}_1^i \beta_i), \text{ with } \mathbf{u} = [u \ v \ w]^T$$

PU based MITC3+
(Enriched MITC3+)

$$\begin{cases} \bar{u} = \sum_{i=1}^3 h_i u_i + \frac{t}{2} \sum_{i=1}^4 a_i f_i (-\mathbf{V}_2^i \alpha_i + \mathbf{V}_1^i \beta_i) + \hat{u} \\ \bar{v} = \sum_{i=1}^3 h_i v_i + \frac{t}{2} \sum_{i=1}^4 a_i f_i (-\mathbf{V}_2^i \alpha_i + \mathbf{V}_1^i \beta_i) + \hat{v} \\ \bar{w} = \sum_{i=1}^3 h_i w_i + \frac{t}{2} \sum_{i=1}^4 a_i f_i (-\mathbf{V}_2^i \alpha_i + \mathbf{V}_1^i \beta_i) \end{cases}$$

MITC3+ shell element

PU approximation

PU based MITC3+ shell element
(Enriched MITC3+)

Proposed remedy to improved membrane response – Partition of unity approximation

Kinematics

Incremental Green-Lagrange strain tensor components :

$${}_0\varepsilon_{ij} = \frac{1}{2}({}^{t+\Delta t}\mathbf{g}_i \cdot {}^{t+\Delta t}\mathbf{g}_j - {}^t\mathbf{g}_i \cdot {}^t\mathbf{g}_j) = \frac{1}{2}(\bar{\mathbf{u}}_{,i} \cdot {}^t\bar{\mathbf{g}}_j + {}^t\bar{\mathbf{g}}_i \cdot \bar{\mathbf{u}}_{,j} + \boxed{\bar{\mathbf{u}}_{,i} \cdot \bar{\mathbf{u}}_{,j}})$$

Finite rotations :

nonlinear part in the GL strain

Linear
$$\bar{\mathbf{u}} = \sum_{i=1}^3 h_i \mathbf{u}_i + \sum_{i=1}^4 \frac{t}{2} a_i f_i (-\mathbf{V}_2^i \alpha_i + \mathbf{V}_1^i \beta_i) + \hat{\mathbf{u}}$$

Nonlinear
$$\bar{\mathbf{u}} = \sum_{i=1}^3 h_i(r,s) \mathbf{u}_i + \frac{t}{2} \sum_{i=1}^4 a_i f_i \left[-\alpha_i {}^t\mathbf{V}_2^i + \beta_i {}^t\mathbf{V}_1^i - \frac{1}{2}(\alpha_i^2 + \beta_i^2) {}^t\mathbf{V}_n^i \right] + \hat{\mathbf{u}}$$

Iterative solution procedures

With the full Newton-Raphson iteration scheme, the equation for the i -th iteration in a finite element model are

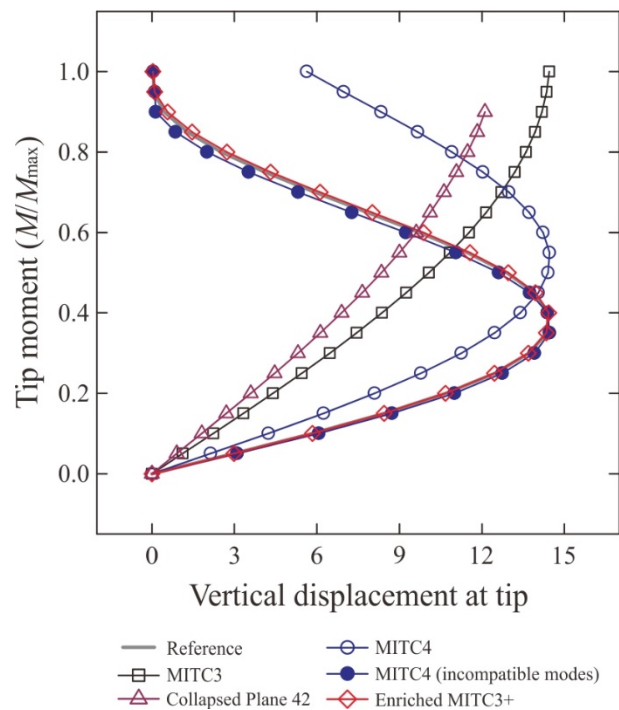
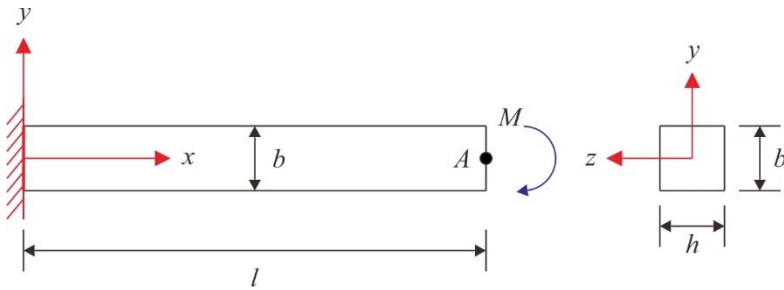
$${}_0\mathbf{K}^{(i-1)} \Delta \mathbf{U}^{(i)} = {}^{t+\Delta t}\mathfrak{R} - {}^t\mathbf{F}^{(i-1)}$$

for the displacement,

$${}^{t+\Delta t}\mathbf{U}^{(i)} = {}^{t+\Delta t}\mathbf{U}^{(i-1)} + \Delta \mathbf{U}^{(i)}$$

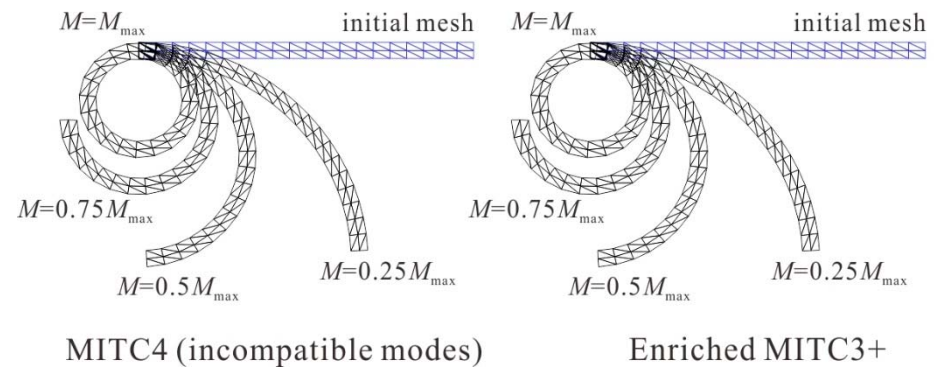
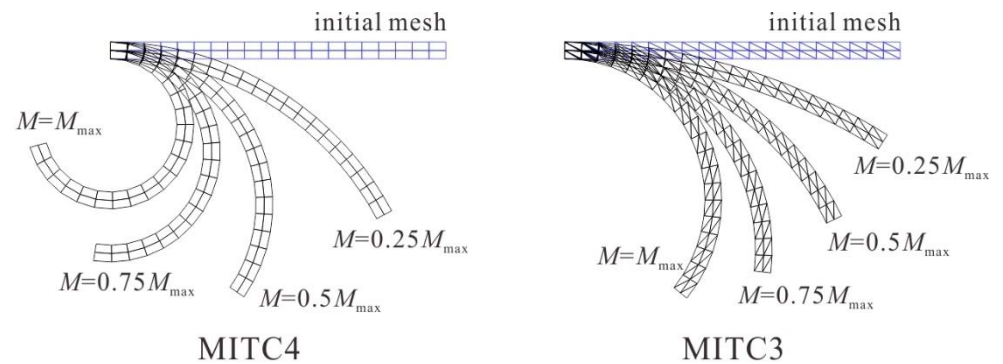
Performances of the PU based MITC3+ element #1

Cantilever beam subjected a tip moment



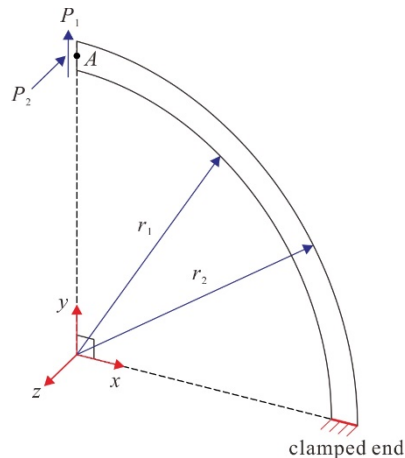
$$l = 20, b = h = 1, M = 10\pi$$

$$E = 1200, \nu = 0.2$$



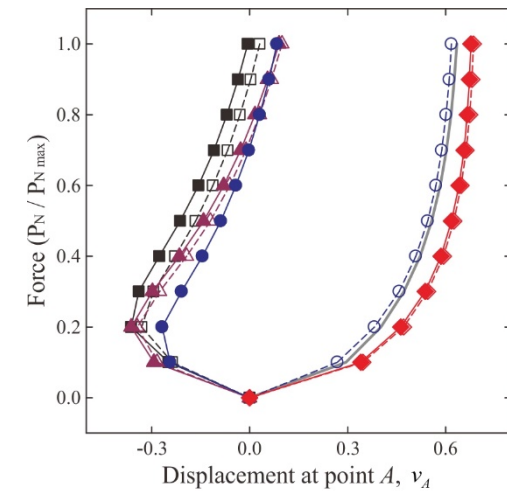
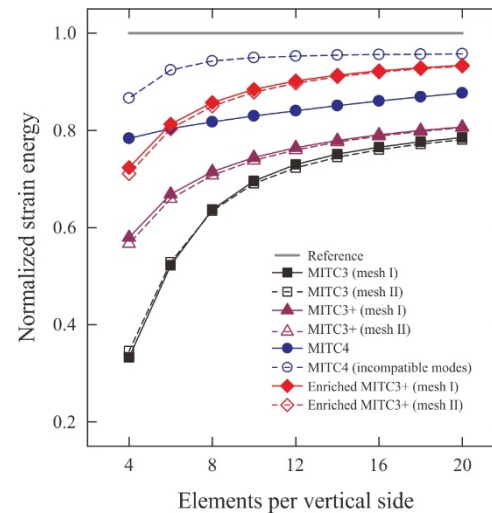
Performances of the PU based MITC3+ element #2

Curved cantilever beam

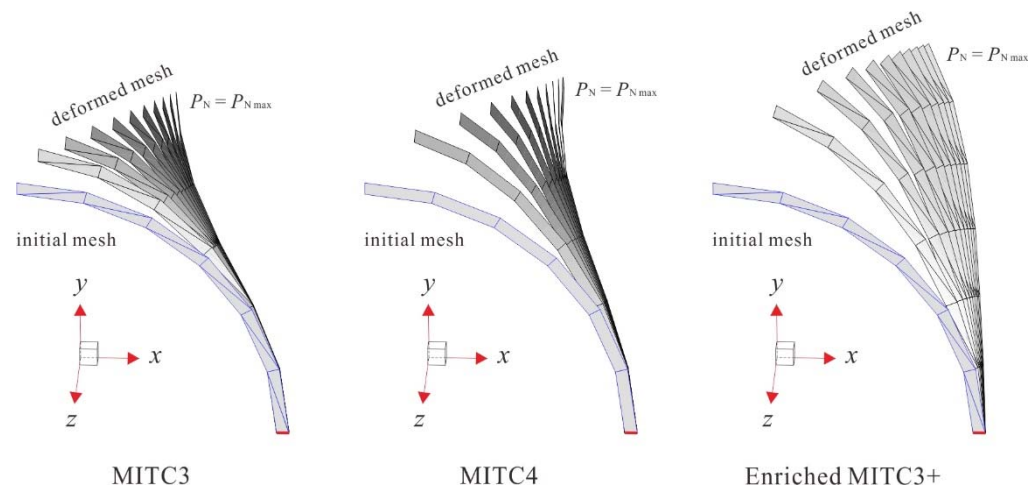


$$R_1 = 4.12, R_2 = 4.32, h = 0.1$$

$$E = 1 \times 10^7, \nu = 0.25$$

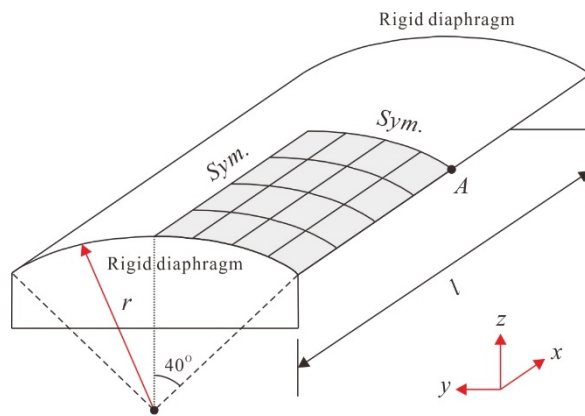


Element	Description
MITC4	4-node quadrilateral shell element (Bathe, 1986)
MITC3	3-node triangular shell element (Lee, 2004)
MITC3+	3-node triangular shell element with a cubic bubble function (Jeon, 2014)
Enriched MITC3+	PU based MITC3+ (present)
Reference	9-node quadrilateral shell element



Performances of the PU based MITC3+ element #3

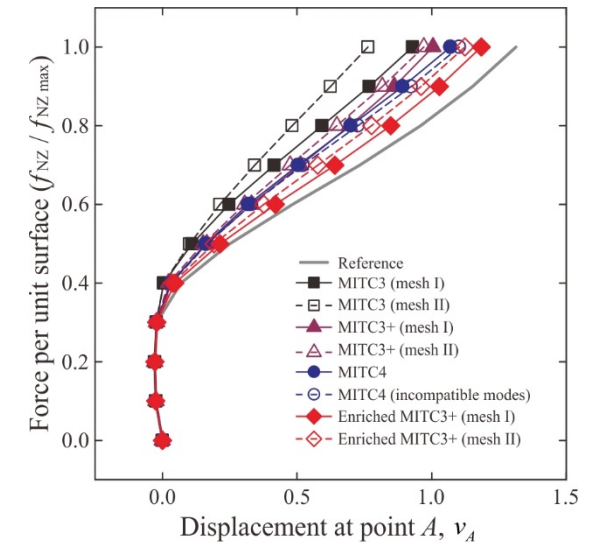
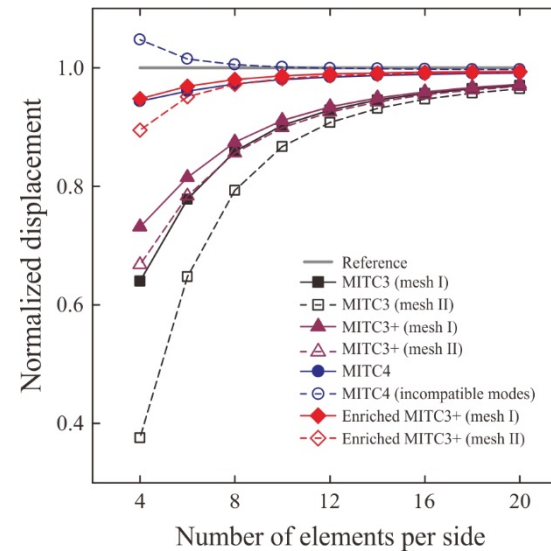
Hemisphere shell problem



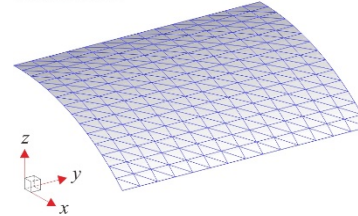
$$L = 50, R = 25, t = 0.25, f_z = 90$$

$$E = 4.32 \times 10^8, \nu = 0$$

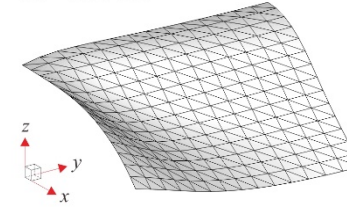
Element	Description
MITC4	4-node quadrilateral shell element (Bathe, 1986)
MITC3	3-node triangular shell element (Lee, 2004)
MITC3+	3-node triangular shell element with a cubic bubble function (Jeon, 2014)
Enriched MITC3+	PU based MITC3+ (present)
Reference	9-node quadrilateral shell element



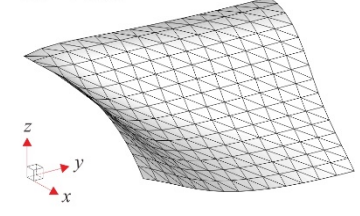
initial mesh



$F_N = 0.7 F_{N \max}$



$F_N = F_{N \max}$



Nonlinear analysis of the MITC3+ shell element published in *Computers and Structures*, Jan 2015



The MITC3+ shell element in geometric nonlinear analysis

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ABSTRACT

In this paper, we present the MITC3+ shell finite element for geometric nonlinear analysis and demonstrate its performance. The MITC3+ shell element, recently proposed for linear analysis [1], represents a further development of the MITC3 shell element. The total Lagrangian formulation is employed allowing for large displacements and large rotations. Considering several analysis problems, the nonlinear solutions using the MITC3+ shell element are compared with those obtained using the MITC3 and MITC4 shell elements. We conclude that the MITC3+ shell element shows, in the problems considered, the same excellent performance in geometric nonlinear analysis as already observed in linear analysis.

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The Enriched MITC3+ shell element will be submitted in *CMAME*

The enriched MITC3+ shell element with improved membrane responses

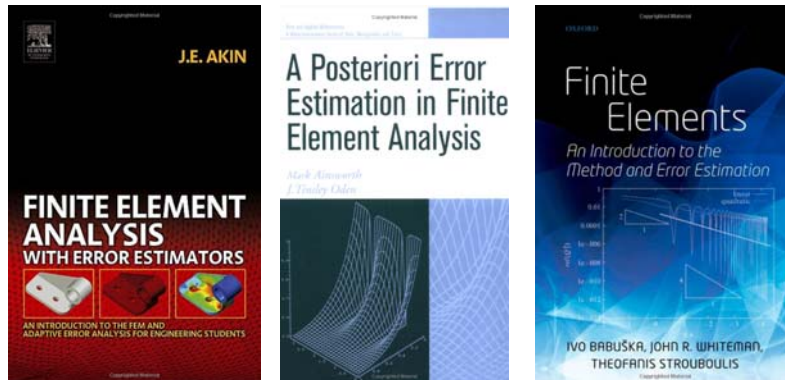
Hyeong-Min Jeon^a, Kyungho Yoon^a, Phill-Seung Lee^{a,*}

^a Division of Ocean Systems Engineering, Korea Advanced Institute of Science and Technology, 291 Daehak-ro, Yuseong-gu, Daejeon, 305-701, Republic of Korea

Abstract

In this paper, we develop a partition of unity based triangular shell finite element with improved membrane behaviors. The proposed shell element is based on the MITC3+ shell element which uses a cubic bubble function for the rotations and assumed shear strain fields to alleviate the transverse shear locking. In order to membrane behavior, the partition of unity approximation are applied only membrane part of the MITC3+ shell element. For geometric nonlinear analysis, the total Lagrangian formulation is employed allowing for large displacements and rotations. The present shell element passes the basic tests (the isotropy, patch and zero mode tests) and shows excellent convergence behavior in several benchmark problems.

Future works #1 – PU based finite element method with the FE error estimation



PROGRAMMING



IFEM

IFEM is a MATLAB software package containing robust, efficient, and easy-following codes for the main building blocks of [adaptive finite element methods](#) on unstructured simplicial grids in both two and three dimensions. Besides the simplicity and readability, sparse matrixization, an innovative programming style for MATLAB, is introduced to improve the efficiency. In this novel coding style, the sparse matrix and its operations are used extensively in the data structure and algorithms.

- A brief [Introduction](#).
- My lecture notes [Programming of Finite Element Methods in Matlab](#)
- Download the latest version from [ifem repository](#) on bitbucket.
- If you have installed Mercurial, you can

```
hg clone https://bitbucket.org/ifem/ifem
```

Integrating the PU based FEM with the error estimator

- Mesh generation
- Adaptive mesh refinement algorithm
- Error estimates
- Coupling method or indicator

Future works #2 – PU based finite element method with reduced integration

4-node partition of unity based finite shell element : *Linear dependency problem*

2D solid element :

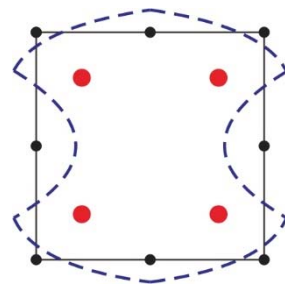
No locking treatments due to the high order interpolation

Shell element :

Absolutely needs to reduce the transverse shear locking even using the high order interpolation

- MITC4-MITC9-MITC16 – Too much computational cost
- MITC4-Reduced Integration – Effectively decrease in computational time
- Reduced Integration – Reduced Integration – Much faster than others

Hourglass mode



8-node serendipity element

$$\bar{e} = \underbrace{e}_{\substack{\text{MITC4} \\ \text{Reduced Integration}}} + \underbrace{\hat{e}}_{\substack{\text{MITC9 / MITC16} \\ \text{Reduced Integration}}}$$

Future works #3 – PU based shell element with high performance computing

*“Parallel programming **is not a trivial task** in most programming languages, and demands a great **theoretical knowledge** about the hardware architecture and good programming skills” - Neto DP-*



Linux Cluster Windows Cluster GPU Cluster WCU Cluster



Tesla Cluster

- 4-Tesla T10 Graphics Processing Units (GPUs)
- 960-cores (240 processor cores for each GPU)
- 16GB high speed memory (4GB for each GPU)
- Host CPU: Intel Xeon x64 QC Dual Processor
- Host RAM: 2G * 12ea = 24GB

Tesla Workstation (2 sets)

- 4-Tesla C1060 Graphics Processing Units (GPU)
- 960-cores (240 processor cores for each GPU)
- 16GB high speed memory (4GB for each GPU)
- Host CPU: Intel Xeon x64 QC Dual Processor
- Host RAM: 4G * 8ea = 32GB

Partition of unity based shell finite element

$$\bar{u}(\mathbf{x}) = u + \hat{u}$$

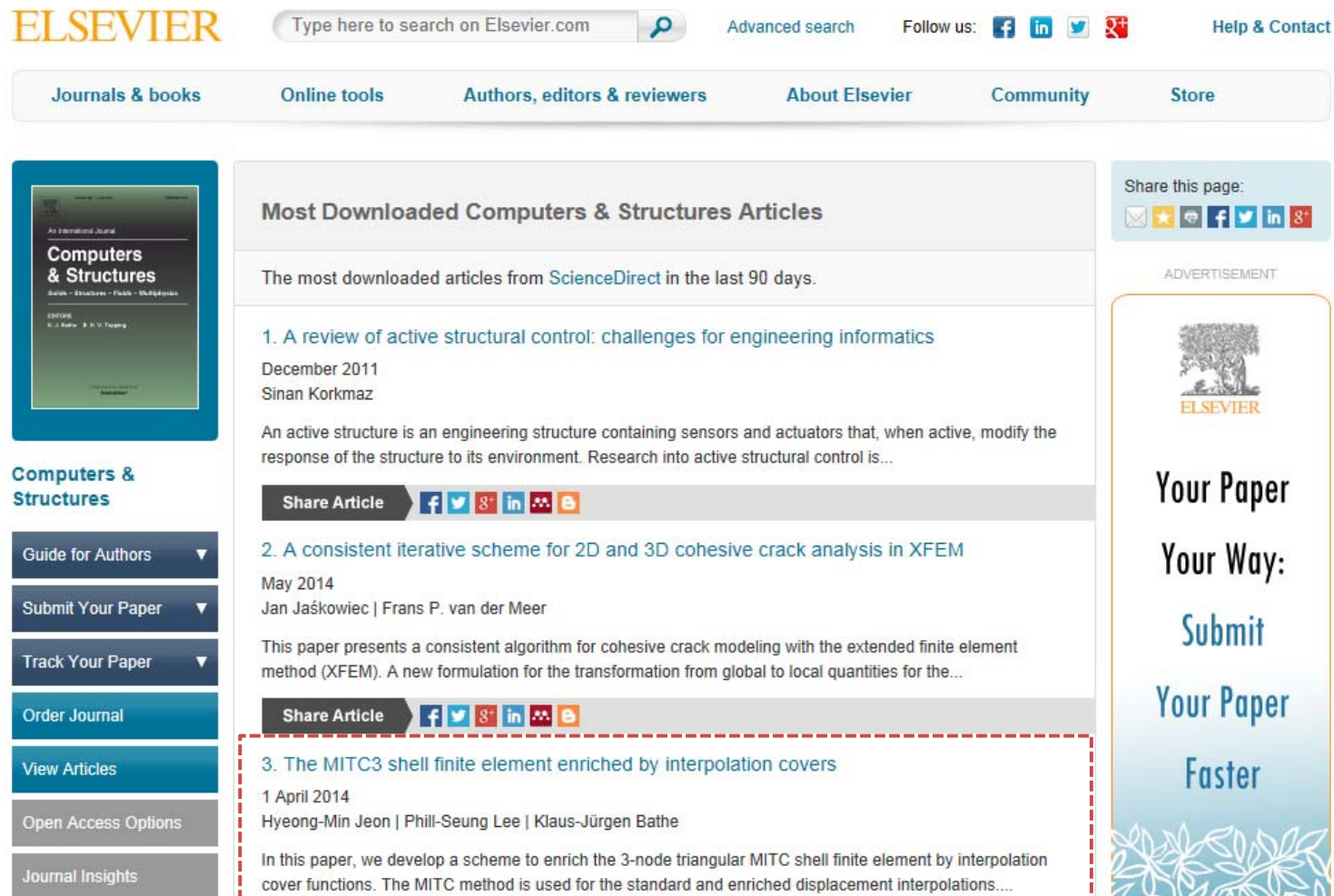
$$u = \sum_{i=1}^3 h_i u_i$$

$$\hat{u} = \sum_{i=1}^3 h_i (x - x_i) a_{2i} + h_i (y - y_i) a_{3i}$$

“The computing of each local matrix is **totally independent** and so parallelization of these computations is straightforward and can be naturally explored”

- We proposed two possibilities of using the partition of unity approximation
 - ✓ Increase solution accuracy without local mesh refinements (Part I)
 - ✓ Improvement membrane behaviors (part II)
- I. PU based MITC3 shell finite element (Enriched MITC3) is reviewed.
 - This element is obtained by applying linear displacement interpolation covers to the standard 3-node shell element and MITC procedures are used.
 - The method increases the solution accuracy **without any local mesh refinement**.
 - We can provide convenience for user by combining PU shell element and FE error estimates.
- II. PU based MITC3+ shell finite element (Enriched MITC3+) is proposed.
 - This shell element is based on the MITC3+ shell element and partition of unity approximations **to improve membrane response**.
 - Membrane behaviors of the triangular shell element efficiently is improved by partition of unity approximation.

Conclusion remarks – Most downloaded papers



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Sinan Korkmaz
An active structure is an engineering structure containing sensors and actuators that, when active, modify the response of the structure to its environment. Research into active structural control is...
- 2. A consistent iterative scheme for 2D and 3D cohesive crack analysis in XFEM**
May 2014
Jan Jaśkowiec | Frans P. van der Meer
This paper presents a consistent algorithm for cohesive crack modeling with the extended finite element method (XFEM). A new formulation for the transformation from global to local quantities for the...
- 3. The MITC3 shell finite element enriched by interpolation covers**
1 April 2014
Hyeong-Min Jeon | Phill-Seung Lee | Klaus-Jürgen Bathe
In this paper, we develop a scheme to enrich the 3-node triangular MITC shell finite element by interpolation cover functions. The MITC method is used for the standard and enriched displacement interpolations....

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Conclusion remarks – Frontier in the analysis of shells



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Conferences and Publications

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Front Page Conferences

- **CST2014**
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 - Sessions
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 - Visas
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 - Submission
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CST2014

The Twelfth International Conference on Computational Structures Technology

Naples, Italy
2-5 September 2014

Introduction

This is the Twelfth Conference in the Computational Structures Technology series that commenced in 1991. Previous venues for the conference have included: Budapest, Edinburgh, Prague, Leuven, Lisbon, Gran Canaria, Athens, Valencia and Dubrovnik. The conference is concerned with the application of the latest computational technology to structural mechanics and engineering. Computational Technology encompasses both the latest hardware and software developments as well as algorithmic and theoretical techniques.

Opening Lecture

The CST2014 Conference will be opened jointly with the ECT2014 Conference with a lecture presented by Professor K.J. Bathe of Massachusetts Institute of Technology, Cambridge MA, USA.

Frontiers in Finite Element Procedures & Applications⁽¹⁾

by

Klaus-Jürgen Bathe
Massachusetts Institute of Technology
Cambridge, MA 02139, USA

2 Frontiers in the analysis of shells

The analysis of shells has been pursued for decades and yet there are still important improvements needed in the effectiveness of shell elements. In practice, low-order elements are much preferred because of their ease of use in meshing, their robustness and computational efficiency, but a significant drawback is the rather low accuracy in the calculated stresses. The stress predictions can be improved by the 'stress improvement scheme' published by Payen and Bathe [5] and by the 'interpolation cover scheme' presented by Kim and Bathe [6, 7]. Both schemes were originally developed for the analysis of solids. In the following, we present the development of the interpolation cover scheme for a 3-node shell element, to obtain an enriched formulation [8], and we present a new more powerful 3-node shell element, the MITC3+ element [9]. Since this element formulation is based on the MITC technique, the extension to nonlinear analyses is directly achieved [10].

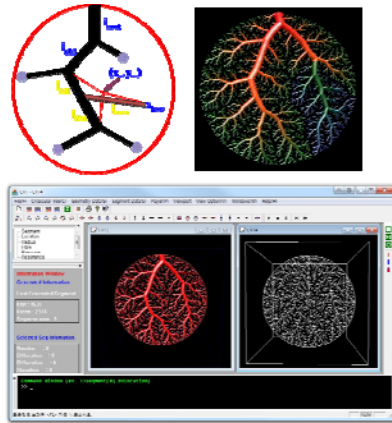
2.1 The use of interpolation covers for the MITC3 shell element

The geometry of the 3-node continuum mechanics based triangular shell finite element is interpolated using [4,11]

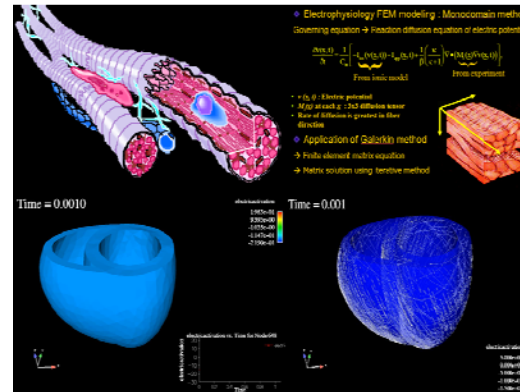
$$\mathbf{x}(r, s, t) = \sum_{i=1}^3 h_i(r, s) \mathbf{x}_i + \sum_{i=1}^3 \frac{t}{2} a_i h_i(r, s) \mathbf{V}_n^i \quad \text{with } h_1 = r, h_2 = s, h_3 = 1 - r - s \quad (1)$$

Previous research (2003 ~ 2012)

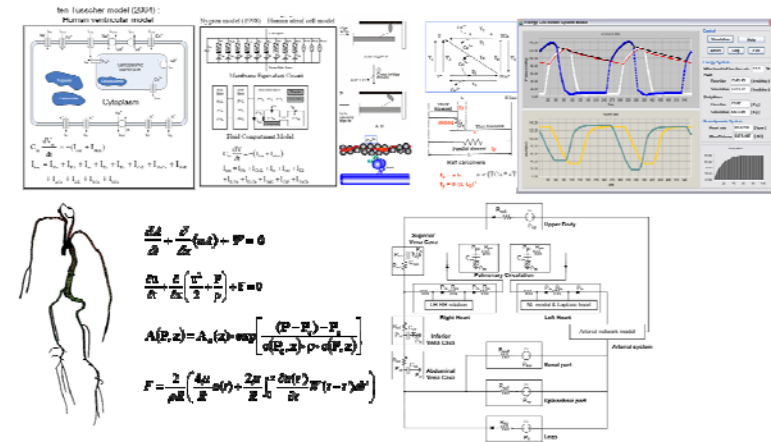
Coronary artery generation



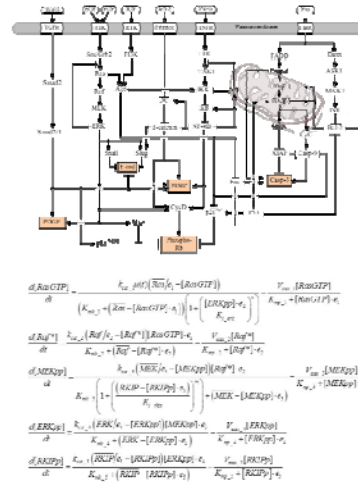
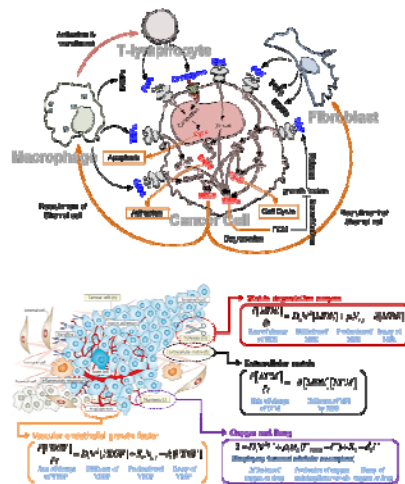
Virtual heart (NRL project)



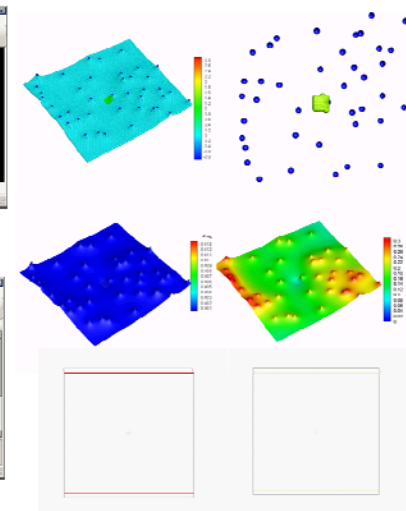
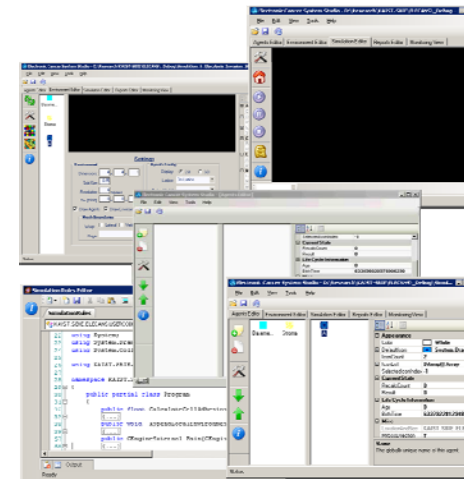
Arterial network with the hemodynamic model



Virtual cancer system (NRL project)



Angiogenesis and vascular tumor growth



Education & Publications

Education

- Ph.D. Student : Korea advanced Institute of Science and Technology (Sep 2010 - present)
- MS : Kangwon National University (Sep 2006 – Aug 2008)
(Thesis : Development of a cell-system coupled model of cardiovascular hemodynamics)
- BS : Kangwon National University (Mar 2000 – Aug 2006) (Summa Cum Laude & Early Graduation)

Experience

- URP program, Kangwon National University (Sep. 2003 ~ Aug 2006), Dep. of Mechanical Engineering
- Visiting Researcher, Kyoto University (Dec 2007- Mar 2008 and Apr 2009- Jun 2009), Graduate School of Medicine
- Visiting Researcher, MIT (Aug 2008 – Oct 2008), Dep. of Health Sciences and Technology
- Visiting Researcher, Oxford (Jul 2010- Aug 2010), Centre for Mathematical Biology
- Research Assistant, KAIST (Jan 2010 – Aug 2012), Dep. of Bio and Brain Engineering, Advisor : Prof. Kwang-Hyun Cho.

International Journals (published : 4)

1. Jeon HM, Yoon K, Lee PS. A partition of unity based triangular shell element with improved membrane response, In preparation
2. Lee Y, Jeon HM, Lee PS, Bathe KJ. On the behavior of the 3-node MITC triangular shell elements, In preparation.
3. Jeon HM, Lee Y, Lee PS, Bathe KJ. The MITC3+ shell element in geometric nonlinear analysis, Computers and Structures, 146, 91-104, Jan 2015.
4. Jeon HM, Lee PS, Bathe KJ. The MITC3 shell finite element enriched by interpolation covers, Computers and Structures, 134, 128-142, Apr 2014.
5. Shim EB, Jun HM, Leem CH, Matusuoka S, Noma A. A new integrated method using a cell-hemodynamics-autonomic nerve control coupled model of the cardiovascular system. Progress in Biophysics and Molecular Biology, 96(1-3), 44-59, Jan 2008.
6. Jun HM, Shim EB. Theoretical analysis of the cross-bridge sliding rate in modulating heart mechanics, International Journal of Vascular Biomedical Engineering, 5(2), 34-45, Oct 2007.

Presentation (27)

- Jeon HM, Yoon K, Lee PS. Development of the enriched MITC3 shell element. KSME Annual Conference, 192-193, Apr 2014.
-
- Computational study on the arterial tree generation based on blood volume optimization. KSME Annual Conference, Oct 2005.