다중 보 유한요소를 이용한 케이블의 모델링 방법 및 이를 이용한 케이블 설계

Modeling of helically stranded cables using multiple beam finite elements and its application to cable design

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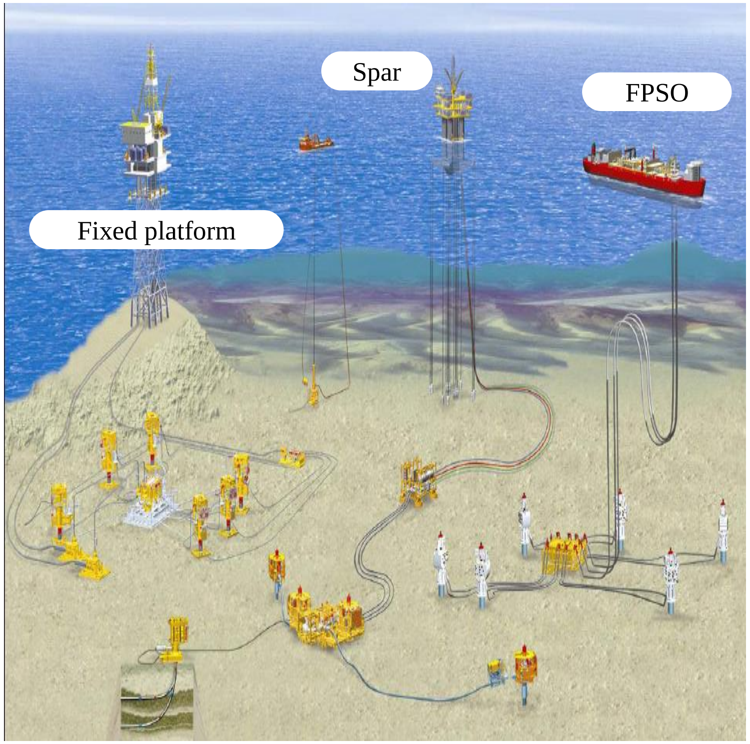
CMSS Computational Mechanics and Structural Systems.

Subsea cable

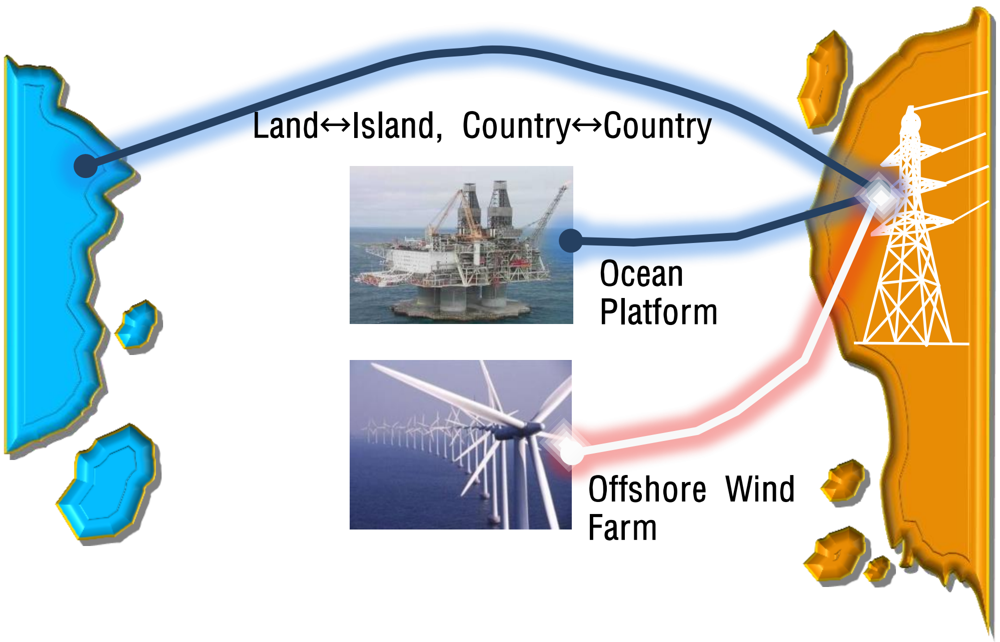
- **URF system :**

  Umbilical, Riser & Flowline



- **Power Transmission system :**

  Submarine Power Cable System

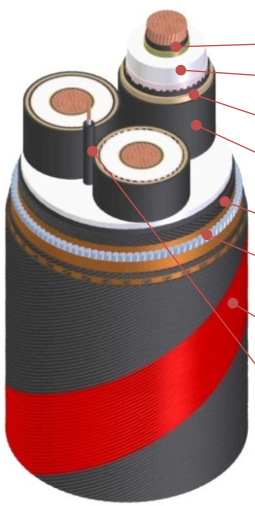


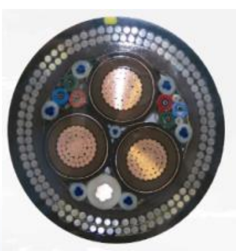
✓ **High system cost, High maintenance fee, High risk when breakdown**

→ Analysis of mechanical behavior & Robust design for cable are very important !!

Cable system

- **Cables are Complex Structures**
  - **A variety of geometries & materials**

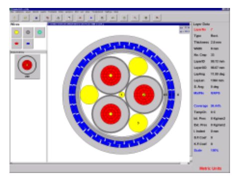
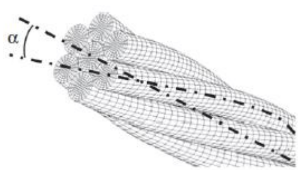
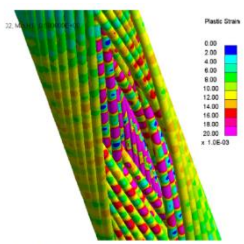




- **Complex Behavior of Cable**
  - **Geometrical nonlinearities**
    - **Large deformations**
  - **Material nonlinearities**
    - **Nonlinear stress-strain curves**
  - **Contact**
    - **Wire slip (effect of internal friction)**

- **Main sources for Cable design**
  - **Experiments**
    - **Expensive & difficult to conduct**
  - **Analytical or Numerical model**
    - **If sufficiently accurate,**
      it can be a good solution.

Analysis of cable mechanical behavior

List	Model	Features
AS IS	**Analytical model (Hruska, Costello, etc.)** - Wire theory - Considering stretch, twist & bending	**Only macro behavior is possible.** - Simple & easy to use - Validity is limited
	Ring Element (Knapp) - Cable CAD **Concise FEM (Jiang, Yao)** 	**Only 2D or Plain strain is possible.** - Not sufficient to solve 3D behavior **Simple beam model with contact**
	3D Solid Element (Judge, Yujie, Stanova) - Full 3D model  	**Great amount of time required.** - Accurate solution - Long solving time - Not easy to carry out parametric study - Element error occurs frequently
TO BE	**Cable FE model** - Multiple beam FE model	**Efficient & Easy to use** - Less computational time - Reasonable accuracy

Research topics

- **Suggestion of multiple beam FE models to predict efficiently the mechanical behavior of the cable**
- **Evaluation of the validity & limitation of analytical models using stiffness comparison method**
- **Proposition of the procedure of torque balance design & Verification of the design result**
- **Suggestion of the generalized torque balance curves**

Study of analytical solution

Mathematical relations of a helically stranded cable

- **Kinematic relations**

$$\begin{bmatrix} F_T \\ M_T \end{bmatrix} = \begin{bmatrix} K_{\varepsilon\varepsilon} \, K_{\varepsilon\theta} \\ K_{\theta\varepsilon} \, K_{\theta\theta} \end{bmatrix} \begin{bmatrix} \varepsilon \\ \delta\theta / h \end{bmatrix}$$

- Axial & shear strain of cable

$$\varepsilon = \frac{\delta h}{h} \qquad \gamma = r(\frac{\delta\theta}{h})\tan\alpha$$

- Axial strain of wire

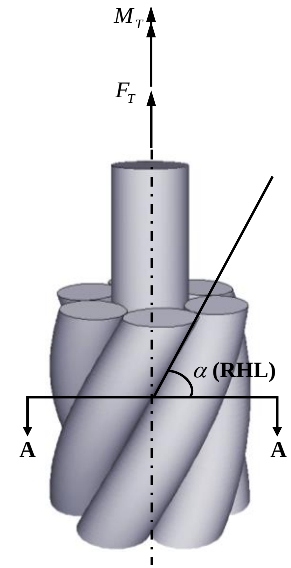
$$\varepsilon_w = \frac{\delta l}{l} = \varepsilon \sin^2\alpha + \gamma\cos^2\alpha$$

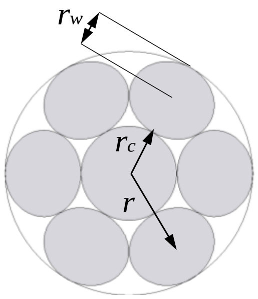
- Final curvature of wire

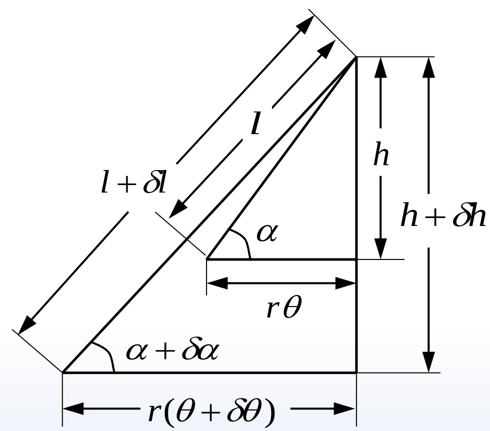
$$\kappa = \kappa_0 + \Delta\kappa = \frac{\cos^2\alpha}{r} - -\sin 2\alpha \sin\alpha\cos\alpha(\varepsilon - \gamma)$$

- Final twist of wire

$$\tau = \tau_0 + \Delta\tau = \frac{(\sin\alpha\cos\alpha)}{r} + \cos 2\alpha \sin\alpha\cos\alpha(\varepsilon - \gamma)$$







Mathematical relations of a helically stranded cable

- **Equilibrium eq. of wire**

$$\frac{dF_x}{ds} - F_y\tau + F_z\kappa_y + f_x = 0$$

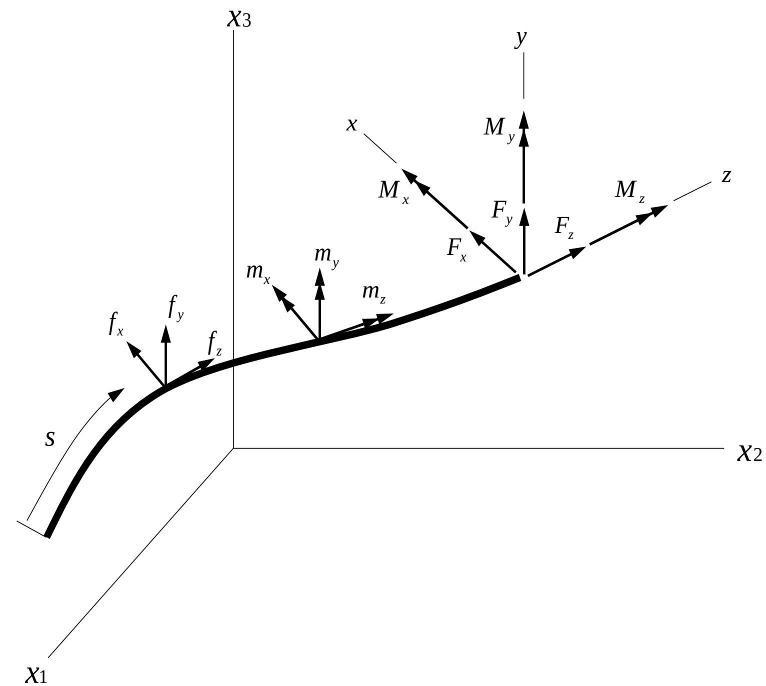
$$\frac{dF_y}{ds} - F_z\kappa_x + F_x\tau + f_y = 0$$

$$\frac{dF_z}{ds} - F_x\kappa_y + F_y\kappa_x + f_z = 0$$

$$\frac{dM_x}{ds} - M_y\tau + M_z\kappa_y + m_x = 0$$

$$\frac{dM_y}{ds} - M_z\kappa_x + M_x\tau + m_y = 0$$

$$\frac{dM_z}{ds} - M_x\kappa_y + M_y\kappa_x + m_z = 0$$



- Tension, twisting & bending can be simply expressed :

$$F_z = EA\varepsilon_w \qquad M_y = EI\Delta\kappa \qquad M_z = GJ\Delta\tau$$

$$F_y = M_z\kappa_y - M_y\tau$$ **(only external tension applied)**

Mathematical relations of a helically stranded cable

- **Equilibrium eq. of stranded cable**

$$F_T = n(F_z \sin\alpha + F_y \cos\alpha) + E_c A_c$$

$$M_T = n(M_z \sin\alpha + M_y \cos\alpha + F_z r \cos\alpha - F_y r \sin\alpha) + G_c J_c$$

- **Stiffness components of various analytical models**

- Hruska's model

$$K_{\varepsilon\varepsilon} = n(EA\sin^3\alpha) + E_c A_c$$

$$K_{\varepsilon\theta} = K_{\theta\varepsilon} = n(EAr\sin^2\alpha\cos\alpha)$$

$$K_{\theta\theta} = n(EAr^2\sin\alpha\cos^2\alpha) + G_c J_c$$

- Machida's model

$$K_{\varepsilon\varepsilon} = n(EA\sin^3\alpha) + E_c A_c$$

$$K_{\varepsilon\theta} = n(EAr\sin^2\alpha\cos\alpha)$$

$$K_{\theta\varepsilon} = n\left[\begin{array}{l} EAr\sin^2\alpha\cos\alpha + \dfrac{GJ\cos 2\alpha\sin^2\alpha\cos\alpha}{r} \\ -\dfrac{EI\sin 2\alpha\sin\alpha\cos^2\alpha}{r} \end{array}\right]$$

$$K_{\theta\theta} = n\left[\begin{array}{l} EAr^2\sin\alpha\cos^2\alpha - GJ\cos 2\alpha\sin^3\alpha \\ + EI\sin 2\alpha\sin^2\alpha\cos\alpha \end{array}\right] + G_c J_c$$

Mathematical relations of a helically stranded cable

- Costello's model

$$K_{\varepsilon\varepsilon} = n\left[EA\sin^3\alpha + \frac{GJ\cos 2\alpha\cos^4\alpha\sin\alpha}{r^2} + \frac{EI\sin 2\alpha\cos^3\alpha\sin^2\alpha}{r^2}\right] + E_c A_c$$

$$K_{\varepsilon\theta} = n\left[EAr\sin^2\alpha\cos\alpha - \frac{GJ\cos 2\alpha\cos^3\alpha\sin^2\alpha}{r} - \frac{EI\sin 2\alpha\cos^2\alpha\sin^3\alpha}{r}\right]$$

$$K_{\theta\varepsilon} = n\left[EAr\sin^2\alpha\cos\alpha + \frac{GJ\cos 2\alpha\sin^4\alpha\cos\alpha}{r} - \frac{EI\sin 2\alpha\cos^2\alpha\sin\alpha(1+\sin^2\alpha)}{r}\right]$$

$$K_{\theta\theta} = n\left[EAr^2\sin\alpha\cos^2\alpha - GJ\cos 2\alpha\sin^5\alpha + EI\sin 2\alpha\cos\alpha\sin^2\alpha(1+\sin^2\alpha)\right] + G_c J_c$$

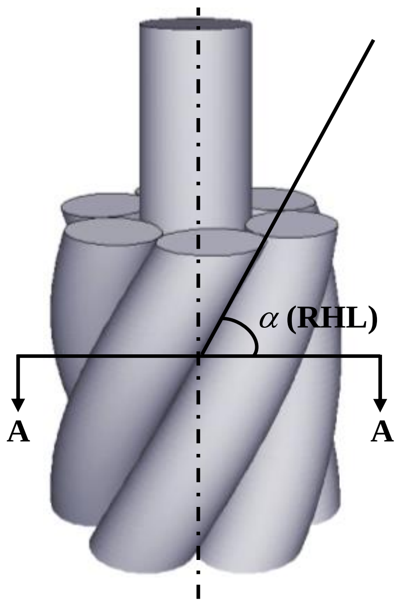
- **Analytical models with their principal features**

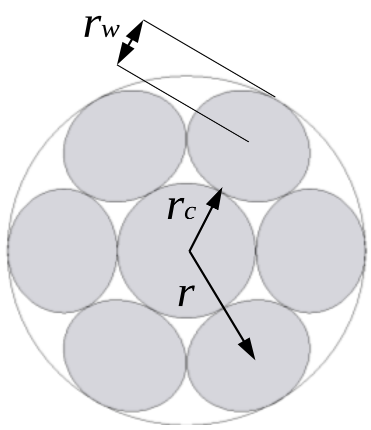
Model	Behavior of wires			Poisson's effect
	Tension	Torsion	Bending	
Hruska	O	×	×	×
Machida	O	O	O	×
Costello	O	O	O	O

Mathematical relations of a helically stranded cable

- **<u>Evaluation & Comparison of stiffness coefficients</u>**

- 7-wire helically stranded cable





Layer No.	No. of Wires	Helical direction & angle	Wire diameter [mm]	Pitch length [mm]	Young's Modulus [GPa]	Plastic Modulus [GPa]	Yield Strength [GPa]	Poisson's ratio
Core	1	-	3.94	-	188	24.60	1.54	0.3
1	6	RHL, 72.97°	3.72	78.67	188	24.60	1.54	0.3

Mathematical relations of a helically stranded cable

- **<u>Evaluation & Comparison of stiffness coefficients</u>**

Model	Helix angle [deg]	$K_{\varepsilon\varepsilon}$ [MN]	$K_{\varepsilon\theta}$ [MN-mm]	$K_{\theta\varepsilon}$ [MN-mm]	$K_{\theta\theta}$ [MN-mm^2]
Hruska	50	7.80	17.71	17.71	58.63
	60	10.26	17.61	17.61	40.65
	70	12.46	14.18	14.18	21.48
	80	14.00	7.91	7.91	7.05
Machida	50	7.80	17.71	16.71	61.57
	60	10.26	17.61	16.69	45.10
	70	12.46	14.18	13.49	27.08
	80	14.00	7.91	7.54	13.34
Costello	50	7.90	17.28	16.26	60.36
	60	10.30	17.33	16.40	42.74
	70	12.48	14.07	13.38	23.69
	80	14.00	7.89	7.53	9.22

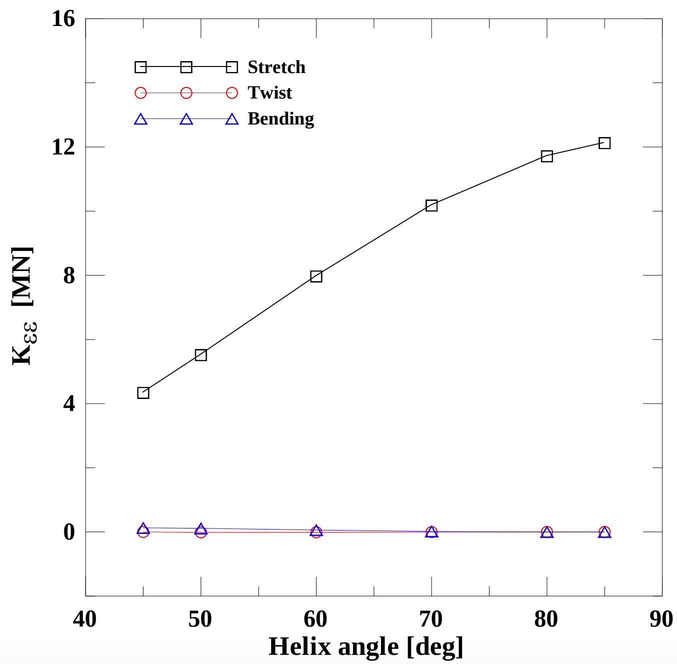
→ The difference in each stiffness coefficient is relatively small.

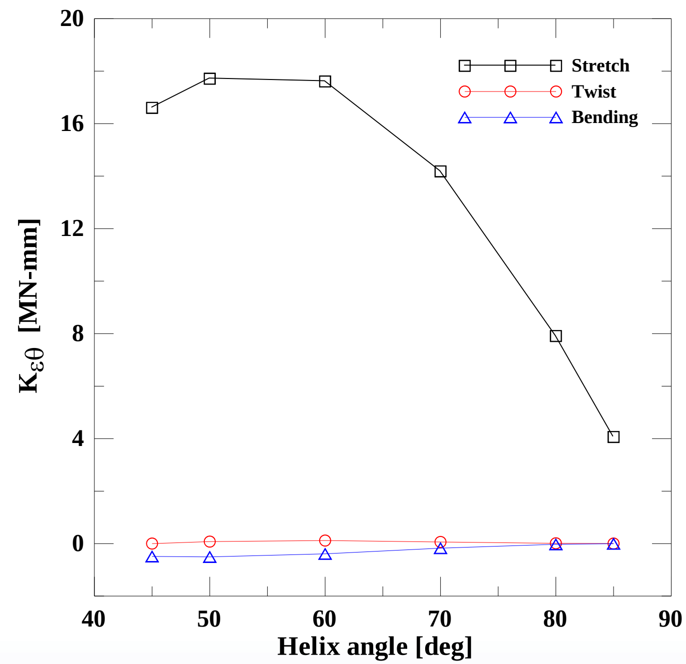
Mathematical relations of a helically stranded cable

- **<u>Evaluation & Comparison of stiffness coefficients</u>**

- Relative significance of individual contributions (for Costello's model)

$$K_{\varepsilon\varepsilon} = n\left[\underbrace{EA\sin^3\alpha}_{\text{Stretch}} + \underbrace{\frac{GJ\cos 2\alpha\cos^4\alpha\sin\alpha}{r^2}}_{\text{Twist}} + \underbrace{\frac{EI\sin 2\alpha\cos^3\alpha\sin^2\alpha}{r^2}}_{\text{Bending}}\right]$$



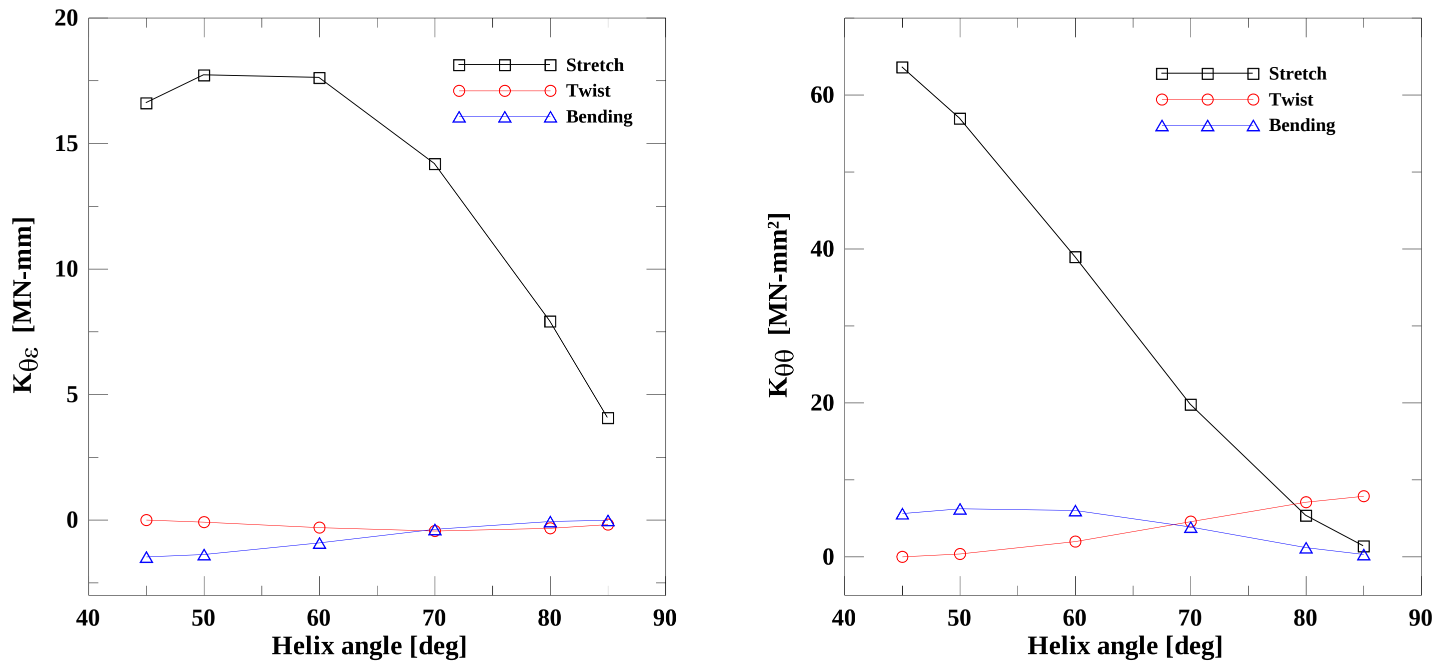


$$K_{\varepsilon\theta} = n\left[\underbrace{EAr\sin^2\alpha\cos\alpha}_{\text{Stretch}} - \underbrace{\frac{GJ\cos 2\alpha\cos^3\alpha\sin^2\alpha}{r}}_{\text{Twist}} - \underbrace{\frac{EI\sin 2\alpha\cos^2\alpha\sin^3\alpha}{r}}_{\text{Bending}}\right]$$

Mathematical relations of a helically stranded cable

- **<u>Evaluation & Comparison of stiffness coefficients</u>**

$$K_{\theta\varepsilon} = n\left[\underbrace{EAr\sin^2\alpha\cos\alpha}_{\text{Stretch}} + \underbrace{\frac{GJ\cos 2\alpha\sin^4\alpha\cos\alpha}{r}}_{\text{Twist}} - \underbrace{\frac{EI\sin 2\alpha\cos^2\alpha\sin\alpha(1+\sin^2\alpha)}{r}}_{\text{Bending}}\right]$$



$$K_{\theta\theta} = n\left[\underbrace{EAr^2\sin\alpha\cos^2\alpha}_{\text{Stretch}} - \underbrace{GJ\cos 2\alpha\sin^5\alpha}_{\text{Twist}} + \underbrace{EI\sin 2\alpha\cos\alpha\sin^2\alpha(1+\sin^2\alpha)}_{\text{Bending}}\right]$$

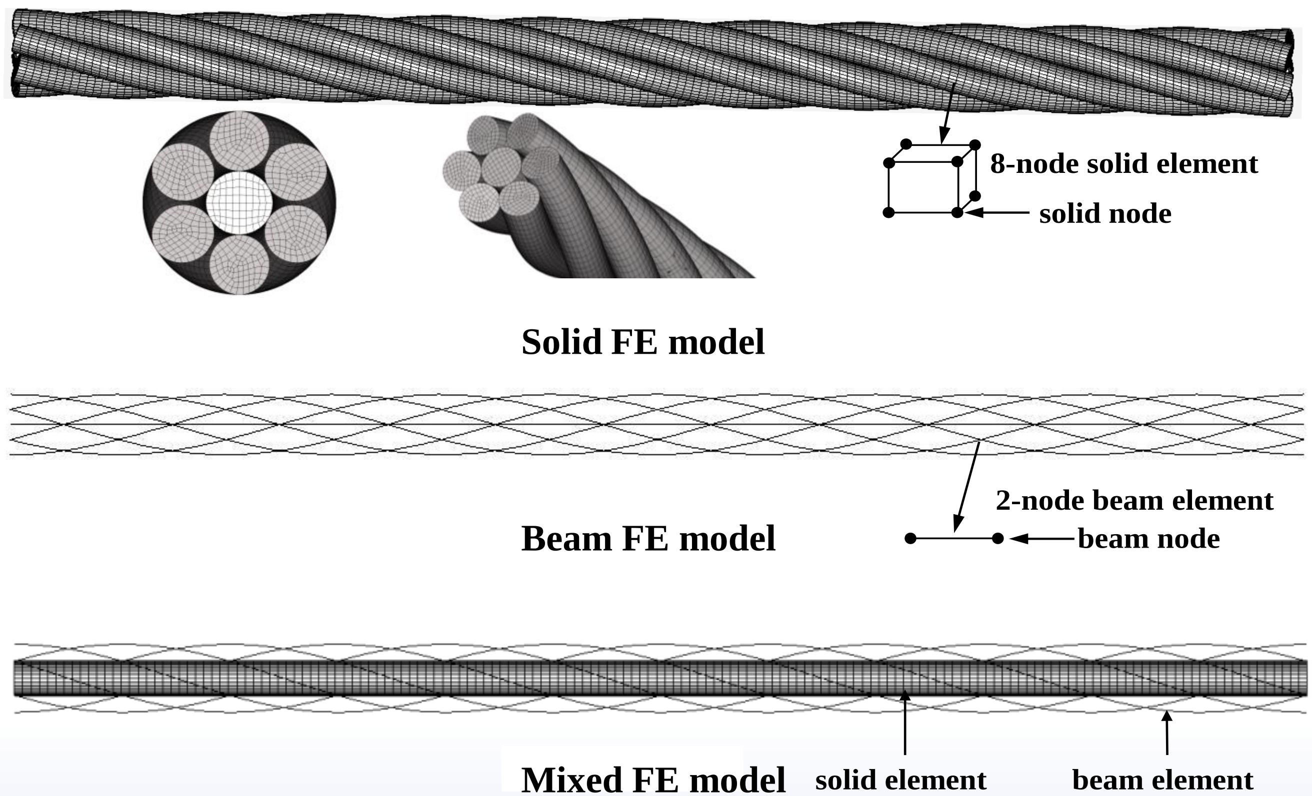
→ Stretch terms are dominant in every stiffness coefficient !!

Finite element models

Finite element models

- **<u>Finite element models for 7-wire helically stranded cable</u>**

- Three different model : 8-node solid FE model, 2-node beam FE model and mixed model (by commercial software CATIA & Marc)



Finite element models

- **<u>Boundary & Loading condition</u>**

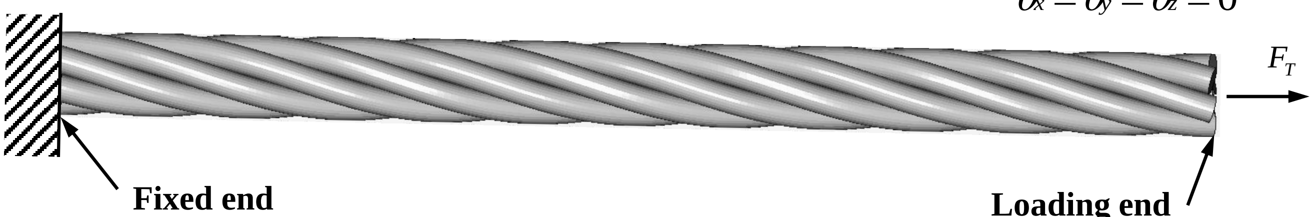
- Axial load case

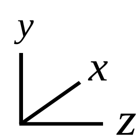
$u_x = u_y = u_z = 0$

$\theta_x = \theta_y = \theta_z = 0$

$u_x = u_y = 0,\ u_z \neq 0$

$\theta_x = \theta_y = \theta_z = 0$

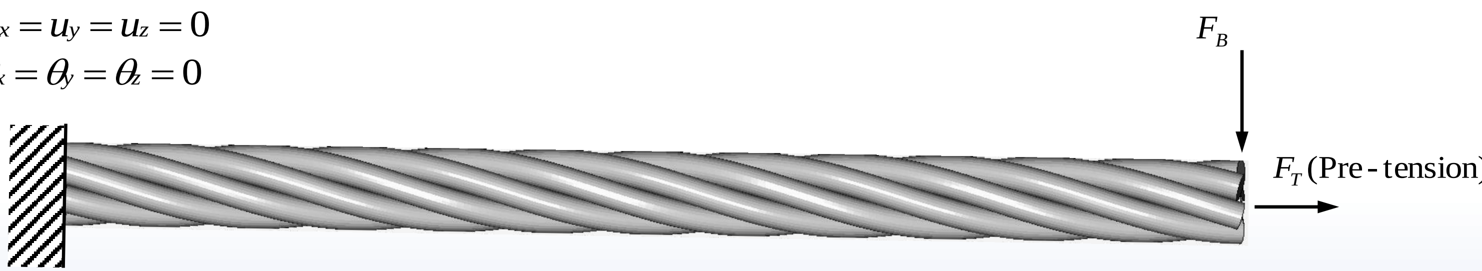




- Transverse load case

$u_x = u_y = u_z = 0$

$\theta_x = \theta_y = \theta_z = 0$



$u_x = 0,\ u_y \neq 0,\ u_z \neq 0$

$\theta_x = \theta_y = \theta_z = 0$

Finite element models

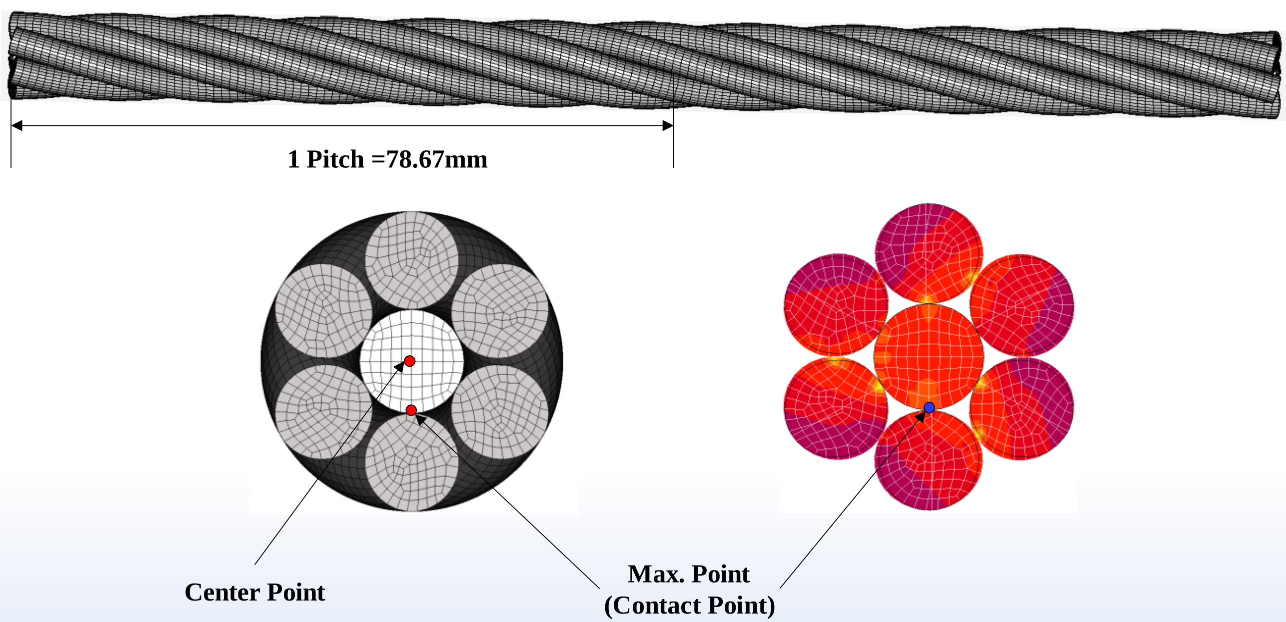
- **Full solid finite element model**

  - Extrusion of the wire cross-section along the helix corresponding to the centroid line

  - Construction of solid elements of each wire positioned around the core

  - To accurately capture the radial contact, a relatively large number (28 elements) of solid elements were used on the circumference direction.

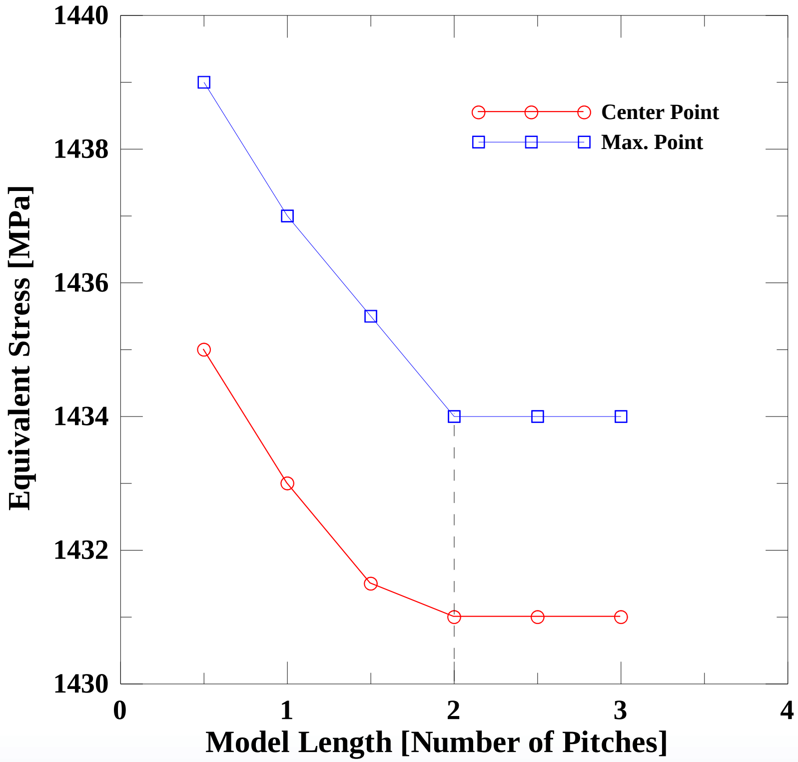
  - Convergence study was performed for model length and mesh refinements.



Finite element models

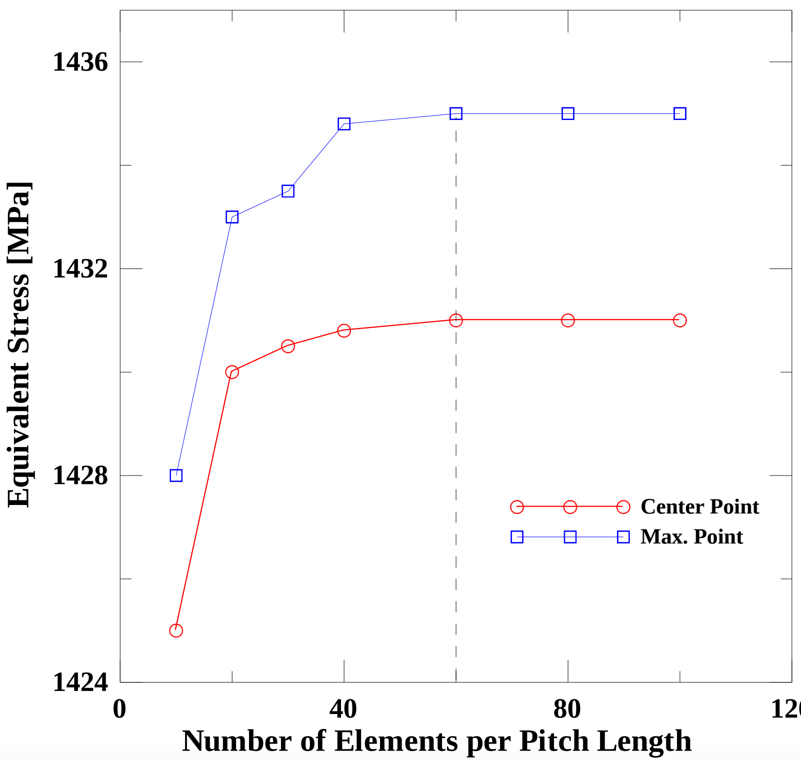
- <u>**Convergence study of solid FE model**</u>

- Model length



Min. 2 pitches

- Mesh refinements



Min. 60 elements per pitch

Finite element models

- **<u>Beam finite element model</u>**

- Beam to beam contact

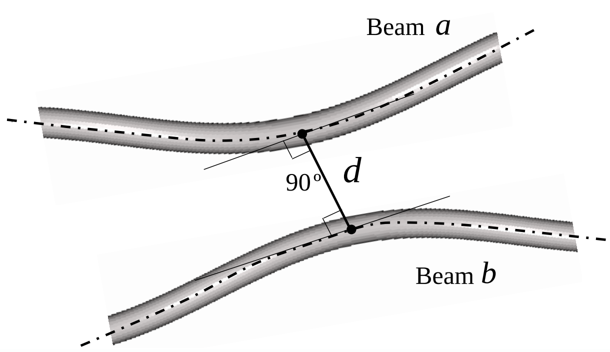
. Not only node to node, but node to element contact

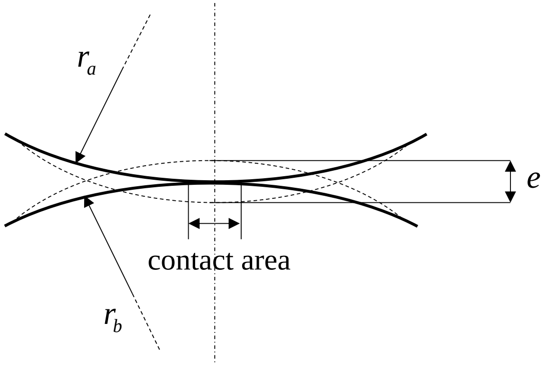
(algorithm creates extra node at contact points)

. Coulomb friction model with a friction coefficient of 0.115

. The friction contact conditions between the wires and between the wires and the core

- Contact condition of circular beam

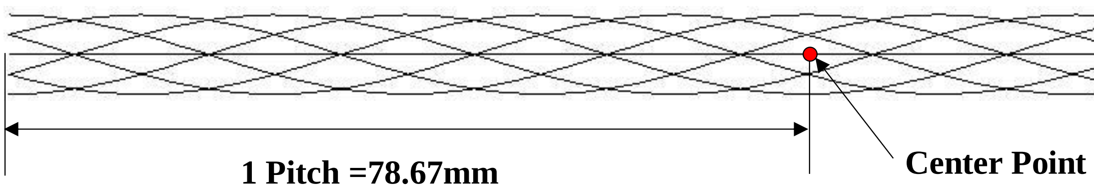




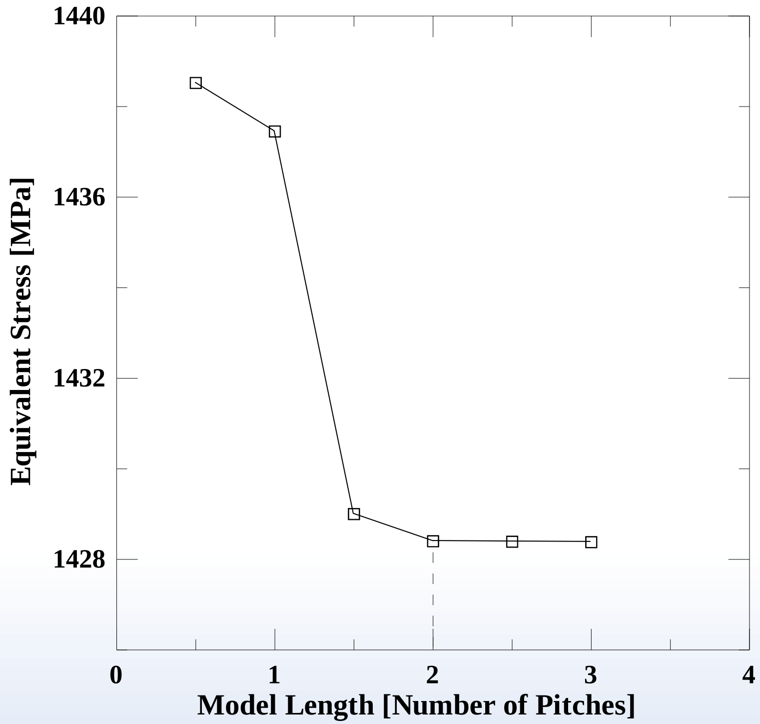
Contact penetration function : $g = d - r_a - r_b + e \leq 0$

Finite element models

- **Convergence study of beam FE model**

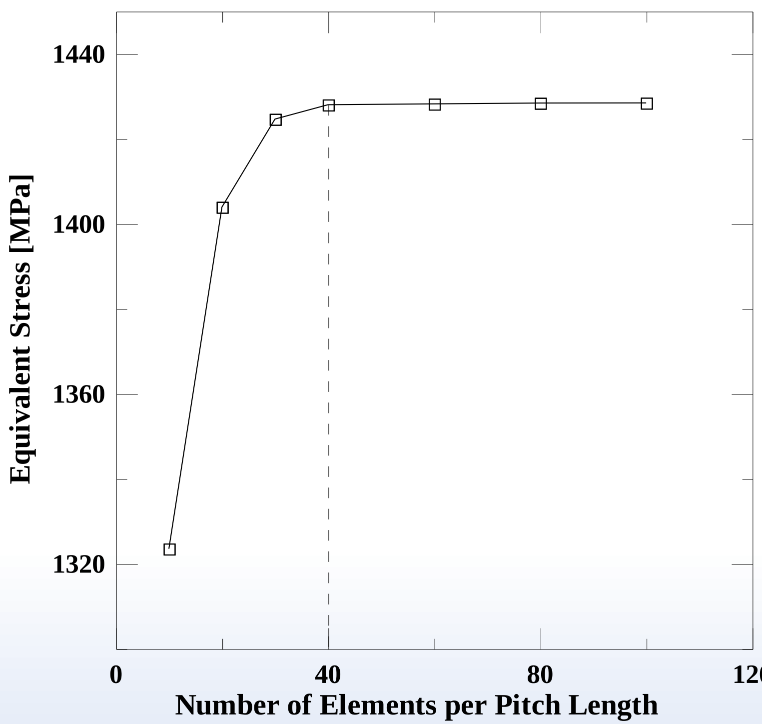


- Model length



Min. 2 pitches

- Mesh refinements

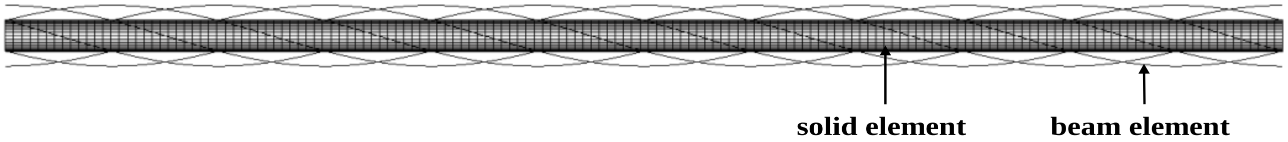


Min. 40 elements per pitch

Finite element models

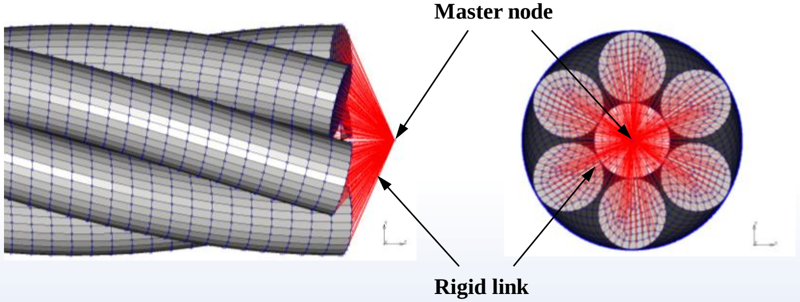
- **Mixed finite element model**

  - Obtaining of the advantages of both the solid FE and beam FE models
  - Consideration of the contact between the solid elements and the beam elements
  - Solid elements : 11,932, Beam elements : 942 , DOFs : 48,825



- **Special B.C : Master node & Rigid link**

  - Loads are applied at the master node and then conveyed to the whole section.
  - The end effects can be greatly reduced.



Result and Discussion

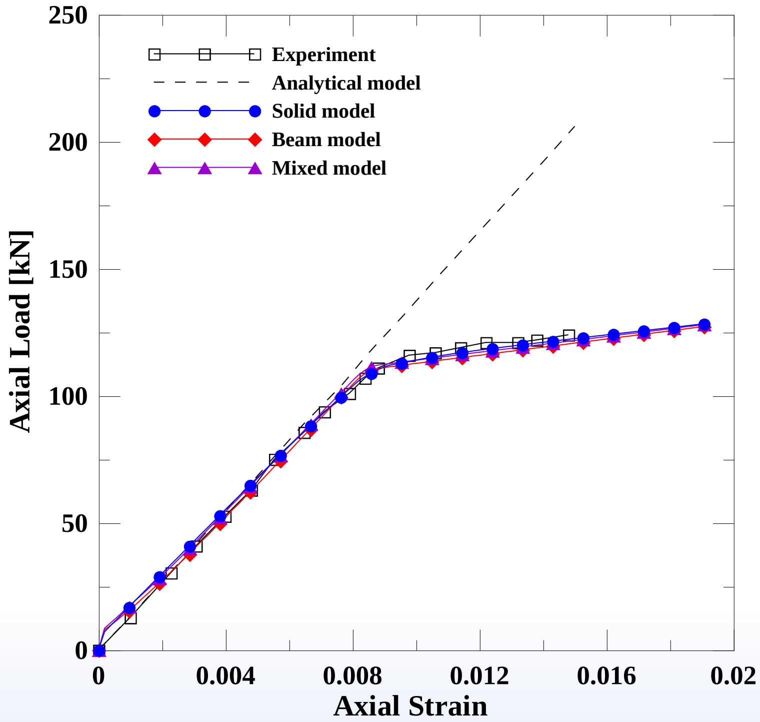
Results and discussion

- **Axial-loading analysis**

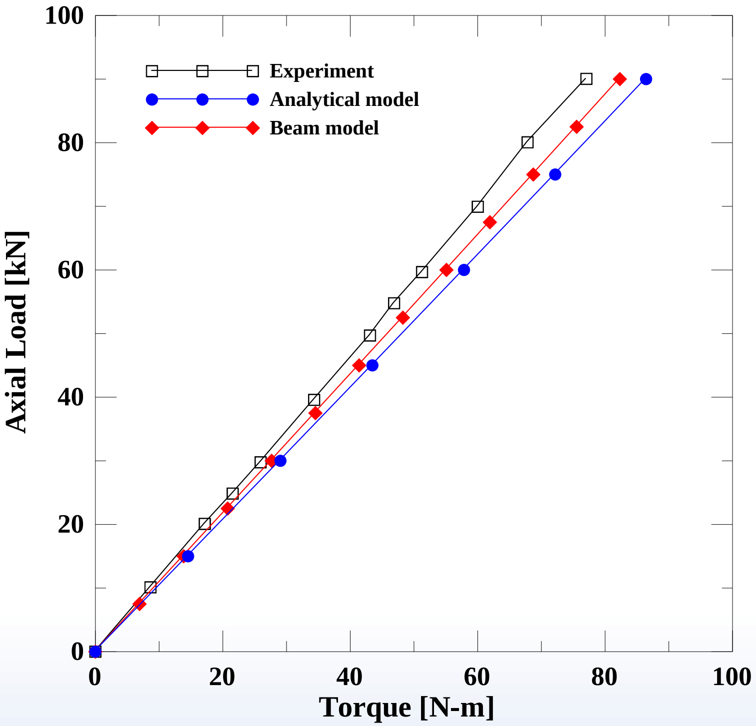
- All of the FE models are in sound agreement with the experimental results.

(Analytical results agreed with experimental results only in the linear-elastic range)

- Axial load/strain curves



- Axial load/torque curves



Results and discussion

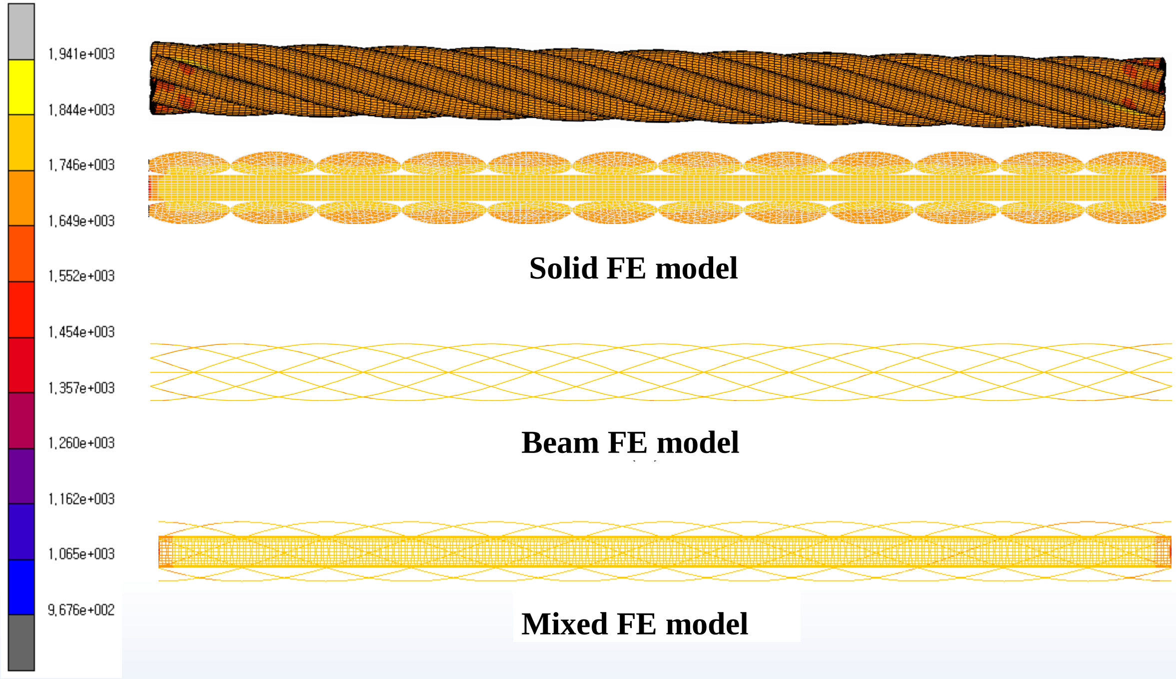
- **<u>Von-Mises stress contours under axial loading</u>**

- The maximum stress occurs in the core (first at yield & the ultimate stress).

: Core is subjected to both axial and transverse contact stress.

: Wires bear less stress because of unwinding.

- Beam model and mixed model have a good agreement with 3D solid model



Von-Mises stress @ ε=0.019, all of wires already yielded (Sy=1540MPa)

Results and discussion

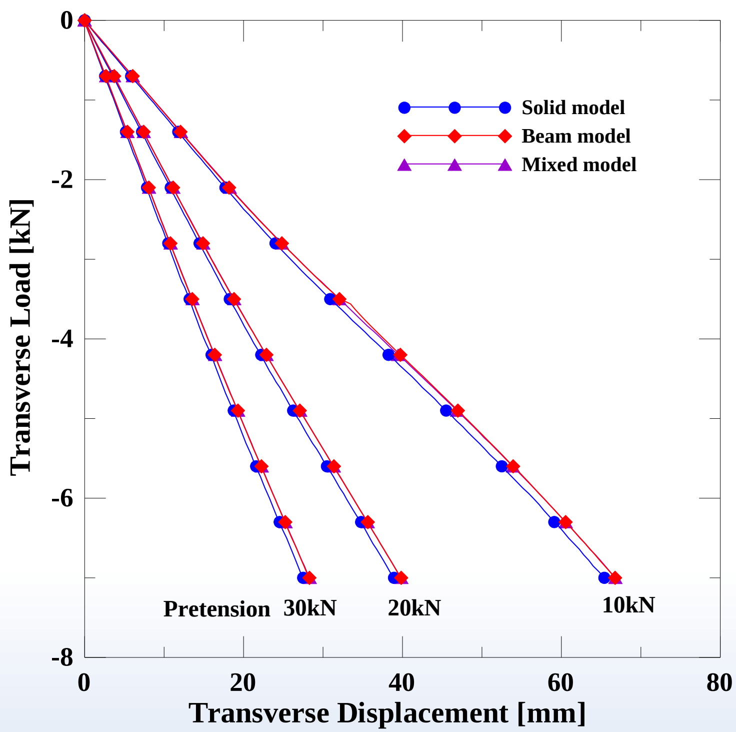
- **Transverse-loading**

- Three different pre-tensions : 10kN, 20kN, and 30kN. The transverse loading : up to 7kN

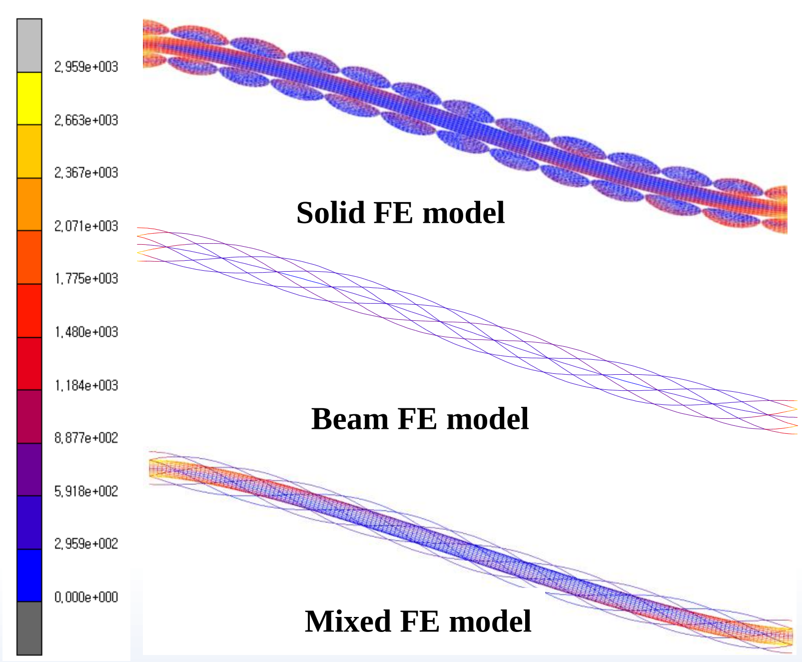
- Almost same results are obtained with all FE models from load/displacement curves.

- The global stress level of beam model is similar with the other FE models (except boundary end).

- Transverse load/displacement curves



- Von-Mises stress (pretension 20kN)



Results and discussion

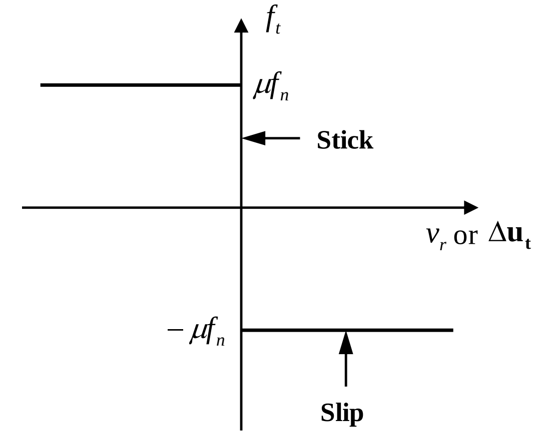
- **Influence of friction**

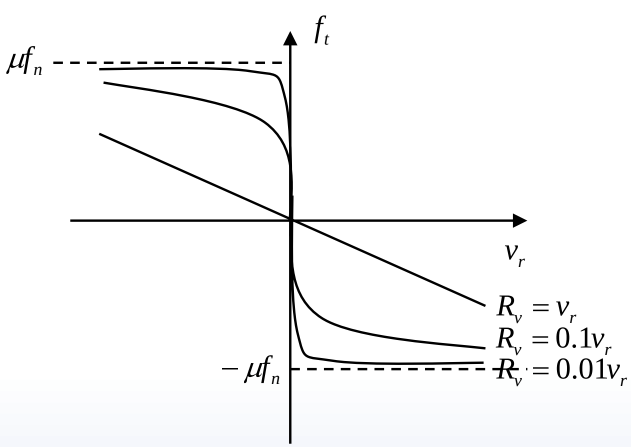
- Influence of friction model (Coulomb friction)

$\| f_t \| < \mu f_n$ **: stick** $\quad\quad$ $f_t = -\mu f_n \mathbf{t}$ **: slip**

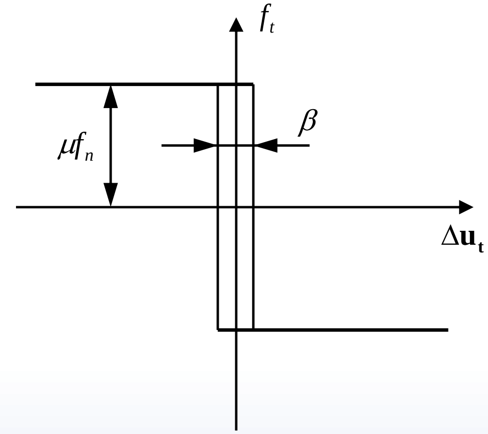
f_t **: tangential force** $\quad$ f_n **: normal force** $\quad$ μ **: friction coefficient**

$\mathbf{t}$ **: tangential direction vector**

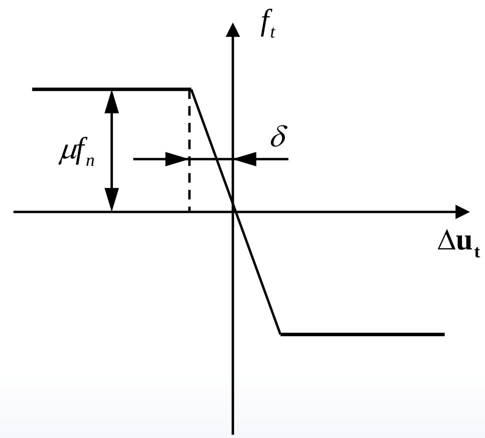




Arctangent model



Stick-slip model

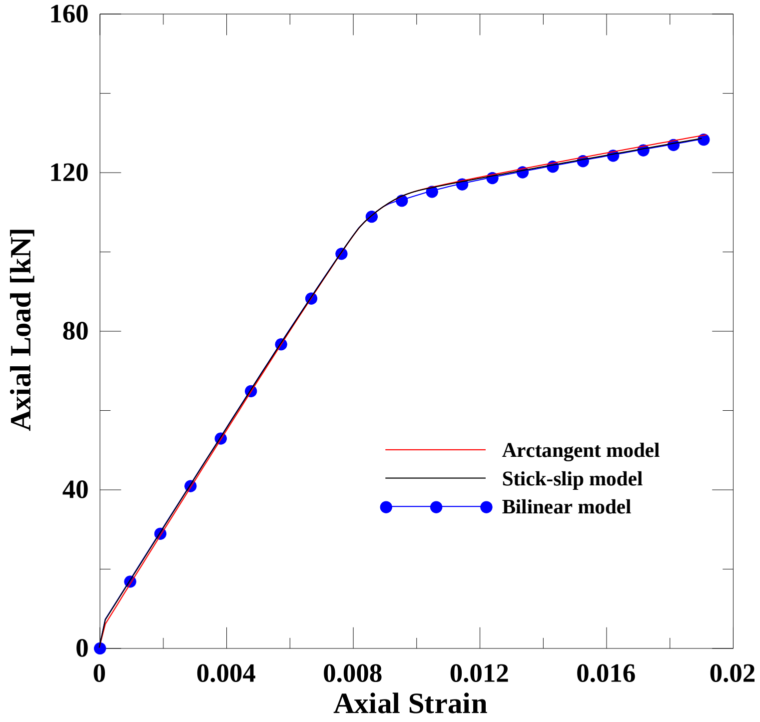


Bilinear model

Results and discussion

- **<u>Influence of friction</u>**

- Axial load/strain



- The global response is almost the same.
- The influence of friction models on the global behavior of the cable is small.
- The bilinear friction model is an effective in terms of computational efficiency.

- Computation time with friction model

Friction model	Arctangent model	Stick-slip model	Bilinear model
Computation time [sec]	26,308 [123%]	23,916 [112%]	21,450 [100%]

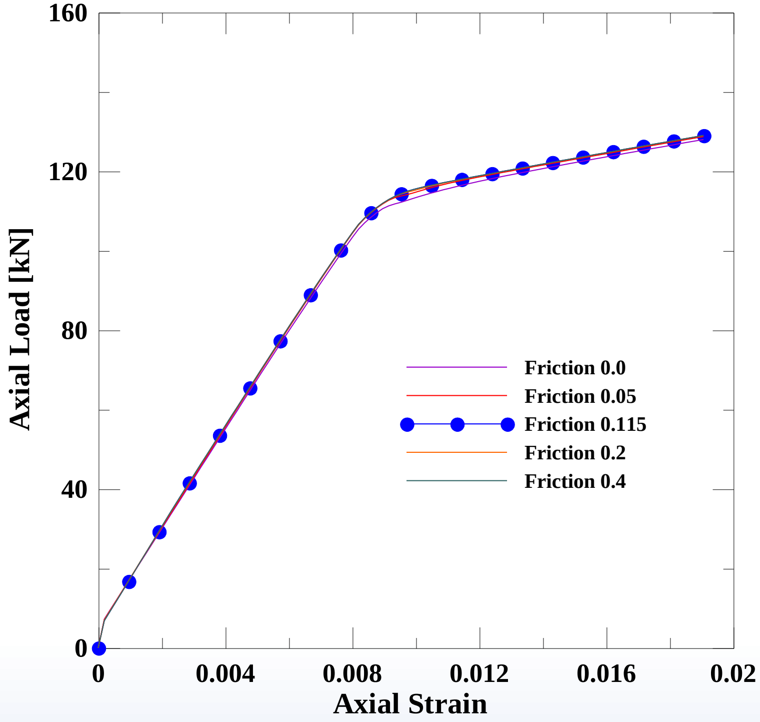
Results and discussion

- <u>**Influence of friction coefficients (under axial load)**</u>

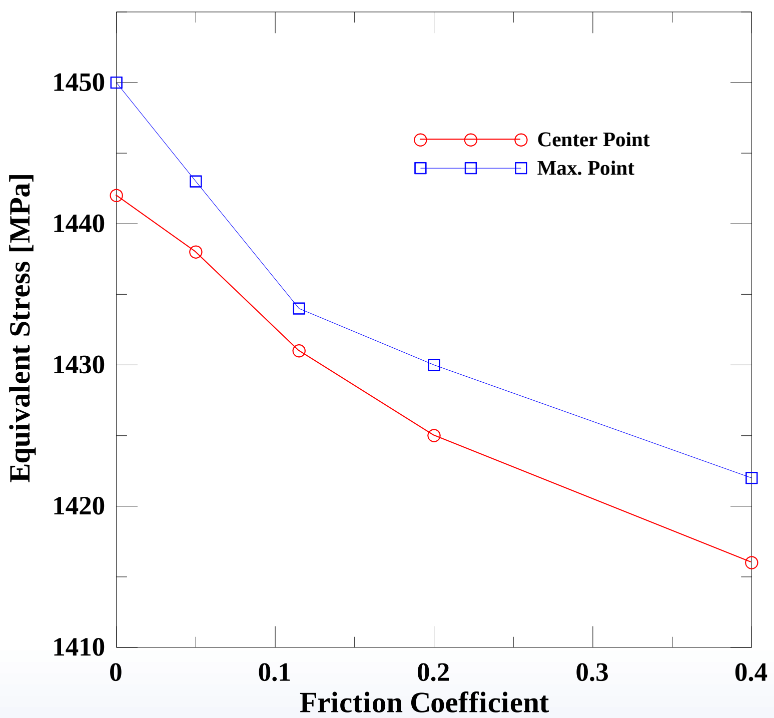
- The influence of friction coefficient on the global behavior of the cable is small.

- The friction coefficient increases, the equivalent stresses decrease.

- Axial load/strain curves



- Von-Mises stress

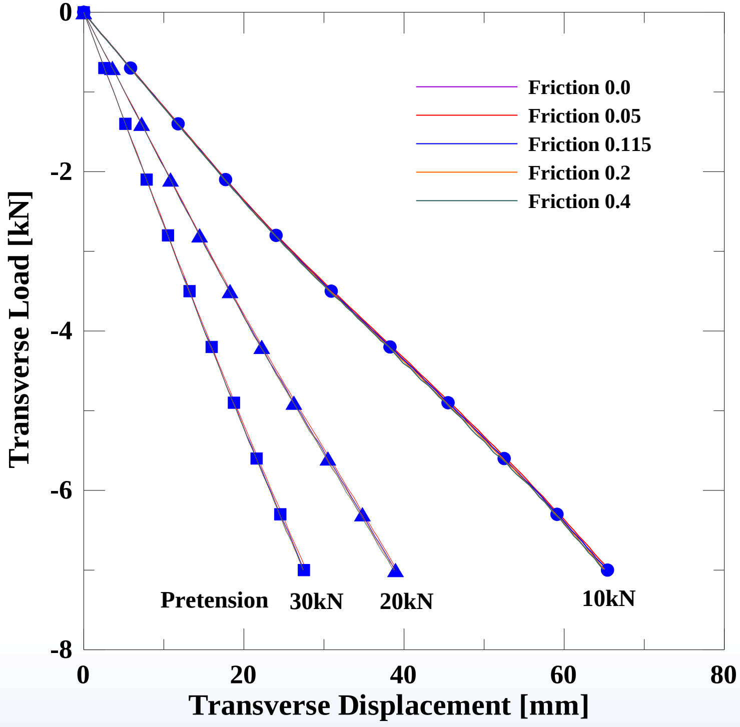


Results and discussion

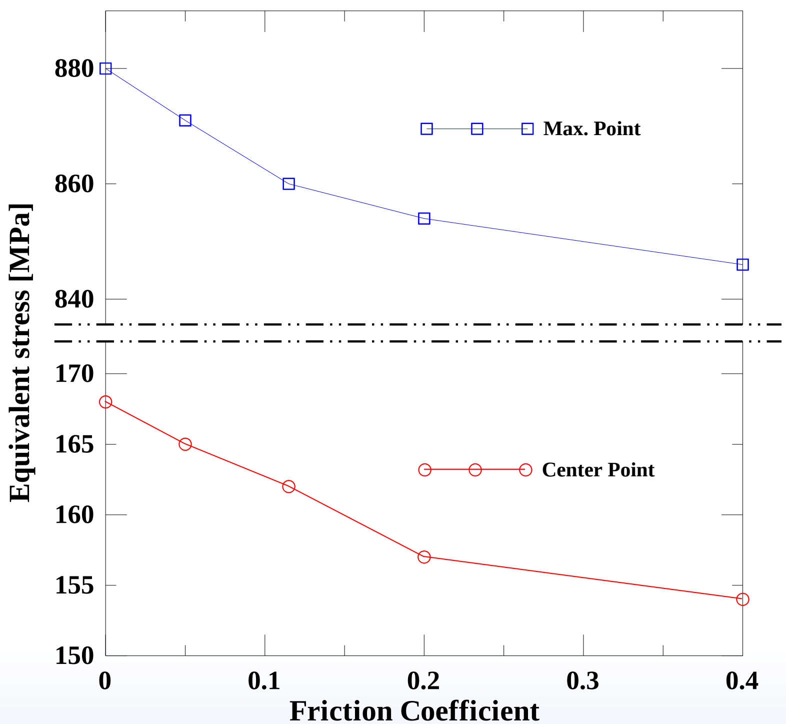
- **Influence of friction coefficients (under transverse load)**

- The friction effect plays a small role in global behavior.

- Axial load/strain curves



- Von-Mises stress



Results and discussion

- **Remarks on computational cost and accuracy**

- Computational time

FE model	Number of Elements	Number of DOFs	Computation time [sec]	
			Axial load	Transverse load
Solid model	96,556 [100%]	343,104 [100%]	21,450 [100%]	13,445 [100%]
Beam model	1,099 [1.1%]	6,636 [1.9%]	999 [4.7%]	1,121 [8.3%]
Mixed model	12,874 [13.3 %]	48,825 [14.2 %]	3,193 [14.9 %]	1,745 [13.0 %]

- Equivalent stresses (Von-Mises stress)

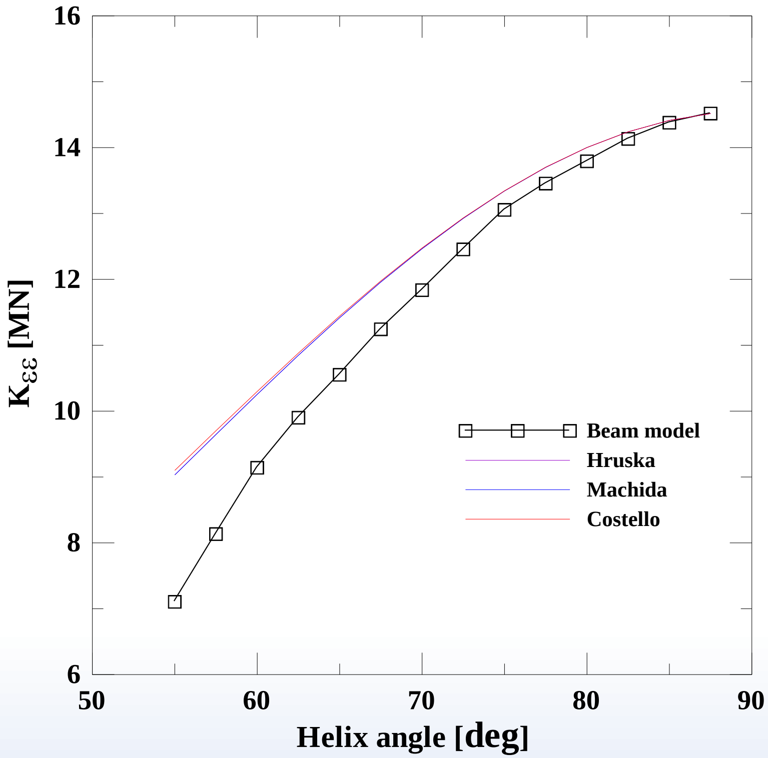
FE model	Center point [MPa]		Maximum point [MPa]	
	Axial load [100 kN]	Transverse load [7 kN]	Axial load [100 kN]	Transverse load [7 kN]
Solid model	1431	162	1434	860
Beam model	1428	159	1428	851
Mixed model	1431	161	1433	858

Results and discussion

- **Validity & Limitation of analytical solutions**

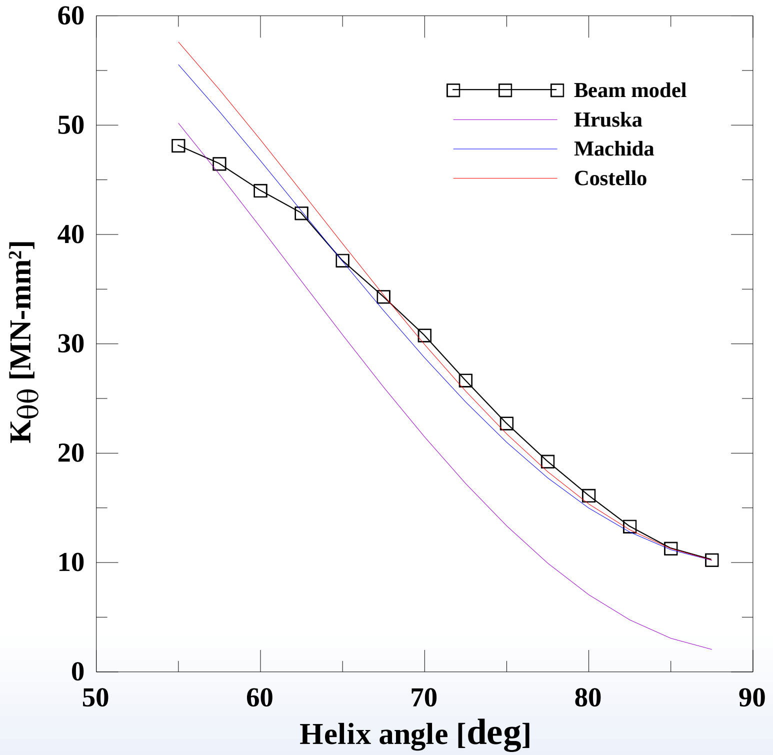
$$\begin{bmatrix} F_T \\ M_T \end{bmatrix} = \begin{bmatrix} K_{\varepsilon\varepsilon} \, K_{\varepsilon\theta} \\ K_{\theta\varepsilon} \, K_{\theta\theta} \end{bmatrix} \begin{bmatrix} \varepsilon \\ \delta\theta / h \end{bmatrix}$$

$F_T \neq 0 \quad \delta\theta = 0$



$\alpha > 75^\circ$ **(within 2.2%)**

$M_T \neq 0 \quad \delta\varepsilon = 0$

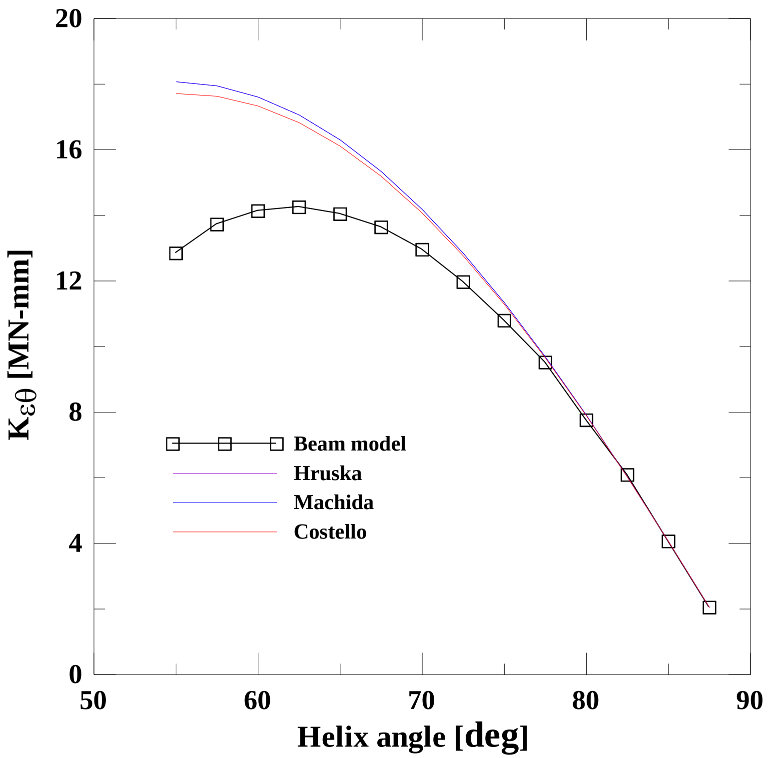


$\alpha > 62.5^\circ$ **(except Hruska's model)**

Results and discussion

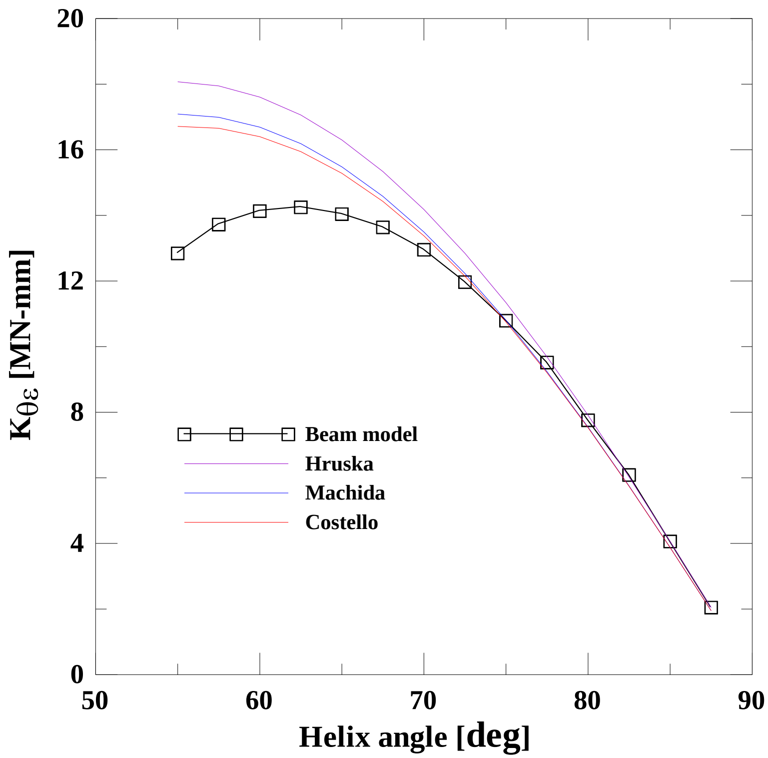
- **<u>Validity & Limitation of analytical solutions</u>**

$F_T \neq 0 \quad \delta\theta \neq 0$



$\alpha > 75^\circ$ **within 5%**

$M_T \neq 0 \quad \delta\varepsilon \neq 0$



$\alpha > 70^\circ$ **within 5%**

Torque balance design

Torque balance design

- **<u>Torque balance design</u>**

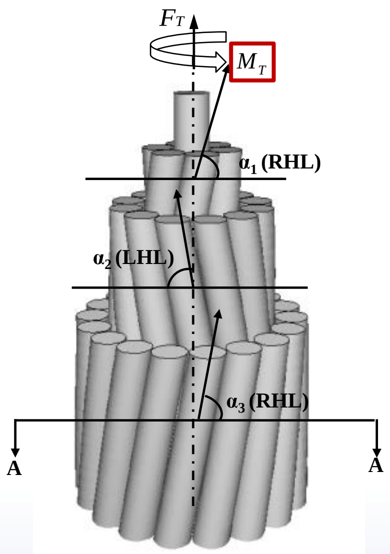
  - During axial loading, the helical wires induce a twisting of the stranded cable.

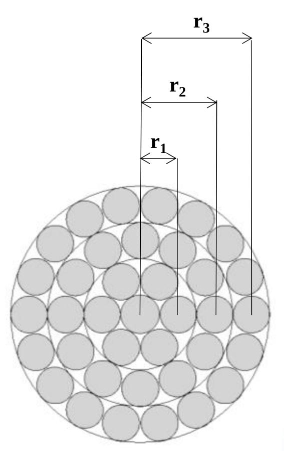
  - Cable twisting may loosen some wires and tighten others depending on the helix directions.

  (Some layers are stressed at higher levels and the breaking strength is considerably reduced.)

  – Slight relaxations of the cable tension can result in hockling (looping) due to the instability.

  → Method to prevent : external torque, minimization of twisting in design stage





<u>Design method</u>
Finding the proper helical angles to make "0" twisting

Torque balance design

- **Concept of torque balance**

$$\begin{bmatrix} F_T \\ M_T \end{bmatrix} = \begin{bmatrix} K_{\varepsilon\varepsilon} & K_{\varepsilon\theta} \\ K_{\theta\varepsilon} & K_{\theta\theta} \end{bmatrix} \begin{bmatrix} \varepsilon \\ \delta\theta / h \end{bmatrix} \qquad \delta\theta / h = 0 \qquad M_T = 0 \qquad \sum K_{\theta\varepsilon} = 0$$

- **Previous study for torque balance**

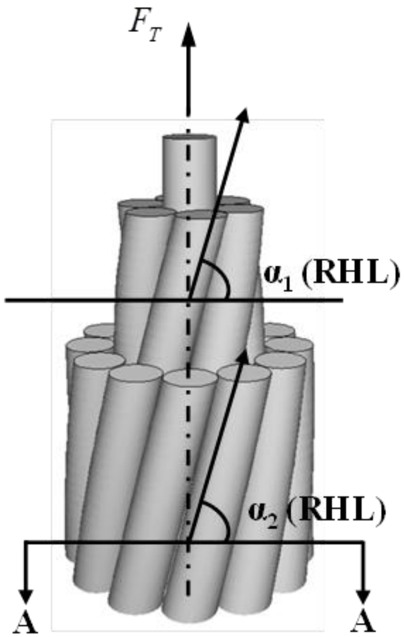
Liu F.C.	R.P. Christian	R.H. Knapp & Shibu G.
Concept of Torque ratio $R_t = \dfrac{n_o d_o^2 r_o \sin\alpha_o}{n_i d_i^2 r_i \sin\alpha_i}$	$T_1 = A_1 E_1 \varepsilon_{w_1} r_1 \sin\alpha_1$ $T_2 = A_2 E_2 \varepsilon_{w_2} r_2 \sin\alpha_2$	**Torque balance curve** $\sum K_{\theta\varepsilon} = \sum_{i=1}^{n} \dfrac{A_i E_i r_i}{L_c} \sin\alpha_i \cos^2\alpha_i = 0$

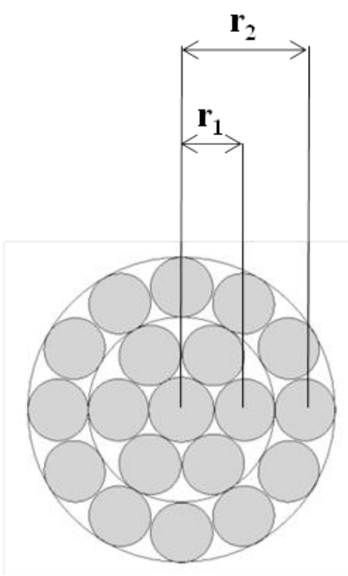
✓ **Torque balance design can be used in the same manner regardless of the static or dynamic condition**

Torque balance design

- **Two layer cable**

- Geometry





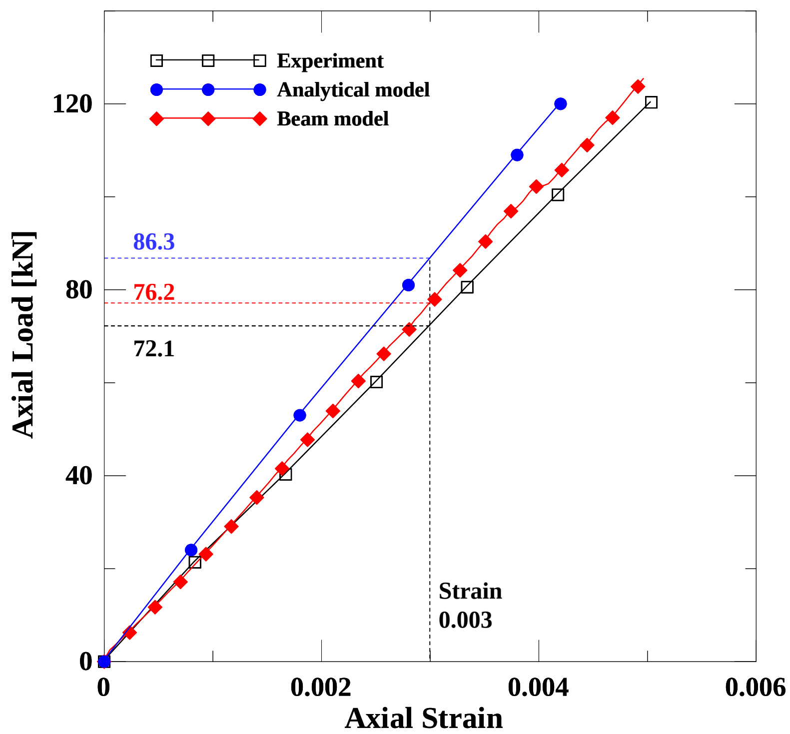
- Geometric and material properties

Layer No.	No. of wires	Helical angle [α]	Wire Diameter [mm]	Pitch length [mm]	Model Length [mm]	Young's Modulus [GPa]	Poisson's ratio
Core	1	-	3.66	-	334.56	198	0.3
1	6	RHL, 75.37°	3.33	84.11	334.56	198	0.3
2	12	RHL, 75.9°	3.33	167.28	334.56	198	0.3

Torque balance design

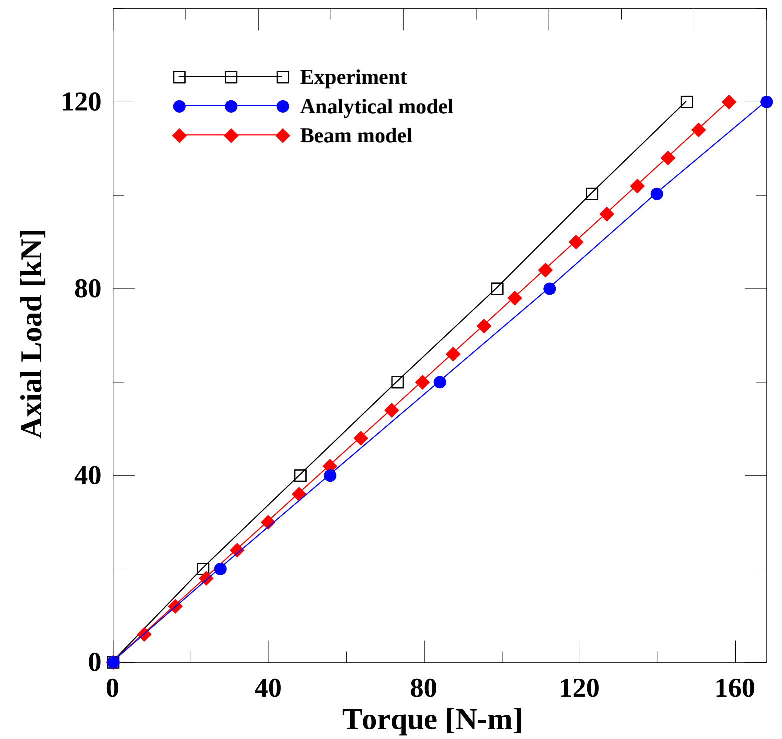
- <u>**Two layer cable**</u>

- Axial load/axial strain curves



- The differences from the experimental results are 20 % and 5.7 % for the analytical model and the beam FE model, respectively.

- Axial load/torque curve

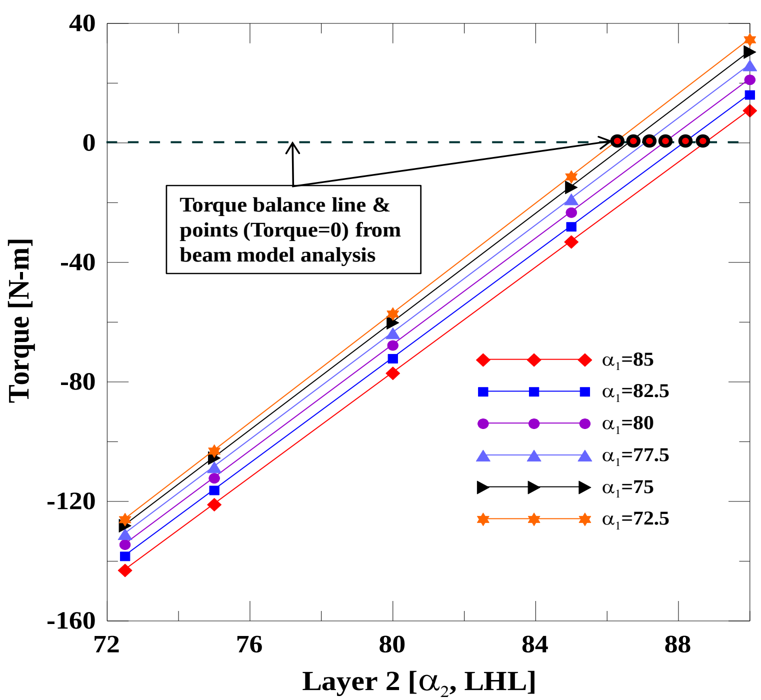


- Cable produces a net torque of 140 N-m at a working load of 120 kN ; that is, the torque in this cable is not balanced.

Torque balance design

- **Two layer cable**

- Torque balance points



- Torque balance design procedure

[Step 1] Select helix angles of each layer (α_1 and α_2)

[Step 2] Perform torque analyses, find zero torque points and plot the torque balance curve

[Step 3] Compute the equivalent stress and plot the equivalent stress curve

[Step 4] Compute the flexural rigidity and plot the flexural rigidity curve

[Step 5] Determine the torque balance point that satisfies the design stress limit and minimizes the flexural rigidity

- Six α_1 values (72.5°, 75°, 77.5°, 80°, 82.5°, and 85° RHL) and five α_2 values (72.5°, 75°, 80°, 85°, and 90° LHL) are selected.

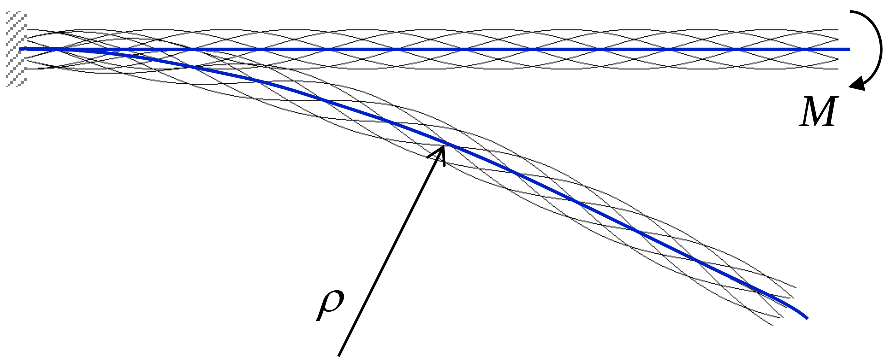
- 30 cases (six $\alpha_1 \times$ five α_2) of torque analyses were performed using beam FE model

Torque balance design

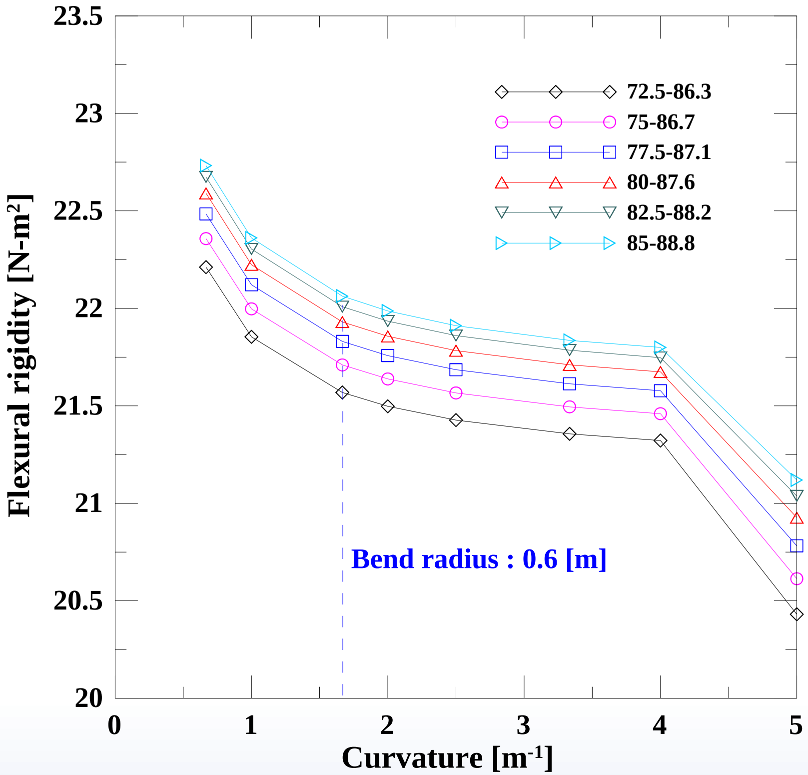
- **Two layer cable**

F.R. (Flexural rigidity) before cable installation is very important factor to handle the cable.

- Bending B.C and deformation



- Flexural rigidity curve



- A lower F.R. → More effective cable handling capability

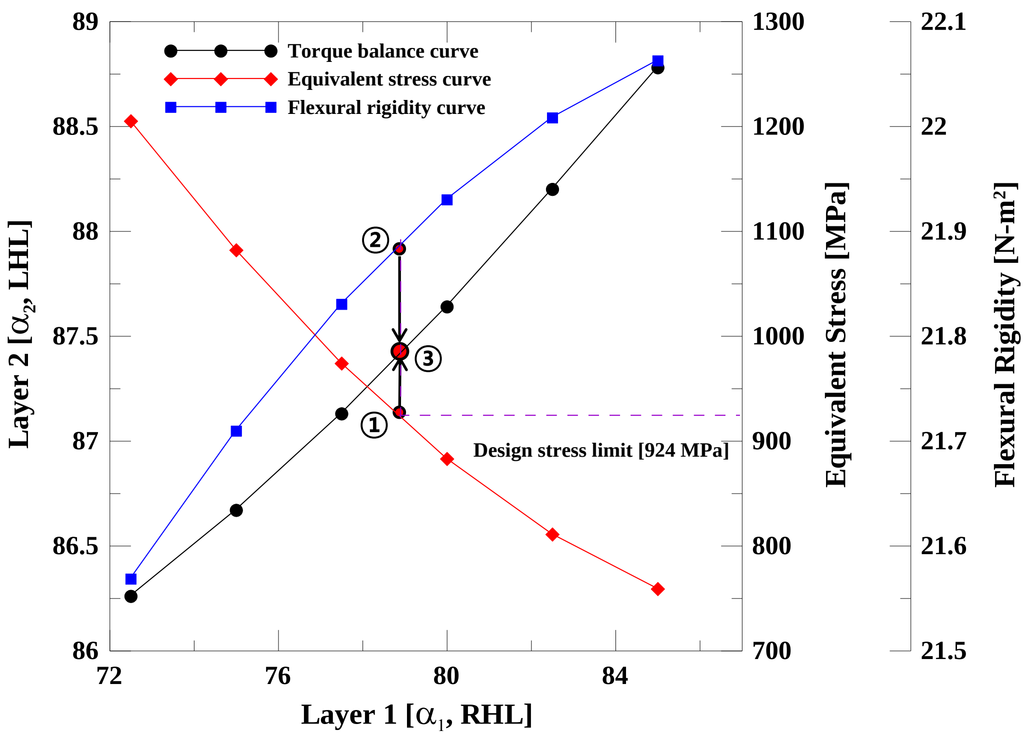
- Nonlinear & friction dependent behavior

- A smaller helix angle → A smaller flexural rigidity

Torque balance design

- **Two layer cable**

- Torque balance



1. Torque balance curves are plotted (the relations between α_1 and α_2)

2. Beyond Point ① $\alpha_1 = 78.85^\circ$ satisfies design stress limit

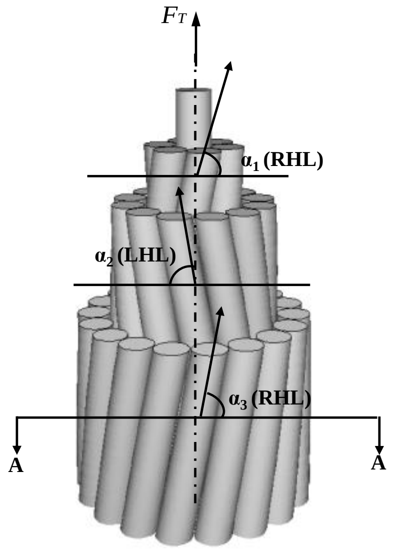
3. Point ② has minimum flexural rigidity

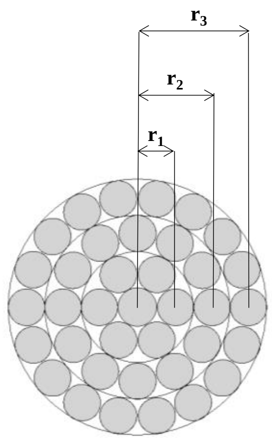
4. Point ③ $\alpha_1 = 78.85^\circ$ $\alpha_2 = 87.41^\circ$ finally determined

Torque balance design

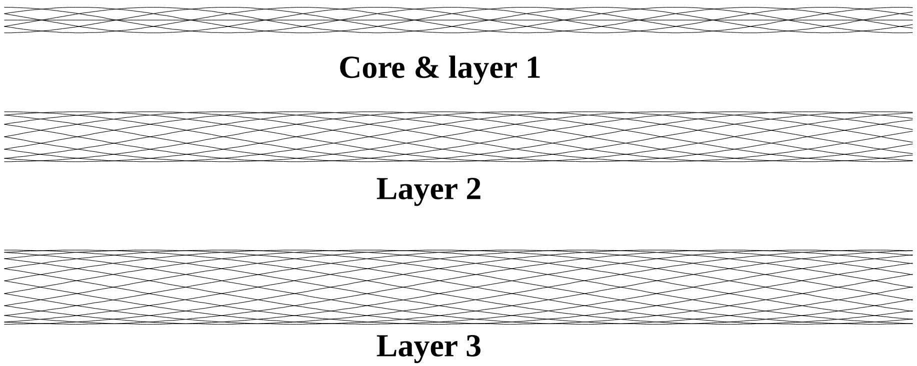
- **Three layer cable**

- Geometry





- Beam FE model

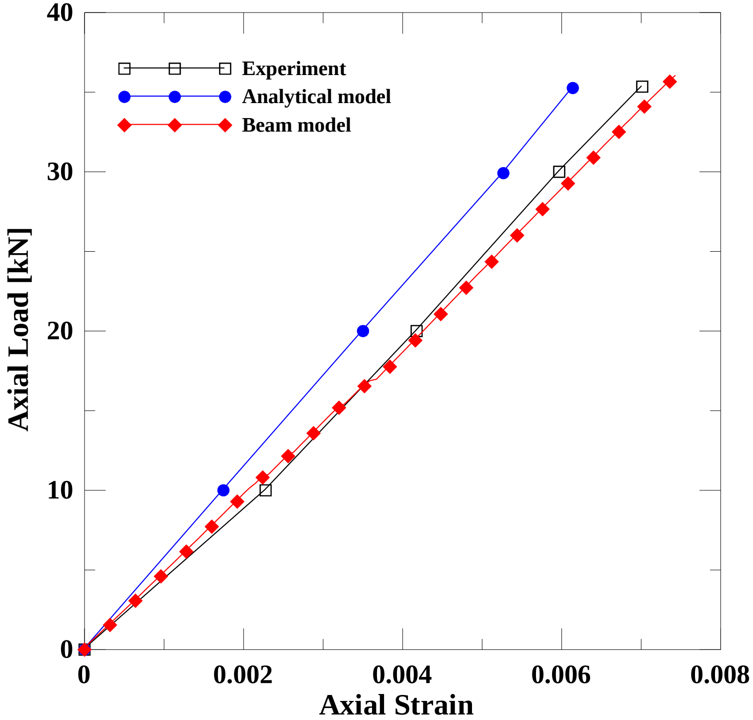


Layer No.	No. of wires	Helical direction and angle [α]	Wire diameter [mm]	Pitch length [mm]	Young's Modulus [GPa]	Poisson's ratio
Core	1	-	1.09	-	190	0.3
1	6	RHL, 79.23°	1.00	34.52	190	0.3
2	12	LHL, 79.23°	1.00	67.55	190	0.3
3	18	RHL, 79.23°	1.00	100.58	190	0.3

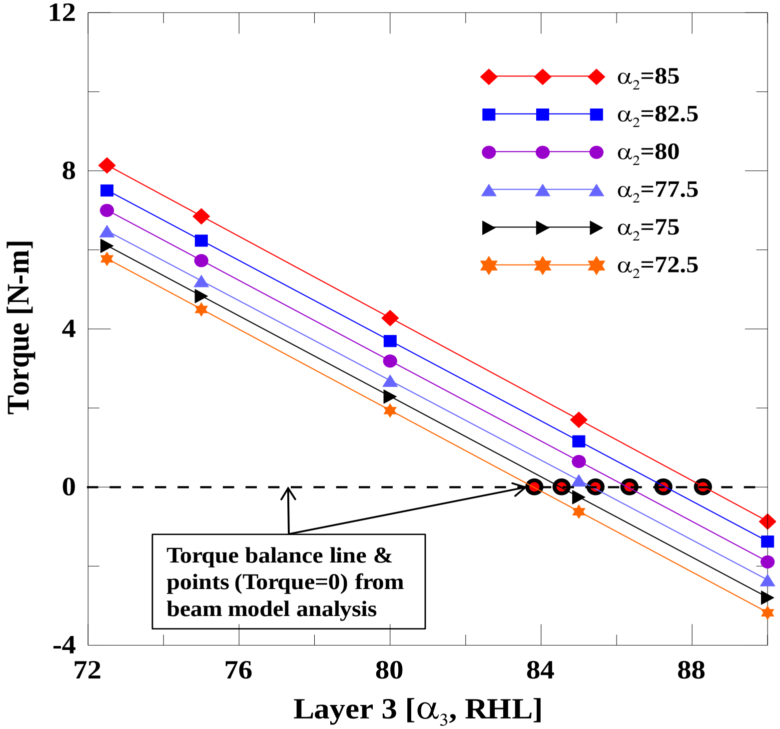
Torque balance design

- **Three layer cable**

- Axial load/strain curves



- Torque balance point (@ $\alpha_1 = 85^\circ$)

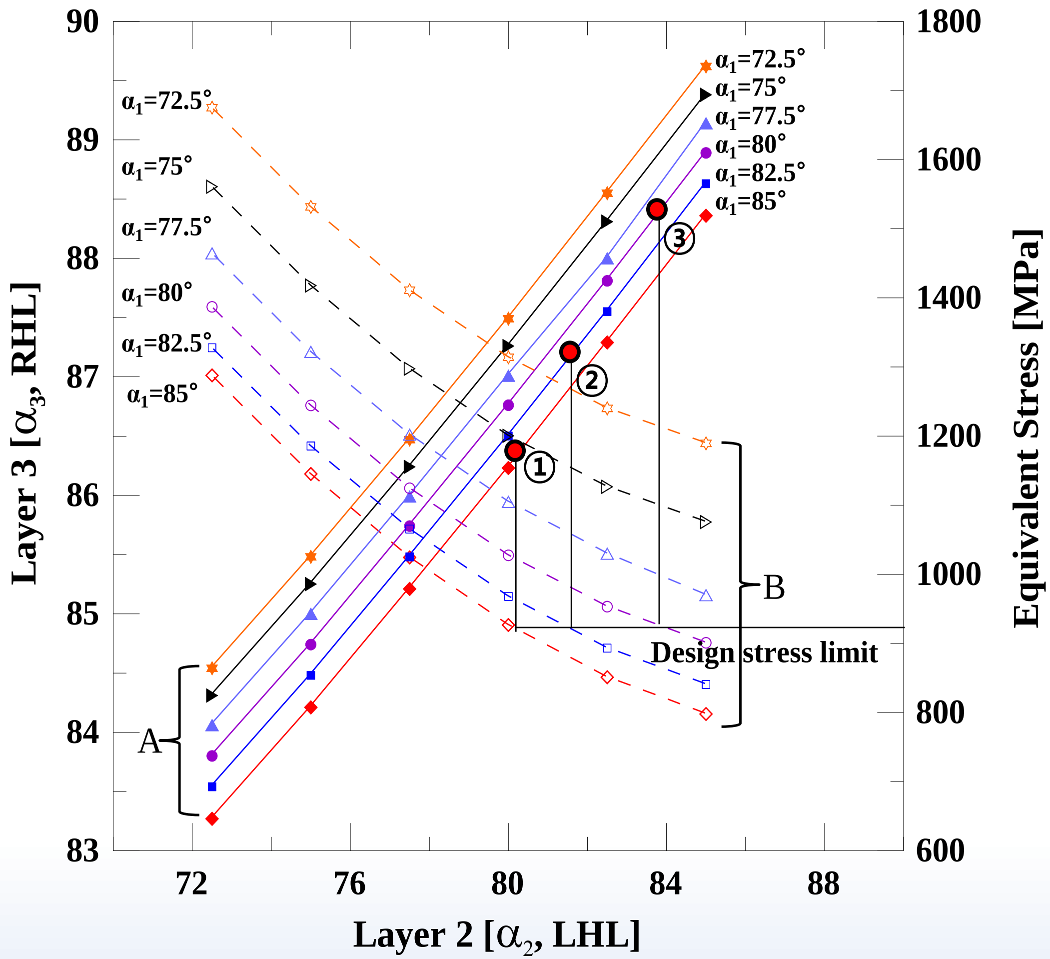


- Six α_1 values (72.5°, 75°, 77.5°, 80°, 82.5°, and 85° RHL), Six α_2 values (72.5°, 75°, 77.5°, 80°, 82.5°, and 85° LHL), and five α_3 values (72.5°, 75°, 80°, 85°, and 90° RHL) are selected.

- 180 cases (six α_1 × six α_2 × five α_3) of torque analyses were performed using beam FE model.

Torque balance design

- **<u>Three layer cable</u>**



1. Torque balance curves are plotted (the relations among α_1 , α_2 and α_3)

2. Stress limit can be reached three α_1 (80°, 82.5° and 85°)

3. Corresponding points in the torque balance curves point ① ② ③

4. Point ① has minimum flexural rigidity (0.342 N-m^2)

5. $\alpha_1 = 85^\circ \quad \alpha_2 = 80.1^\circ \quad \alpha_3 = 86.3^\circ$ **finally determined**

Torque balance design

- **<u>Multilayered helically-stranded cables (four or more layers)</u>**

- Nicolas K. suggests analytical model to minimize the torque up to five layers' cable ('15)

. Difference from previous analytical model : the radial deformation is considered

- Torque balance can be achieved by designing the total torque before the last layer is same as that of last layer with opposite helical direction regardless of the numbers of layers.

$$T_{layer1} + T_{layer2} + T_{layer3} \cdots\cdots + T_{layer\_n} = 0 \qquad \sum_{i=1}^{n-1} T_i = -T_n$$

$$\begin{bmatrix} F_T \\ M_T \end{bmatrix} = \begin{bmatrix} K_{\varepsilon\varepsilon} \, K_{\varepsilon\theta} \\ K_{\theta\varepsilon} \, K_{\theta\theta} \end{bmatrix} \begin{bmatrix} \varepsilon \\ \delta\theta / h \end{bmatrix}$$

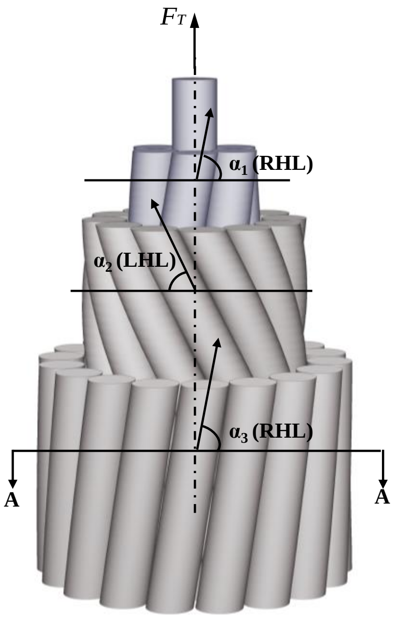
$$(K_{\theta\varepsilon})_{layer1} + (K_{\theta\varepsilon})_{layer2} \cdots\cdots + (K_{\theta\varepsilon})_{layer\_n} = 0 \qquad \sum_{i=1}^{n-1} (K_{\theta\varepsilon})_{i-1} = -(K_{\theta\varepsilon})_n$$

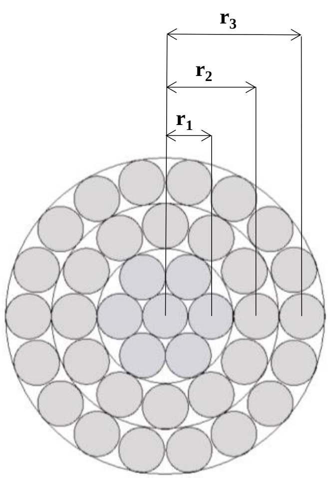
- When using beam FE model, for practical and simple method, it could be a good method to determine only two helical angles, α_{n-1} and α_n , for multilayer torque balance.

Case study

Case study 1

- **<u>ACSR(Aluminum conductor steel reinforced) Cable</u>**





1. Core & Layer 1 : Main structural part

(aluminum-clad steel)

2. Layer 2 & 3 : Conductor part

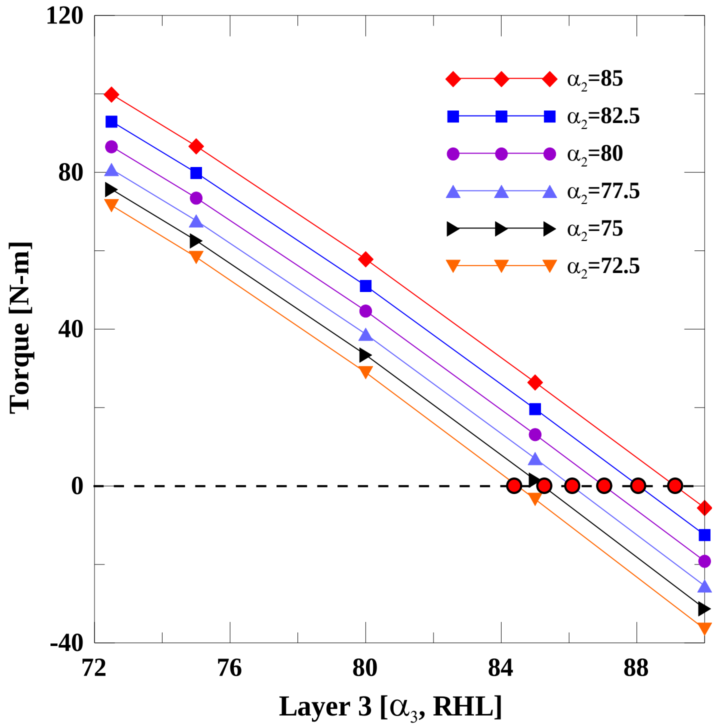
(aluminum alloy)

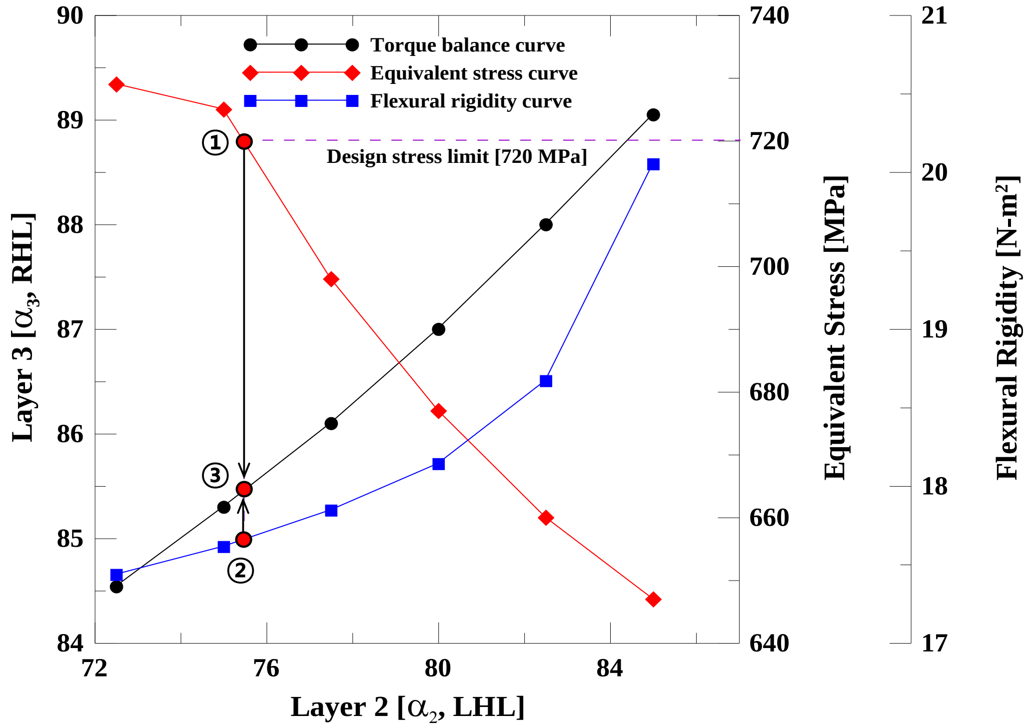
Layer No.	No. of wires	Helical direction and angle [α]	Wire diameter [mm]	Pitch length [mm]	Young's Modulus [GPa]	Poisson's ratio
Core	1	-	3.25	-	162	0.3
1	6	RHL, 85°	3.25	233.4	162	0.3
2	12	LHL, 75.5°	3.25	157.9	69	0.33
3	18	RHL, 85.4°	3.25	761.4	69	0.33

Case study 1

- **Design of ACSR Cable**

- Torque balance points (@ $\alpha_1 = 85^\circ$)





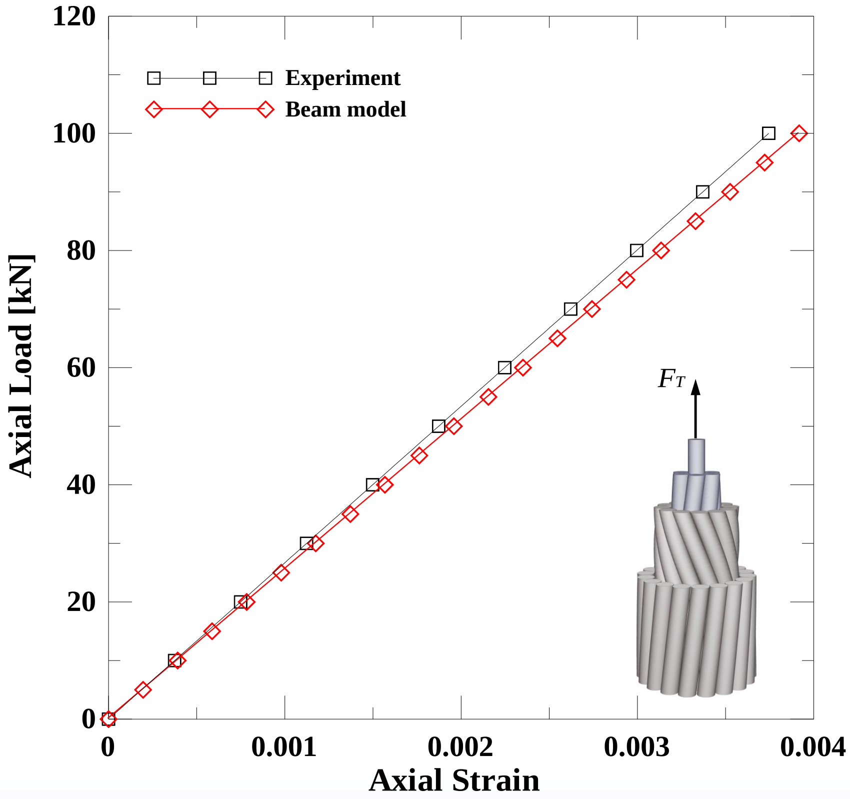
1. Beyond Point ① $\alpha_2 = 75.5^\circ$ satisfies design stress limit ⇨ **2. Point ② has minimum flexural rigidity** ⇨ **3. Point ③ $\alpha_2 = 75.5^\circ$ $\alpha_3 = 85.4^\circ$ finally determined**

Case study 1

- **<u>Comparison between test and design of ACSR Cable</u>**

- Experimental test

- Axial load/strain curves



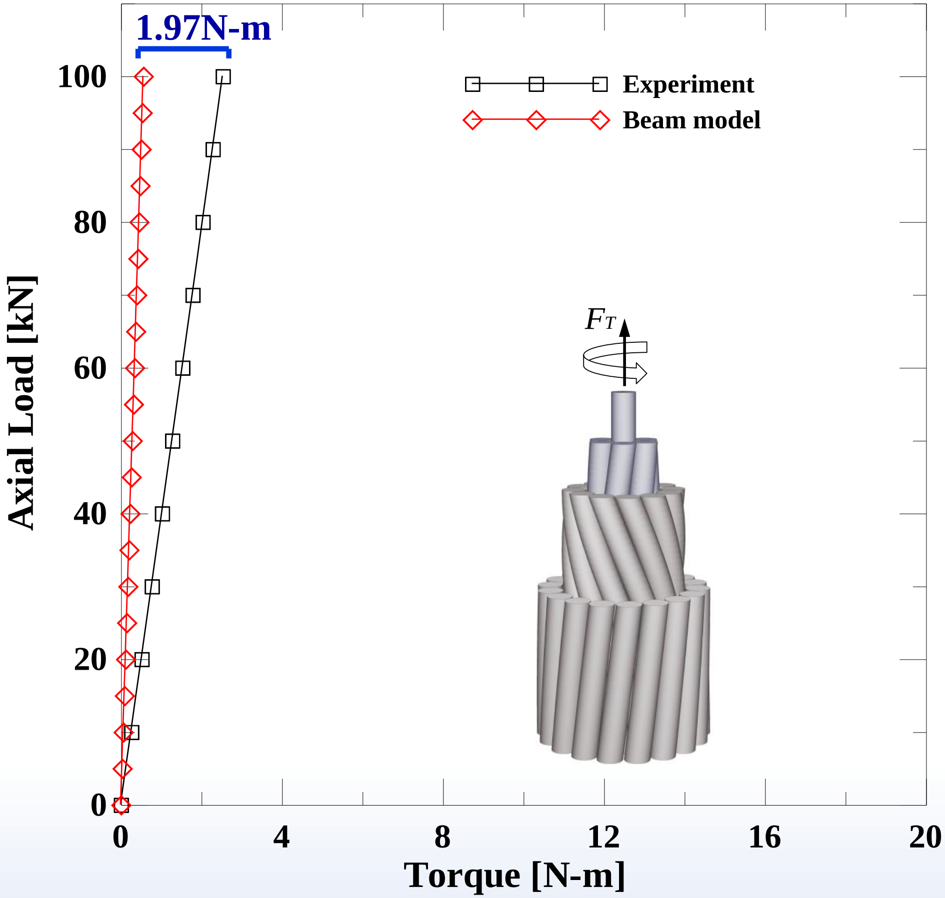
- Both ends socketed and fixed against rotation

- One end was attached to a torque meter

Case study 1

- <u>**Comparison between test and design of ACSR Cable**</u>

- Axial load/torque curves



- Axial load applied 100kN, torque generated :

. Experiment : 2.53 [N-m]

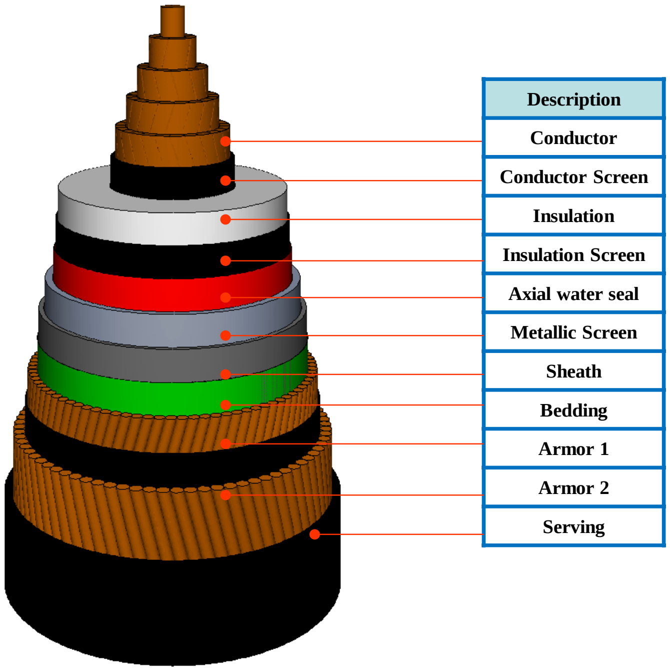
. Beam model : 0.56 [N-m]

→ Difference is due to unequal load sharing among wires (wires have unequal lengths between sockets).

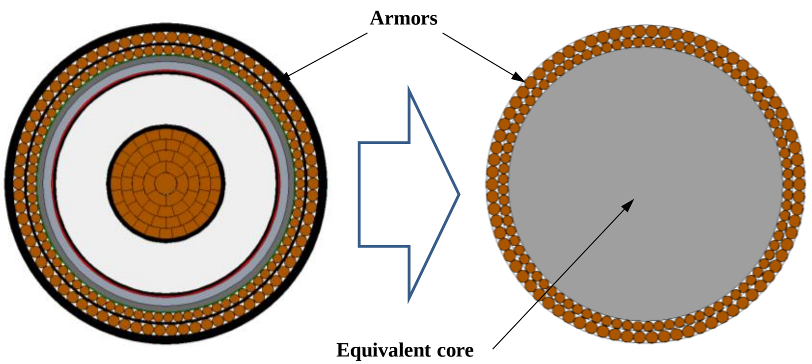
→ The torque value from experiment is relatively small.

Case study 2

- **Submarine power cable**



- Modified model



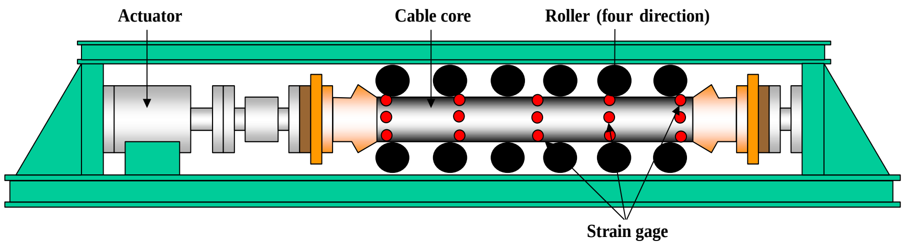
- Equivalent core properties should be defined

Layer	No. of wires	Helix angle [α]	Outer Diameter [mm]	Pitch [mm]	Young's modulus [GPa]	Poisson's ratio
Conductor	-	-	58.8	-	117	0.36
Insulation	-	-	117.2	-	0.30	0.46
Metallic screen	-	-	130.6	-	9.78	0.45
Sheath	-	-	137.0	-	0.88	0.46
Armor 1	85	RHL, 78.8°	148.5 (5mm : wire)	2270	117	0.36
Armor 2	78	LHL, 82.4°	162.7 (6mm : wire)	3629	117	0.36

Case study 2

- **Submarine power cable**

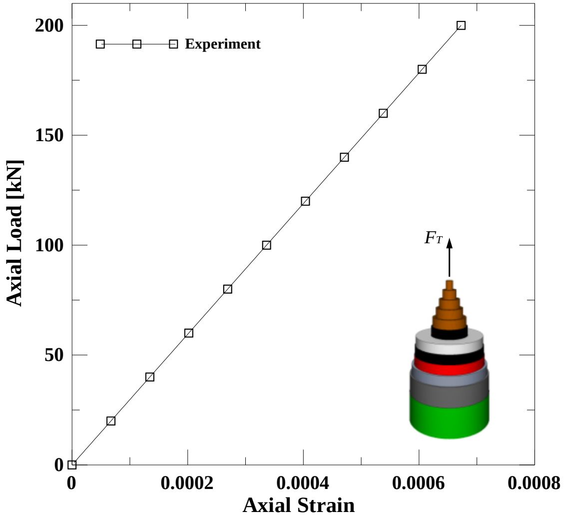
- Experimental test (tensile test) for core



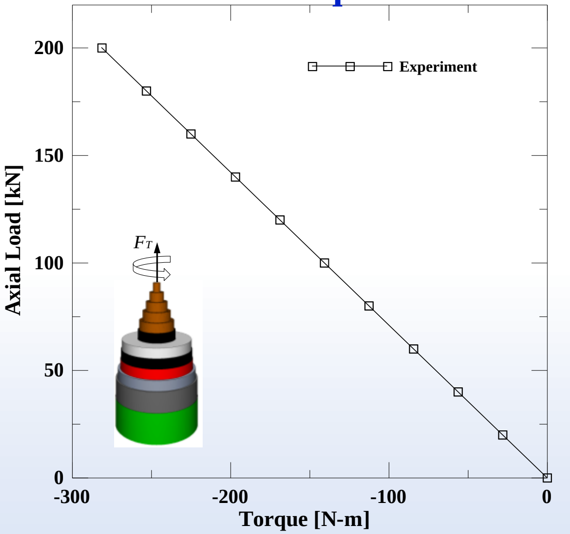
- Axial rigidity can be measured.

- Torque (281 N-m) was generated due to conductor structure (stranded flat wire).

- Axial load/strain curve



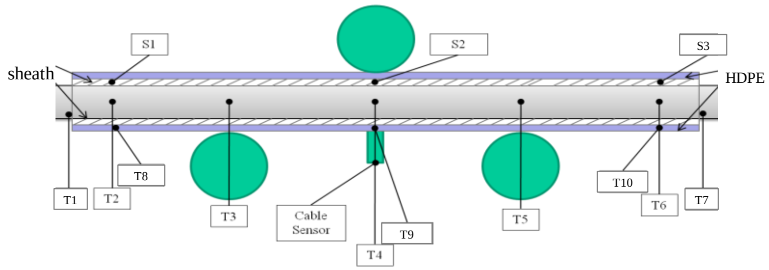
- Axial load/torque curve



Case study 2

- **Submarine power cable**

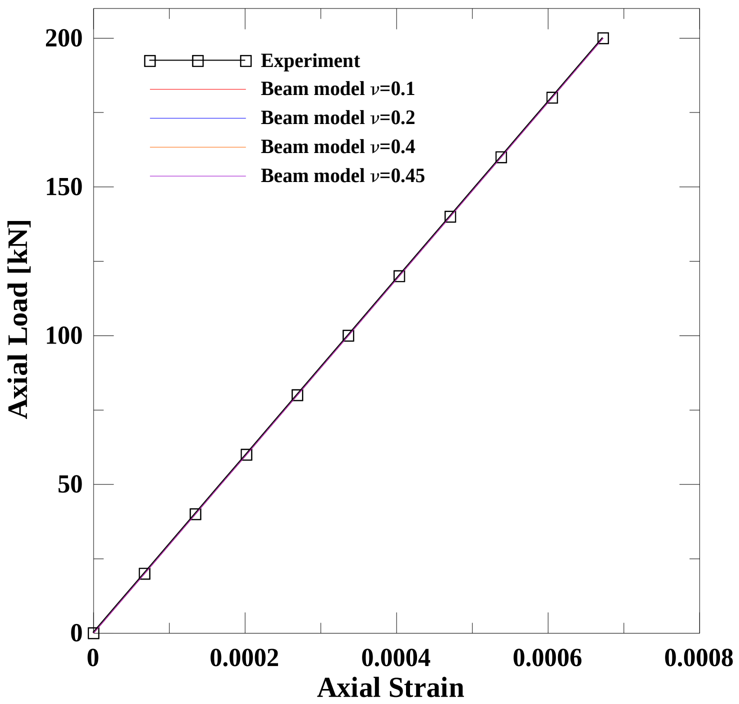
- Experimental test (bending test) for core



- Bending(Flexural) rigidity

$$EI = \frac{WL^3}{48\delta}$$

- Poisson's ratio



- The effect of Poisson's ratio is so small

Case study 2

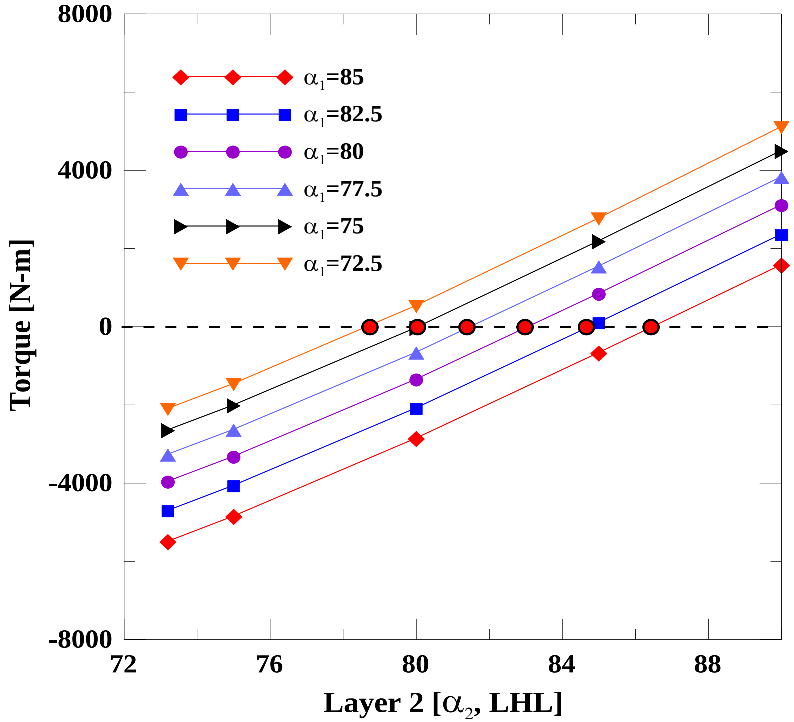
- **Submarine power cable**

- Properties for equivalent model

Layer	No. of wires	Helix angle [α]	Wire Dia. [mm]	Pitch length [mm]	Young's Modulus * [GPa]	Poisson's ratio
Eq. core	-	-	138.12	-	19.84 (A) 2.36 (B)	0.4
A 1	85	RHL, 78.8°	5	2270	117	0.36
A 2	78	LHL, 82.4°	6	3629	117	0.36

*** Two Young's modulus, A : for axial rigidity, B : for flexural rigidity**

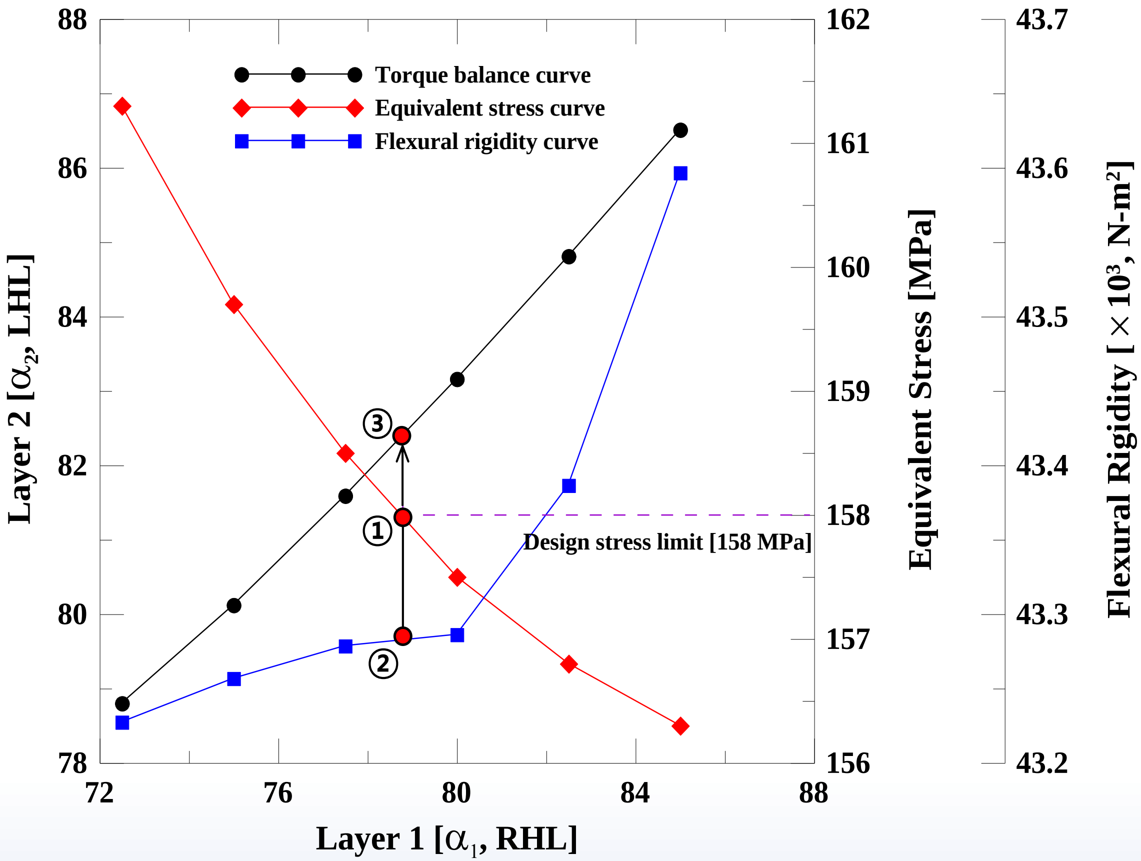
- Torque balance points



Case study 2

- **Submarine power cable**

- Torque balance point for cable design



1. Beyond Point ① $\alpha_1 = 78.8^\circ$ **satisfies design stress limit**

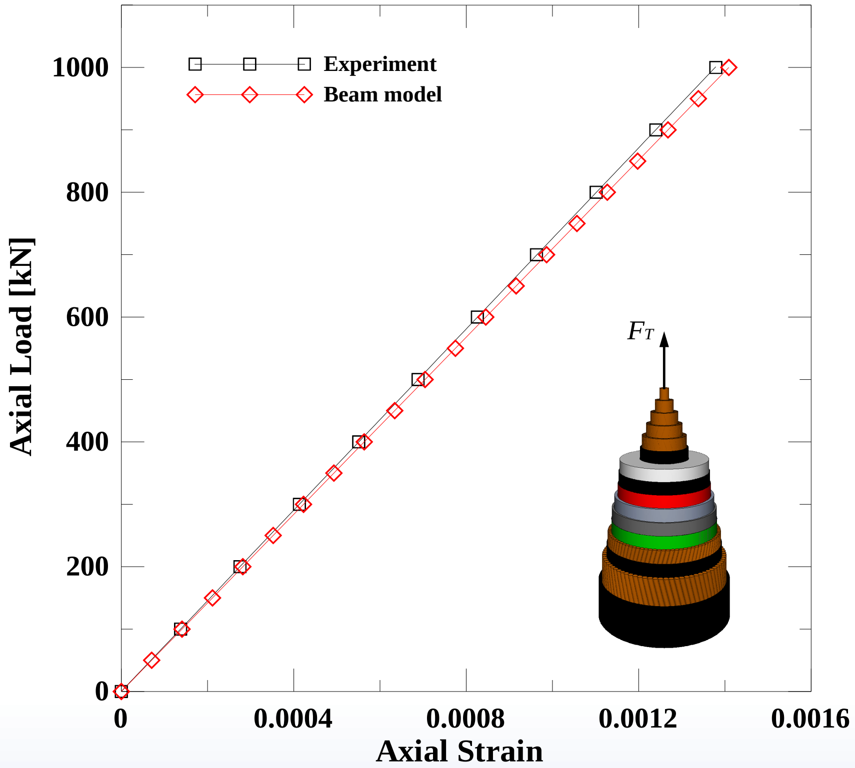
2. Point ② has minimum flexural rigidity

3. Point ③ $\alpha_1 = 78.8^\circ$ $\alpha_2 = 82.4^\circ$ **finally determined**

Case study 2

- **<u>Submarine power cable</u>**

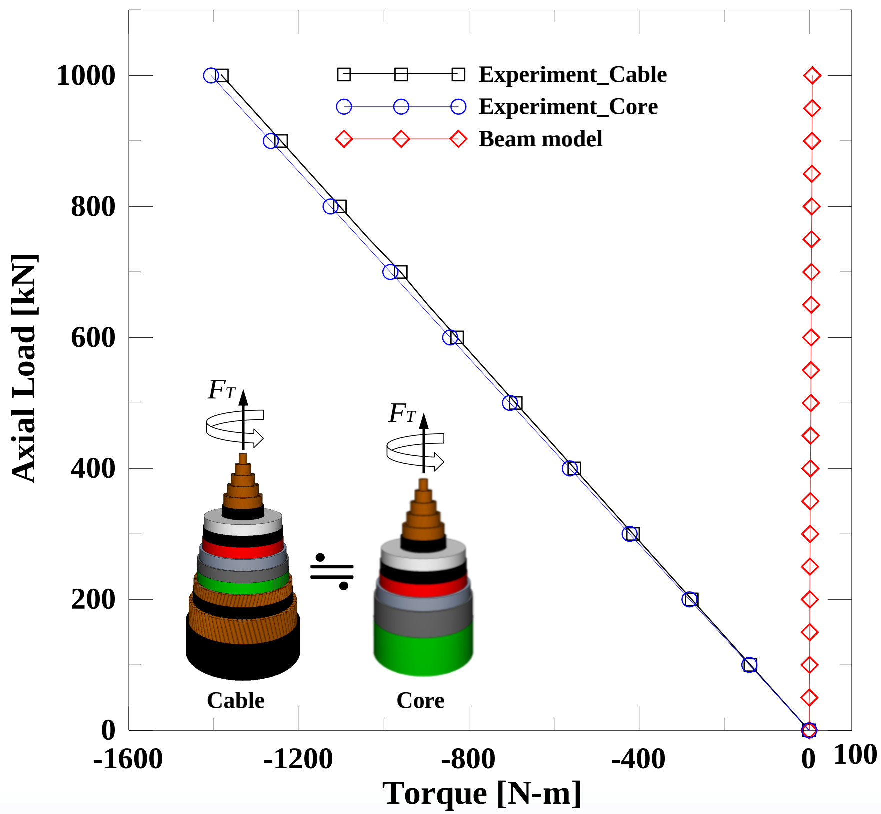
  - Experimental test

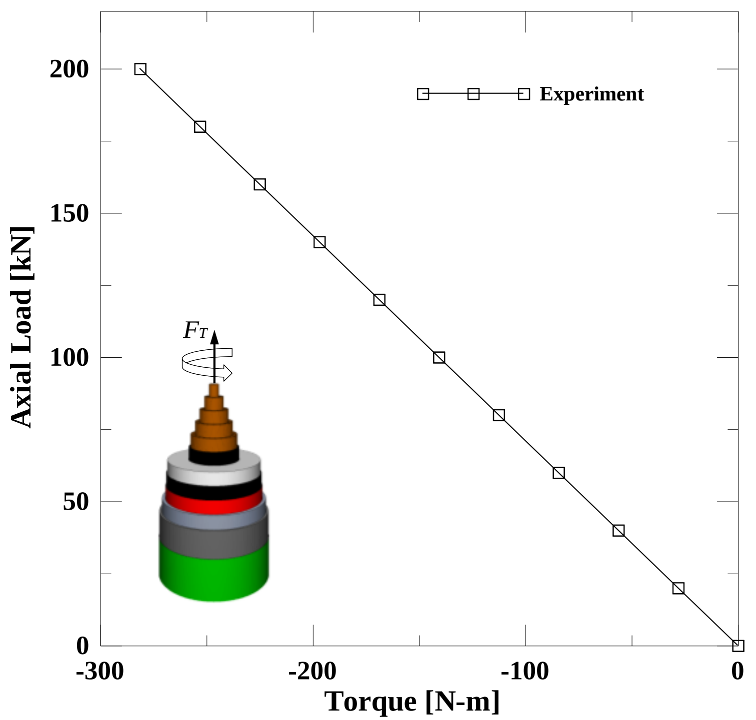


Case study 2

- **Submarine power cable**

- Comparison between design & test considering torque from core





- Torque 1,381 N-m (1,000kN applied)

- Torque 281 N-m (200kN applied to core)

- Torque 1,405 N-m may be generated if 1,000kN applied.

Generalized torque balance curves

Generalized torque balance curves

- **Dimensionless parameter**

R_t can be introduced from $K_{\theta\varepsilon}$

Hint : Stretch terms are dominant

$$K_{\theta\varepsilon} = n(EAr\sin^2\alpha\cos\alpha)$$

$$K_{\theta\varepsilon} = n\left[EAr\sin^2\alpha\cos\alpha + \frac{GJ\cos 2\alpha\sin^2\alpha\cos\alpha}{r} - \frac{EI\sin 2\alpha\sin\alpha\cos^2\alpha}{r}\right]$$

$$K_{\theta\varepsilon} = n\left[EAr\sin^2\alpha\cos\alpha + \frac{GJ\cos 2\alpha\sin^4\alpha\cos\alpha}{r} - \frac{EI\sin 2\alpha\cos^2\alpha\sin\alpha(1+\sin^2\alpha)}{r}\right]$$

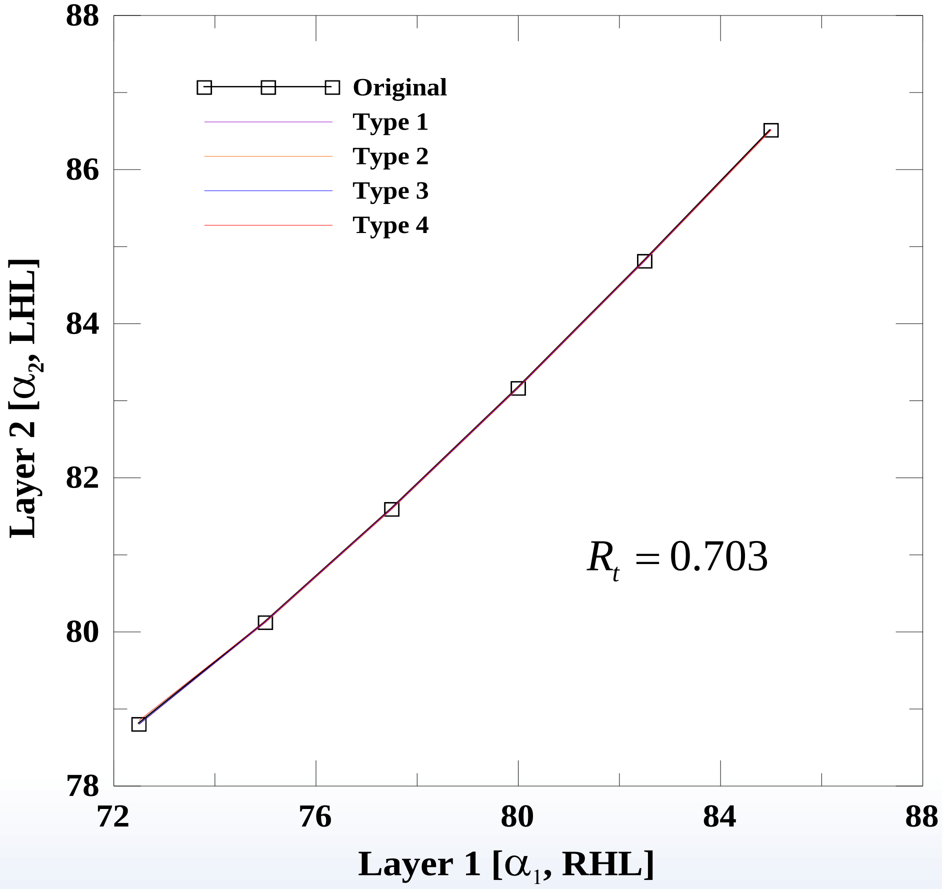
$$R_t = \frac{n_1E_1A_1r_1}{n_2E_2A_2r_2}$$

Model	Layer	No. of wires	Wire diameter [mm]	Center line radius [mm]	Young's Modulus [GPa]	R_t
Original	Core	-	138.12	69.06	19.84	0.703
	Armor 1	85	5	71.56	117	
	Armor 2	78	6	77.06	117	
Type 1	Core	-	138.12	69.06	19.84	0.703
	Armor 1	85	5	71.56	117	
	Armor 2	57	7	77.56	117	
Type 2	Core	-	138.12	69.06	19.84	0.703
	Armor 1	59	6.02	72.07	117	
	Armor 2	78	6	78.08	117	
Type 3	Core	-	138.12	69.06	19.84	0.703
	Armor 1	85	5	71.56	100	
	Armor 2	66	6.03	77.08	117	
Type 4	Core	-	69.06	34.53	19.84	0.703
	Armor 1	45	5.09	37.08	117	
	Armor 2	40	6	42.62	117	

Generalized torque balance curves

- **Comparison of torque balance curves with R_t**

  - Torque balance analysis performed using beam model

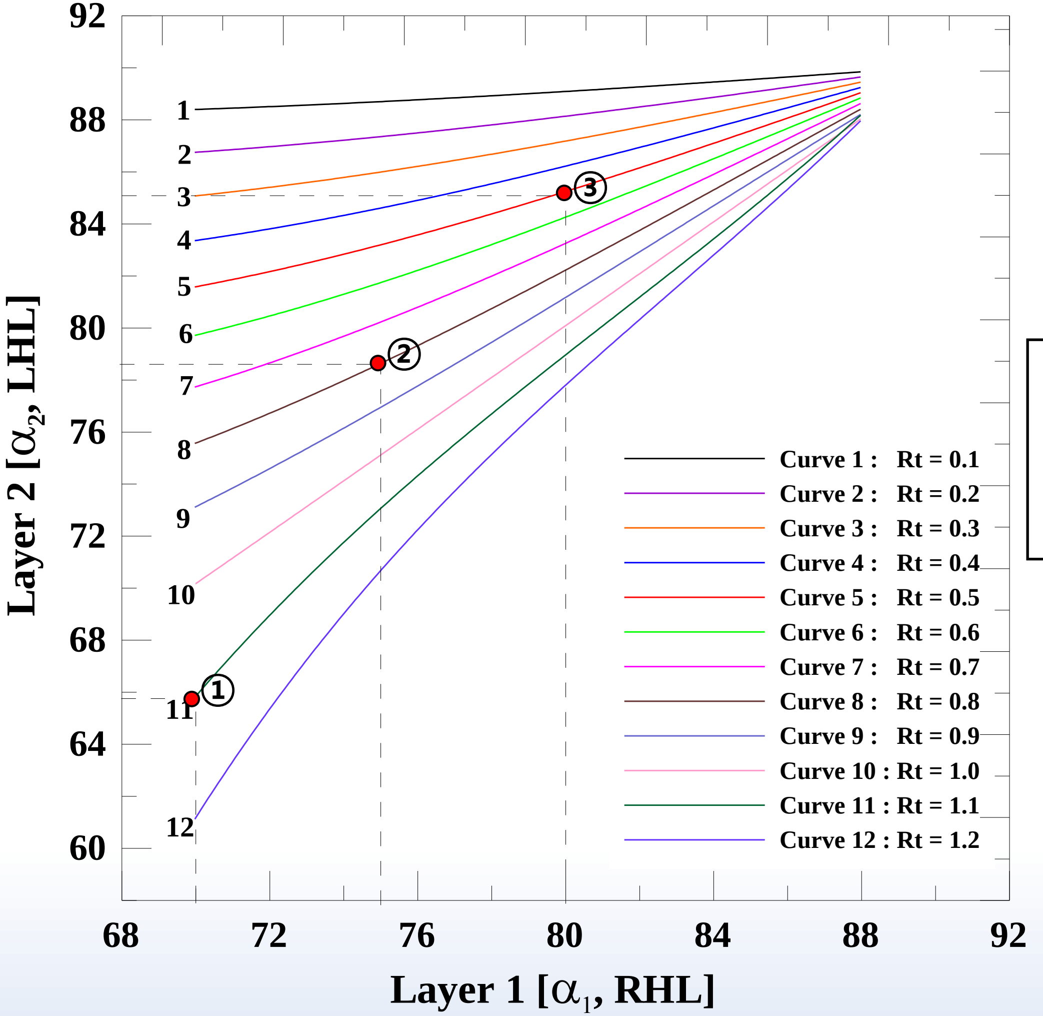


- Same R_t gives same torque balance curves

- Generalized torque balance curves can be plotted by R_t

Generalized torque balance curves

- **Generalized torque balance curves**



No.	R_t	α_1	α_2
①	1.1	70°	65.8°
②	0.8	75°	78.6°
③	0.5	80°	85.2°

This can be design guidance instead of experiments or analysis tools

Conclusion

- **Conclusion**

◆ **Multiple beam FE models was proposed to predict the behavior of the cable.**

- Beam FE model can be the most effective solution due to accuracy and computation cost.

- Optimal FE model & friction effect were determined by sensitivity analysis.

- The validity and limitation of analytical models were evaluated.

- This model can be used a variety of cable analyses, dynamic, buckling, birdcaging, etc.

◆ **A new procedure for the torque balance design was proposed.**

- Helix angles of each of the layers satisfy the design stress limit and the minimum flexural rigidity as well as the satisfaction of torque balance.

- The design procedure was verified by experimental test.

◆ **The generalized torque balance curves were proposed**, which can be a good design guide instead of relying on experimental test or analysis tools.

THANK YOU