Ph.D. dissertation

부유식 구조물의 유탄성 해석에 관한연구
: 정적/동적 통합해석 및 응력 전달함수의 직접계산 방법

Hydroelastic analysis of floating structures: integrated static and dynamic analysis and direct calculation of stress RAO

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Contents

1. Introduction

Introduction

- **Various loads effected on a floating structure**
  - Static loads
  - Hydrodynamic loads(Wave induced loads)
  - Impact loads (bottom slamming, bow flare impacts, sloshing)
  - Cyclic loads (main engine, propeller)
  - Ice loads
  - Others

Introduction

- **Importance of the hydroelastic behavior**




Effect of hydro elasticity becomes more important.

Brief history

▪ Methods for hydroelastic analysis

Rigid-body motion analysis

- Pioneering works on the motion of floating rigid bodies in frequency and time domain
- Dealing with various problems of potential flows and hydrodynamics
- Lamb(1945), John (1950), Stoker(1956), Wehausen (1960), Cummins(1962), Newman(1977), …

1980s

- Development of numerical methods for rigid body - wave interactions in frequency domain
- WAMIT - WaveAnalayisMIT(MIT, 1987)

1990s

- Development of numerical methods for rigid body - wave interactions in time domain
- TiMIT(MIT,1999)

2000s

- Improve numerical algorithms for complex structure – nonlinear wave interactions

Hydroelastic analysis

- 2D hydroelastic analysis of ships in frequency domain developed by Bishop (1979)

1980s

- 3D hydroelastic analysis of ships extended by Wu (1984)

1990s

- Research projects for very large floating structures(VLFS)
- Mega-float(Japan, 1995-2001)
- MOB - Mobile Offshore Base(USA, 1997-2000)

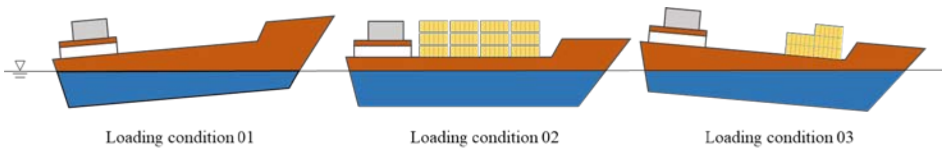
2000s

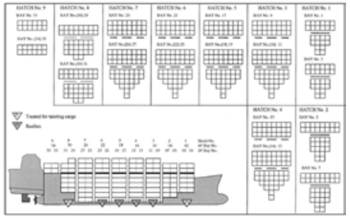
- Development of numerical methods for hydro-elastic analysis in frequency and time domain
- Kashiwagi(2000), Khabakhpasheva(2002), Taylor(2007), WISH(Kim, 2008),…
- 3D floating structures – wave interactions in frequency domain
- HYDRAN(Riggs, 2003)
- PADO(Kim, 2013)

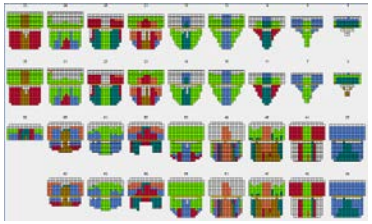
In this work

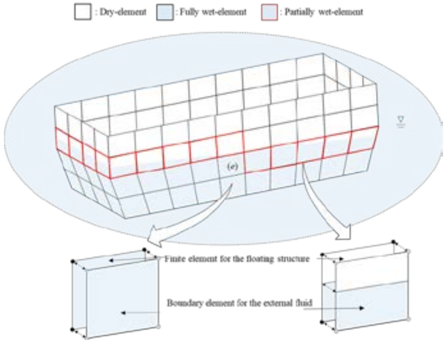
An effective numerical method to Hydroelastic analysis of floating structures

1. Integrated hydro–static and dynamic analysis

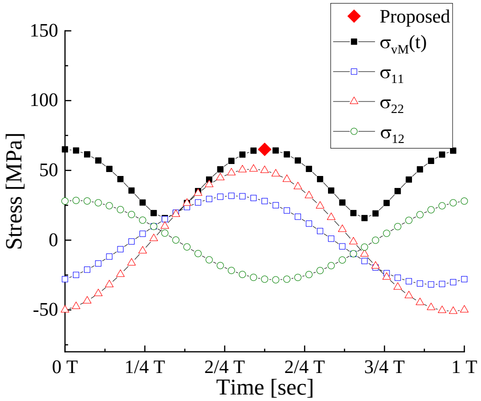


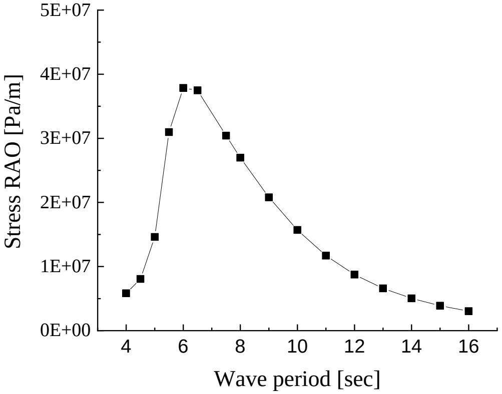


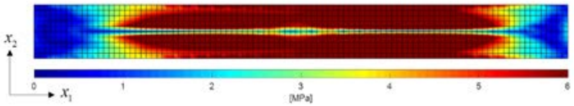




2. Direct calculation of stress RAOs in hydroelastic analysis

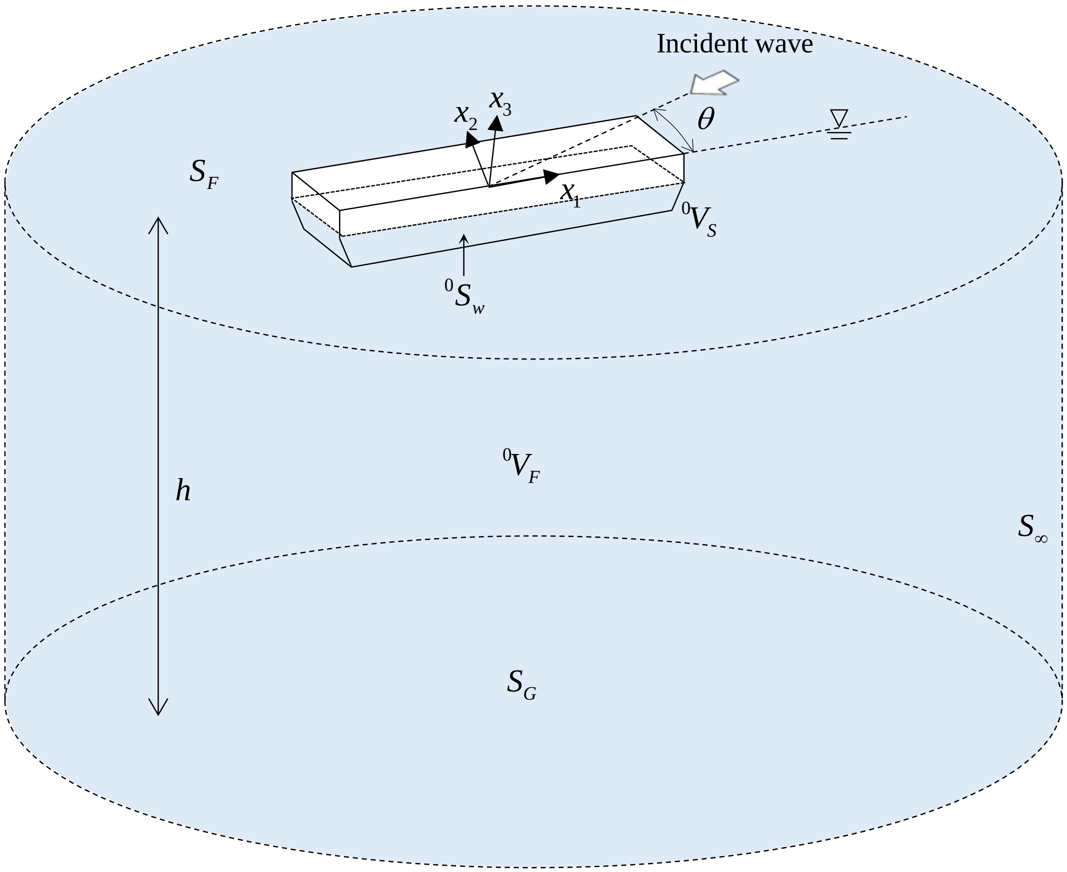






2. Integrated hydro-static and -dynamic analysis

- **Problem description**



Fixed Cartesian coordinate system (x_1, x_2, x_3)

Floating structures

- V_S : Volume of structure
- S_w : Wet surface
- S_d : Dry surface

External fluid

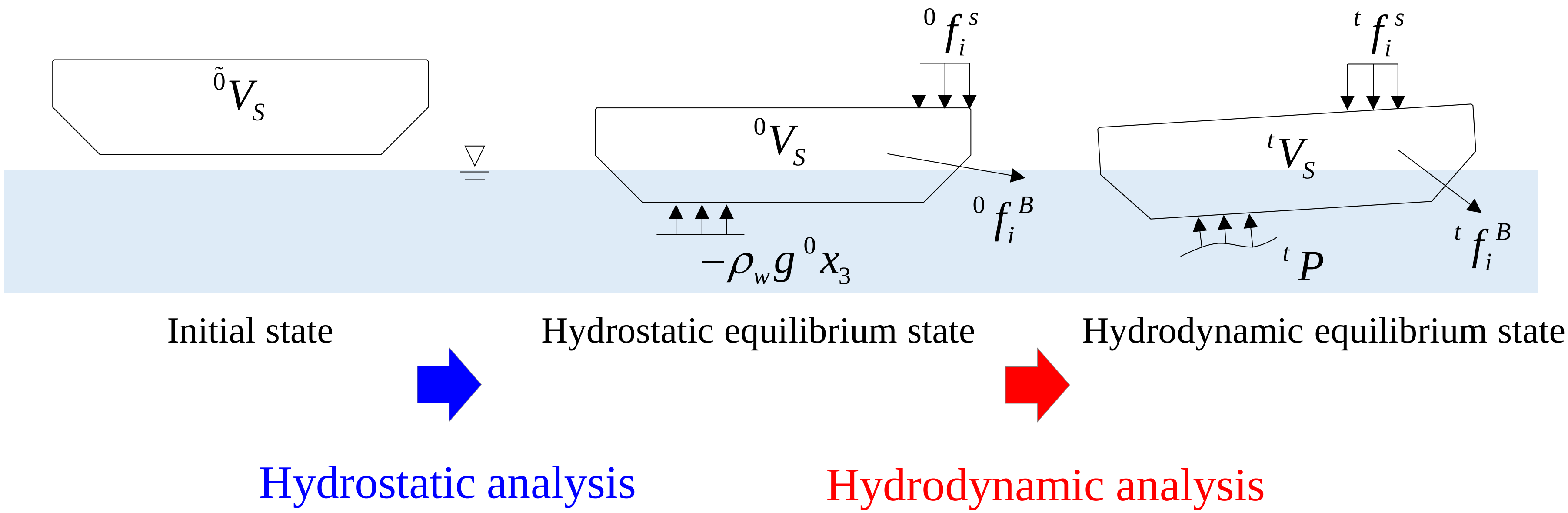
- V_F : volume of external fluid
- S_∞ : infinite boundary surface
- S_G : flat bottom surface
- S_F : free surface
- S_W : wet surface

Others

- h : Water depth
- θ : Incident wave angle

Formulation of the floating structure

- **Solution procedure of the integrated analysis**



S_S : the surface of floating structure

V_S : the volume occupied by the structure

f_i^s : the surface force

^0P : hydrostatic pressure($^0P = -\rho_w g x_3$)

^tP : total pressure($^tP = -\rho_w g x_3 + {}^tP_D$)

Formulation of the floating structure

- **Assumption**

  Homogeneous, isotropic, and linear elastic material

- **Strong form**

  The equilibrium equations at time $\tau + \Delta\tau$ (updated Lagrangian formulation employed)

$$\frac{\partial\, {}^{\tau+\Delta\tau}\sigma_{ij}}{\partial\, {}^{\tau+\Delta\tau}x_j} - \rho_S\, {}^{\tau+\Delta\tau}\ddot{x}_i - \rho_S g \delta_{i3} + {}^{\tau+\Delta\tau}f_i^B = 0 \quad \text{in} \quad {}^{\tau+\Delta\tau}V_S$$

$${}^{\tau+\Delta\tau}\sigma_{ij}\, {}^{\tau+\Delta\tau}n_j = {}^{\tau+\Delta\tau}f_i^S \quad \text{on} \quad {}^{\tau+\Delta\tau}S_S$$

$${}^{\tau+\Delta\tau}\sigma_{ij}\, {}^{\tau+\Delta\tau}n_j = -{}^{\tau+\Delta\tau}P\, {}^{\tau+\Delta\tau}n_i \quad \text{on} \quad {}^{\tau+\Delta\tau}S_W \qquad {}^{\tau+\Delta\tau}P = -\rho_W g\, {}^{\tau+\Delta\tau}x_3 + {}^{\tau+\Delta\tau}P_D$$

σ_{ij} : Cauchy stress tensor

g : Gravitational acceleration

n_i : unit normal vector

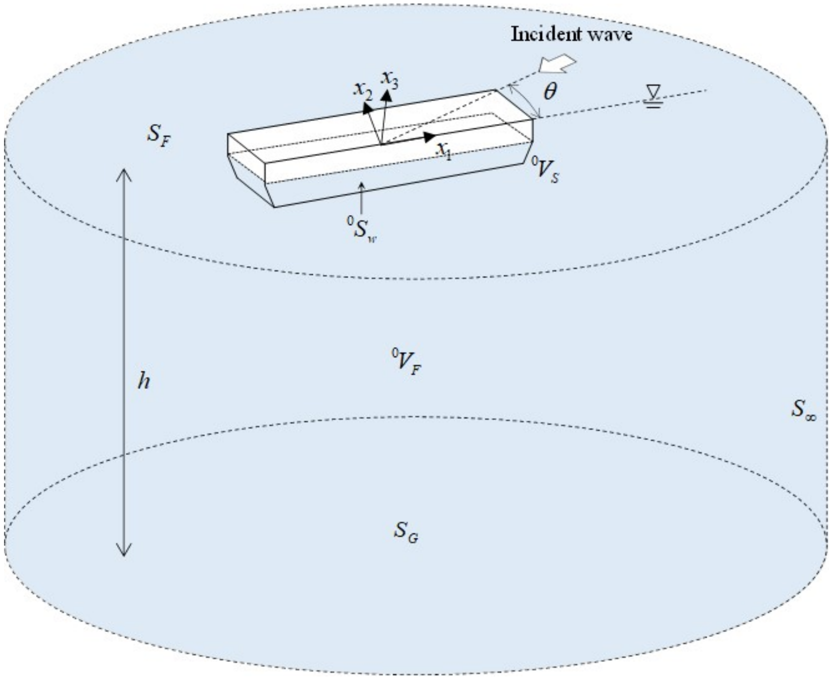
ρ_s : density of the floating structure

P : pressure affected on wet surface

δ_{ij} : Kronecker delta

Formulation of the fluid

- **Assumption**
  - ✓ Incompressible, inviscid and irrotational flow



- **Governing equations**

$$\phi(t) = \mathrm{Re}\{\hat{\phi}({}^0x_i)e^{\hat{j}\omega t}\}$$

$$\nabla^2\hat{\phi} = 0 \quad \text{in} \quad {}^0V_F$$

$$\frac{\partial\hat{\phi}}{\partial x_3} = \frac{\omega^2}{g}\hat{\phi} \quad \text{on} \quad S_F\,(x_3 = 0)$$

$$\frac{\partial\hat{\phi}}{\partial x_3} = 0 \quad \text{on} \quad S_G\,(x_3 = -h)$$

$$\sqrt{R}\left(\frac{\partial}{\partial R} + \hat{j}\hat{k}\right)(\hat{\phi} - \hat{\phi}^I) = 0 \quad \text{on} \quad S_\infty\,(R \to \infty)$$

$$\frac{\partial\hat{\phi}}{\partial\,{}^0n} = \hat{j}\omega\hat{u}_i\,{}^0n_i \quad \text{on} \quad {}^0S_W$$

ϕ : Velocity potential

∇^2 : Laplace operator

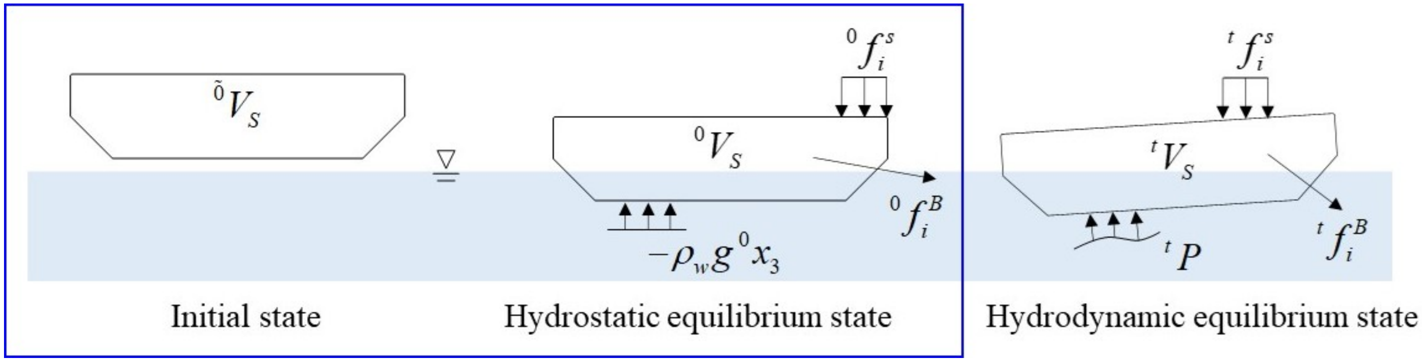
ϕ^I : Incident wave potential

k : wave number

Element discretization

- **Element discretization for hydrostatic analysis**

$$[{}^{\tau}\mathbf{S}_K + {}^{\tau}\mathbf{S}_{KN} - {}^{\tau}\mathbf{S}_{HD} - {}^{\tau}\mathbf{S}_{HN}]\mathbf{U} = -{}^{\tau+\Delta\tau}\mathbf{R}_B + {}^{\tau+\Delta\tau}\mathbf{R}_{HS} + {}^{\tau+\Delta\tau}\mathbf{R}_S - {}^{\tau}\mathbf{F}$$

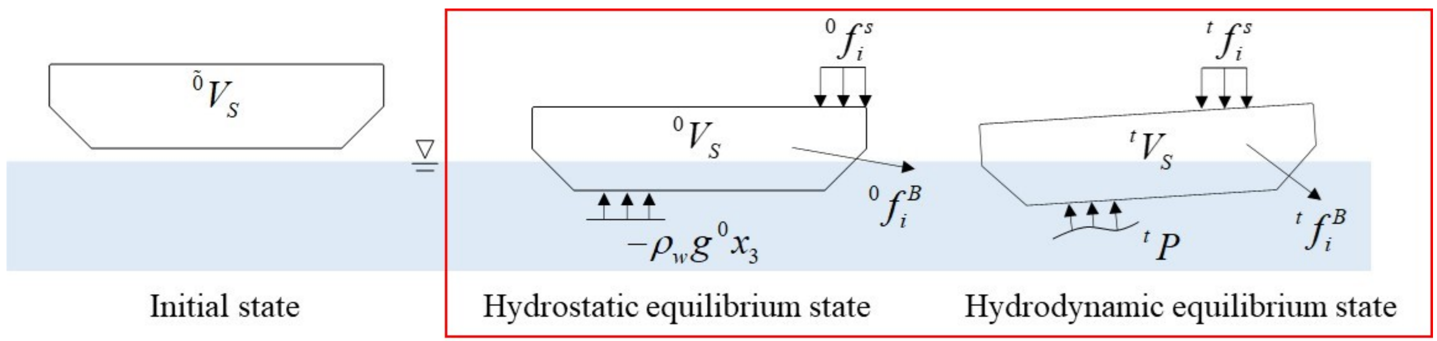


- **Element discretization for hydrodynamic analysis**

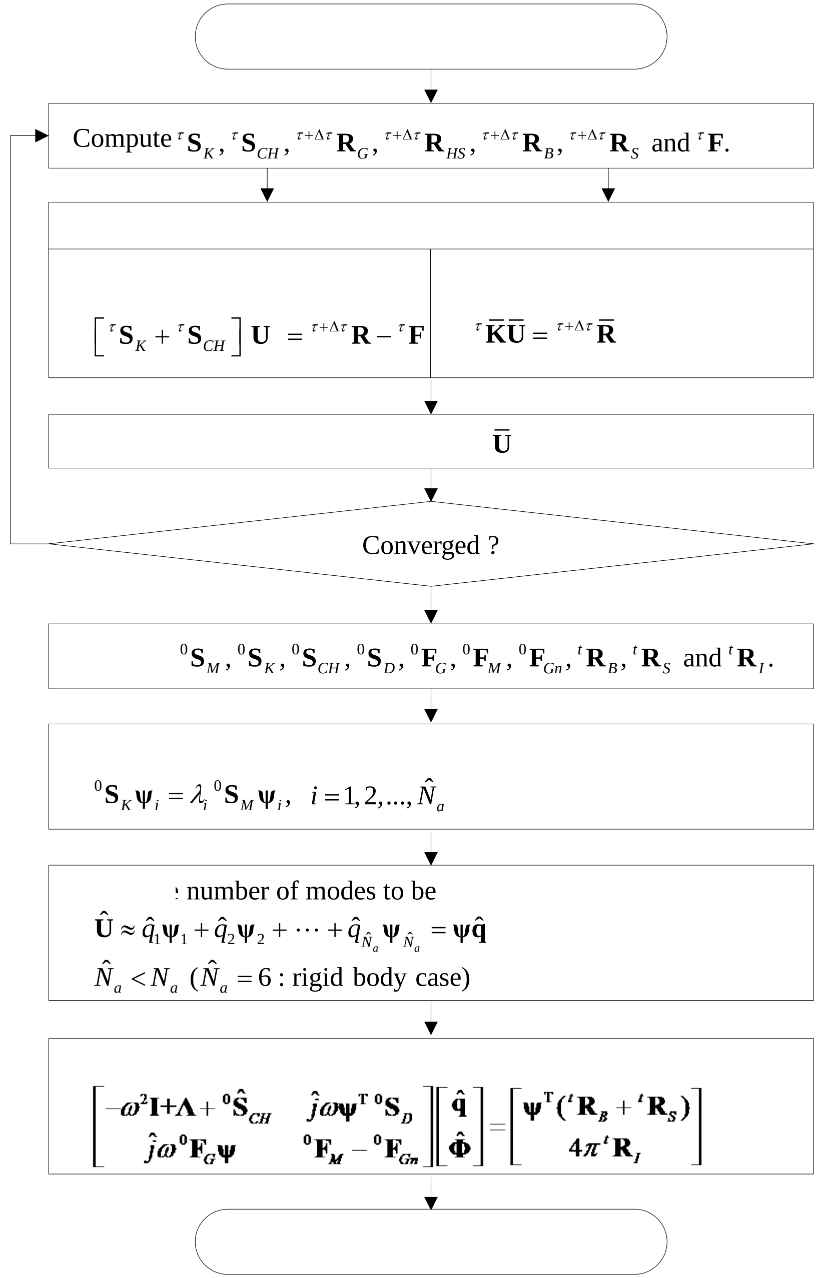
$$\begin{bmatrix} -\omega^2\,{}^{0}\mathbf{S}_M + {}^{0}\mathbf{S}_K + {}^{0}\mathbf{S}_{CH} & \hat{j}\omega\,{}^{0}\mathbf{S}_D \\ \hat{j}\omega\,{}^{0}\mathbf{F}_G & {}^{0}\mathbf{F}_M - {}^{0}\mathbf{F}_{Gn} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{U}} \\ \hat{\mathbf{\Phi}} \end{bmatrix} = \begin{bmatrix} {}^{t}\mathbf{R}_B + {}^{t}\mathbf{R}_S \\ 4\pi\,{}^{t}\mathbf{R}_I \end{bmatrix}$$

: coupling term

$${}^{0}\mathbf{S}_{CH} = {}^{0}\mathbf{S}_{KN} - {}^{0}\mathbf{S}_{HD} - {}^{0}\mathbf{S}_{HN}$$



Integrated hydro –static and -dynamic analysis procedure



Research purpose

▪ Integrated hydro-static and dynamic analysis

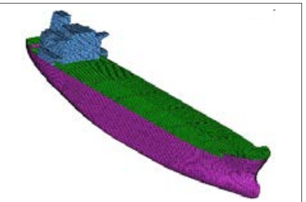
Hull geometry
→ Hydrostatic panel modeling
↓
Hydrostatic analysis
↓
Hydrostatic response
(wet surface, waterline, pressure, ……)
⬇
Hydrodynamic panel modeling
w.r.t wet surface
↓
Hydrodynamic analysis
↓
Hydrodynamic response
(motion, pressure, ……)
⬇
Structural mesh modeling
(pressure mapping)
↓
Structural analysis
↓
Structural response
(stress, strain, ……)

Hydrostatic panel model

NAPA,
ORCA 3D,
etc.

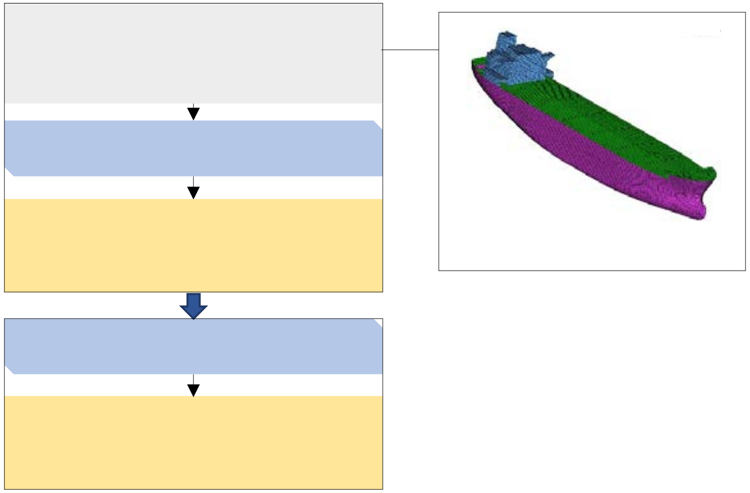
Hydrodynamic panel model

AQWA ,
HYDROSTAR,
WAMIT, etc.

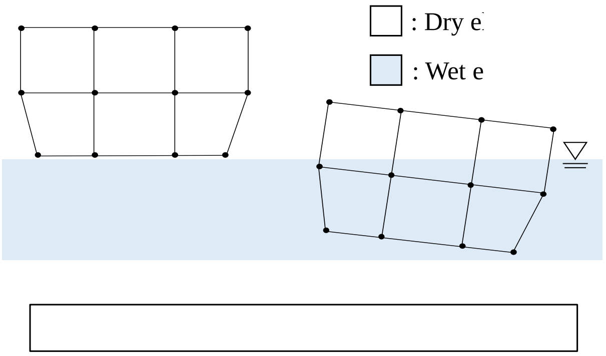


Structural mesh model

Commercial
FE software



(a)

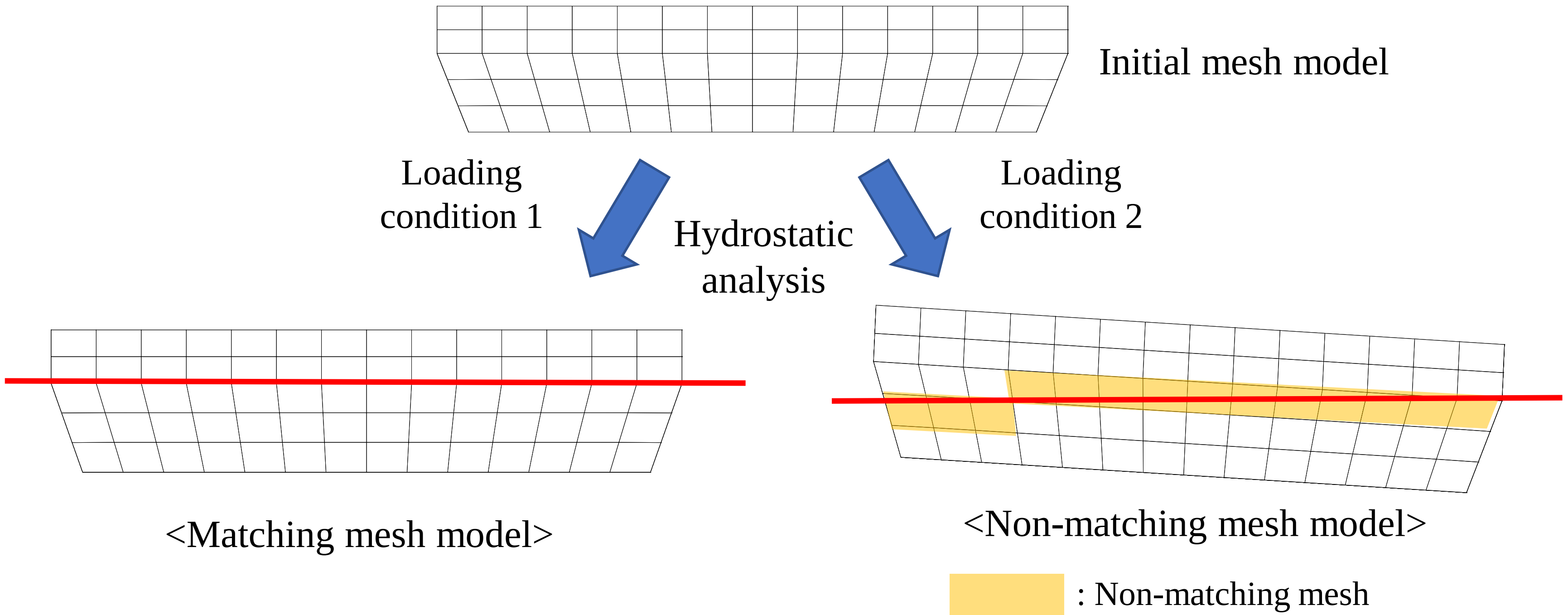


(b)

- We propose the integrated hydro –static and –dynamic analysis using a single integrated mesh model

Motivations

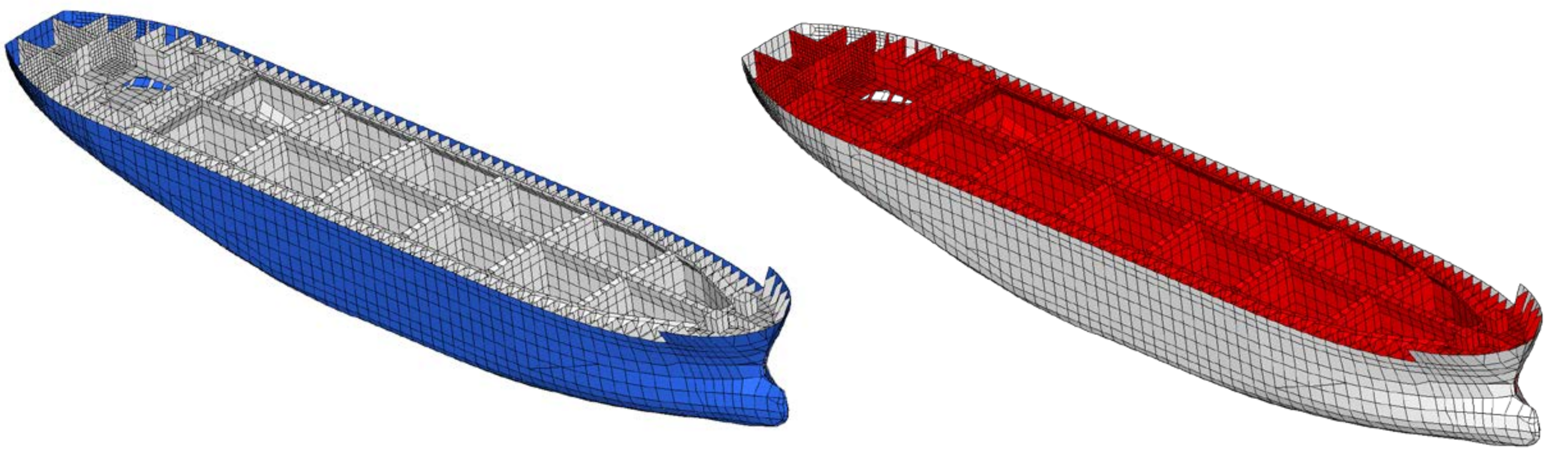
- **Non-matching mesh after hydrostatic analysis**



A non-matching with free surface problem occurs in the initial mesh model at the hydrostatic equilibrium.

Motivations

- **Non-matching mesh problem**
  - In the case of a floating structure model that includes internal members, it is very difficult to modify the mesh or create a new mesh model.

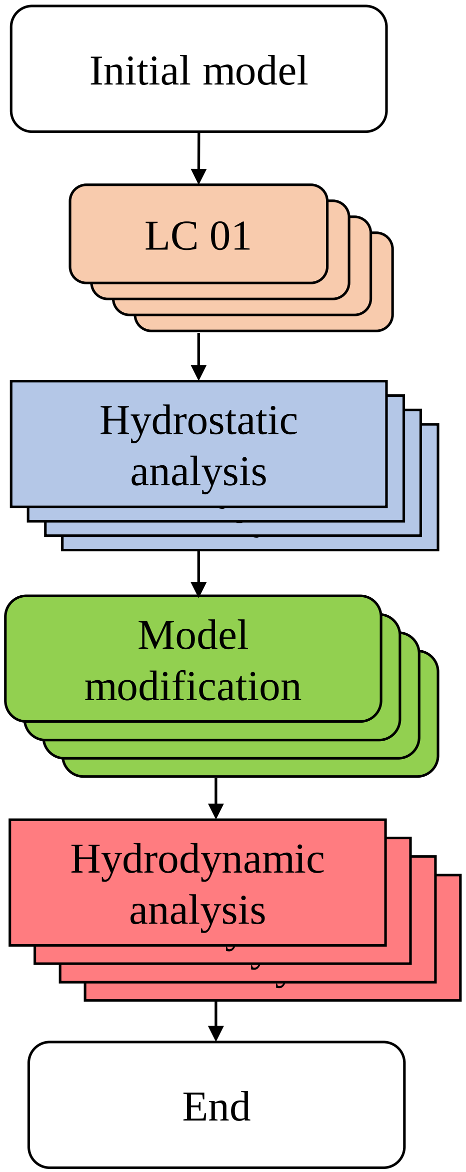


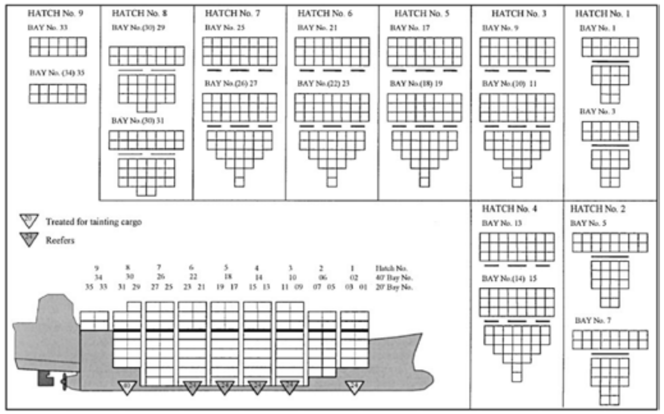
outer shell mesh change → internal mesh change

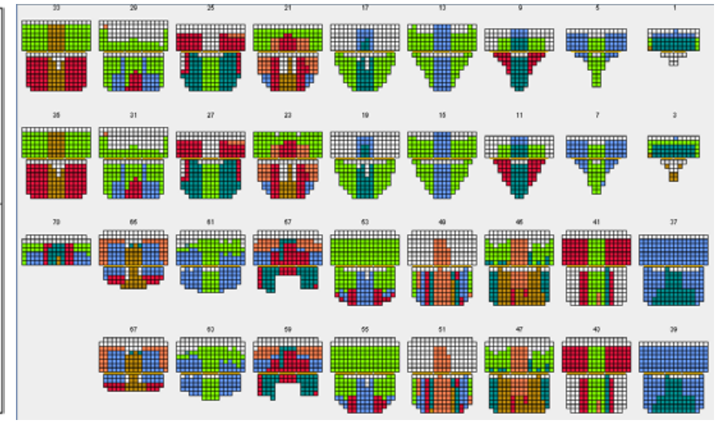
Remeshing process increases structural DOFs and computational cost.

Motivations

- **Non-matching mesh problem**







- Hydrostatic analysis is required as many as the number of loading conditions to be considered.

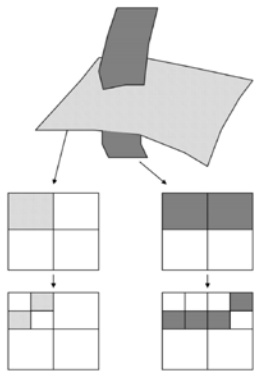
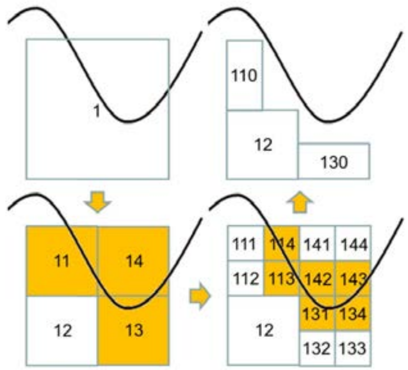
- In the case of ships structures having many loading conditions, it is very complicated to modify the mesh model one by one.

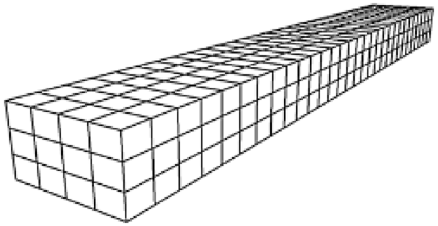
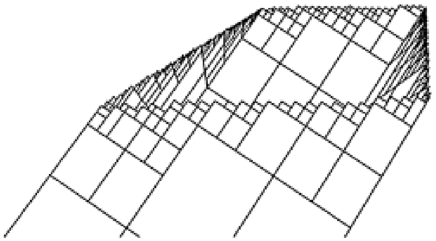
Related works

- **Mesh adjustment**
  - ✓ Mesh model modification according to wet surface at the hydrostatic equilibrium state.

- **Remeshing algorithm**
  - ✓ **Ko KH, et al. (2011)**
    - Development of panel generation system for seakeeping analysis.

  - ✓ **Rodrigues JM, Guedes Soares C(2017)**
    - Froude-Krylov forces from exact pressure integrations on adaptive panel meshes in a time domain partially nonlinear model for ship motions.

Related works

- **Numerical integration method**

  ✓ **Lee et al. (2016)**
  - Nonlinear hydrostatic analysis of flexible floating structures

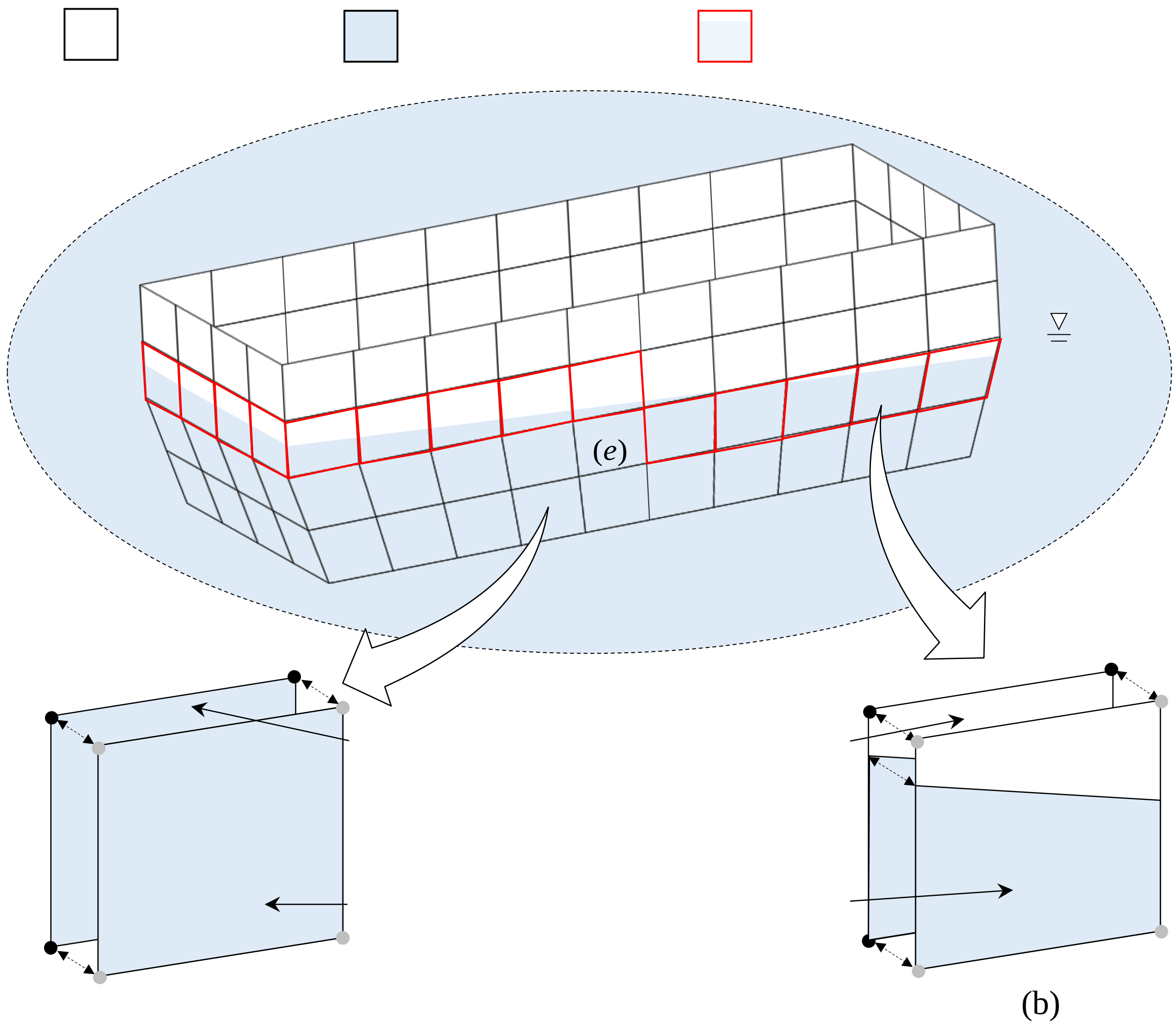
  ✓ **Hoareau C, Deü JF. (2019)**
  - Nonlinear equilibrium of partially liquid-filled tanks: A finite element/level-set method to handle hydrostatic follower forces.

  ✓ **Narayanan NK, et al. (2020)**
  - Monolithic and partitioned approaches to determine static deformation of membrane structures due to ponding.

- **These studies are focused only hydrostatic equilibrium state**

Numerical methods(proposed)

- **Non-matching mesh treatment**



Non-matching mesh treatment without remeshing process

Numerical methods(proposed)

- **Non-matching mesh treatment terms**

$$\begin{bmatrix} -\omega^2\,{}^0\mathbf{S}_M + {}^0\mathbf{S}_K + \boxed{{}^0\mathbf{S}_{CH}} & \boxed{\hat{j}\omega\,{}^0\mathbf{S}_D} \\ \boxed{\hat{j}\omega\,{}^0\mathbf{F}_G} & \boxed{{}^0\mathbf{F}_M - {}^0\mathbf{F}_{Gn}} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{U}} \\ \hat{\mathbf{\Phi}} \end{bmatrix} = \begin{bmatrix} {}^t\mathbf{R}_B + {}^t\mathbf{R}_S \\ \boxed{4\pi\,{}^t\mathbf{R}_I} \end{bmatrix}$$

$${}^0\mathbf{S}_M = \mathop{\mathsf{A}}_{e=1}^{N} \int_{{}^0V_S^{(e)}} {}^0\rho_S \mathbf{H}_i^{(e)\mathrm{T}} \mathbf{H}_i^{(e)} d\,{}^0V_S \qquad {}^0\mathbf{S}_{CH} = {}^0\mathbf{S}_{KN} - {}^0\mathbf{S}_{HD} - {}^0\mathbf{S}_{HN}$$

$${}^0\mathbf{S}_D = \mathop{\mathsf{A}}_{e=1}^{M} \int_{{}^0S_W^{(e)}} \rho_W \mathbf{H}_i^{(e)\mathrm{T}}\,{}^0n_i^{(e)} \mathbf{P}^{(e)} d\,{}^0S_W \qquad {}^0\mathbf{F}_M = \mathop{\mathsf{A}}_{e=1}^{M} \int_{{}^0S_W^{(e)}} \alpha^{(e)} \mathbf{P}^{(e)\mathrm{T}} \mathbf{P}^{(e)} d\,{}^0S_W$$

$${}^0\mathbf{F}_G = \mathop{\mathsf{A}}_{e=1}^{M} \int_{{}^0S_W^{(e)}} \mathbf{P}^{(e)\mathrm{T}} \left\{ \mathop{\mathsf{A}}_{\bar{e}=1}^{M} \int_{S_W^{(\bar{e})}} G(x_i;\xi_i)\,{}^0n_i \mathbf{H}_i^{(\bar{e})} d\,{}^0S_\xi \right\} d\,{}^0S_x$$

$${}^0\mathbf{F}_{Gn} = \mathop{\mathsf{A}}_{e=1}^{M} \int_{{}^0S_W^{(e)}} \mathbf{P}^{(e)\mathrm{T}} \left\{ \mathop{\mathsf{A}}_{\bar{e}=1}^{M} \int_{{}^0S_W^{(\bar{e})}} \nabla_\xi G(x_i;\xi_i) \mathbf{P}^{(\bar{e})} d\,{}^0S_\xi \right\} d\,{}^0S_x$$

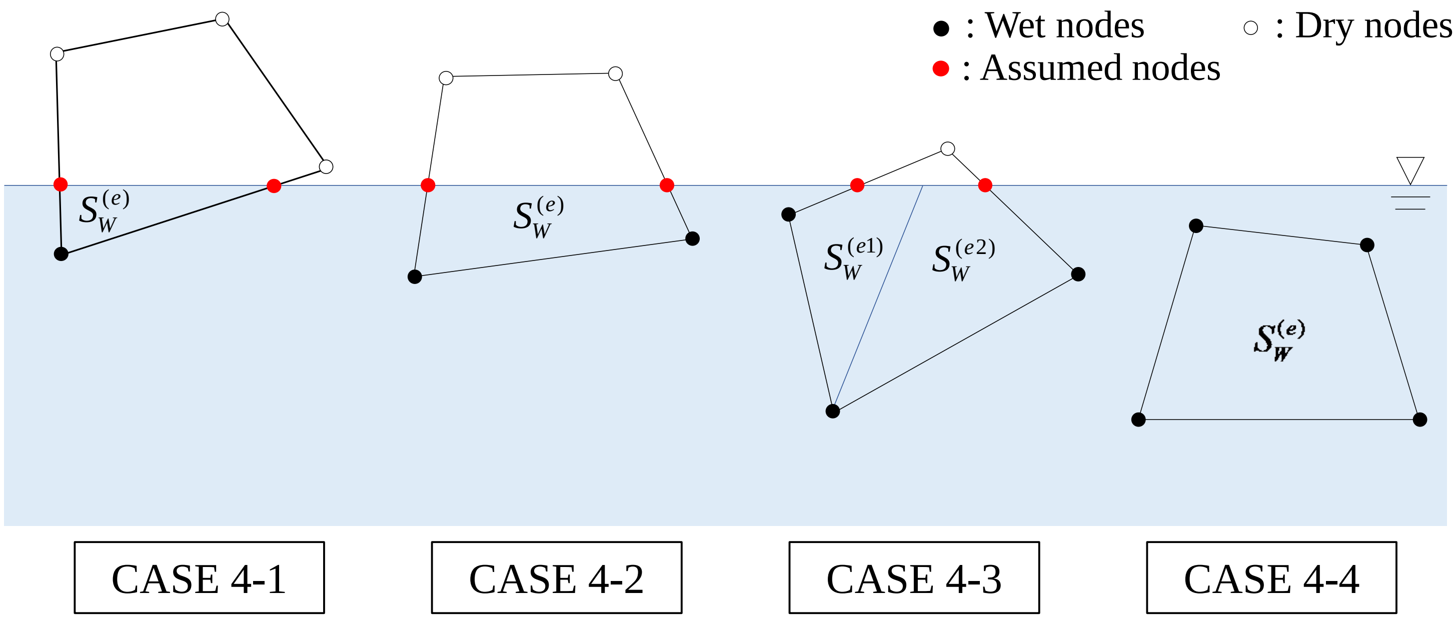
$${}^t\mathbf{R}_B = \mathop{\mathsf{A}}_{e=1}^{N} \int_{{}^0V_S^{(e)}} \mathbf{H}_i^{(e)\mathrm{T}}\,{}^t\hat{\mathbf{f}}_i^{B(e)} d\,{}^0V_S \qquad {}^t\mathbf{R}_S = \mathop{\mathsf{A}}_{e=1}^{N} \int_{{}^\tau S_S^{(e)}} \mathbf{H}_i^{(e)\mathrm{T}}\,{}^t\hat{\mathbf{f}}_i^{S(e)} d\,{}^0S_S$$

$${}^t\mathbf{R}_I = \mathop{\mathsf{A}}_{e=1}^{M} \int_{{}^0S_W^{(e)}} \mathbf{P}^{(e)\mathrm{T}} \hat{\phi}^I d\,{}^0S_W$$

Numerical integration method applied the terms related wet surface.

Numerical methods(proposed)

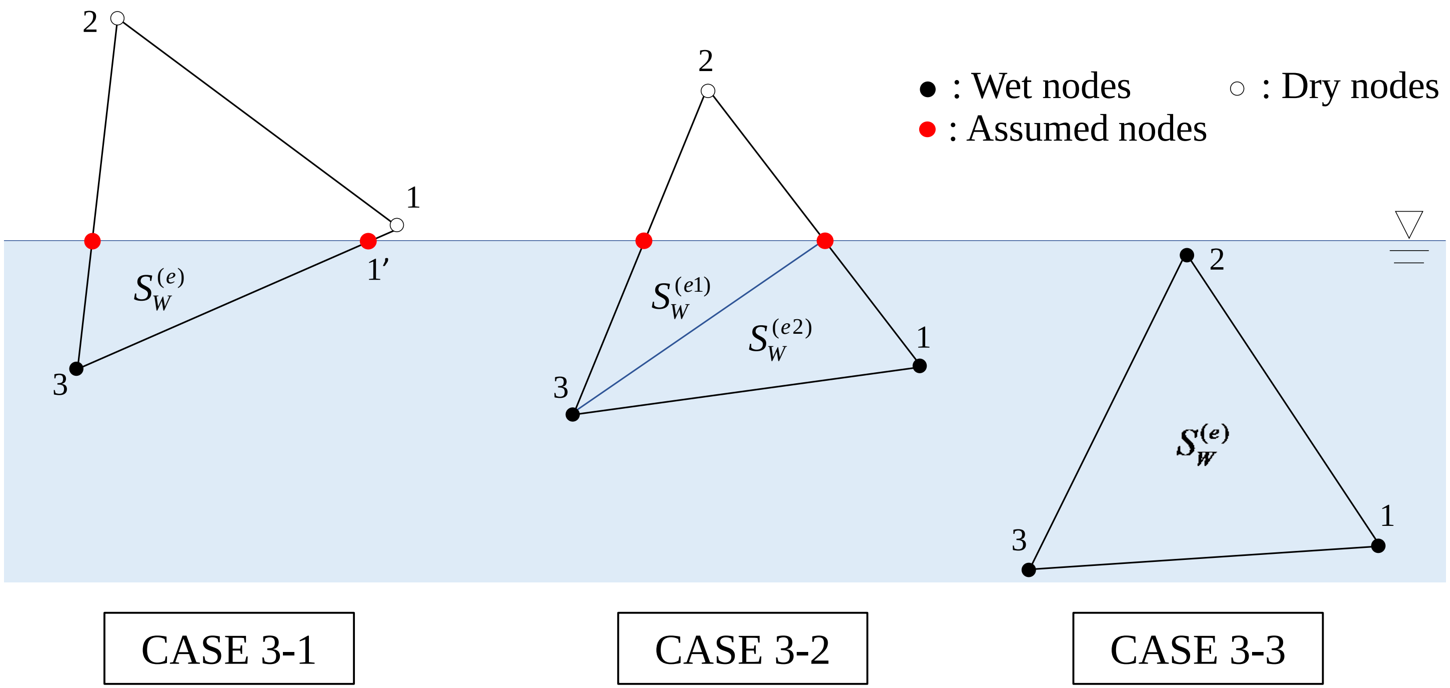
- **Non-matching mesh treatment(4 node)**



- Numerical integration is performed considering four different cases according to the number of wet nodes.
- In the case of CASE 4-1, numerical integration was performed on the triangular area.
- CASE 4-3 is the 3 nodes are below the free surface, the pentagon-shape wetted part is divided into two rectangular subparts.

Numerical methods(proposed)

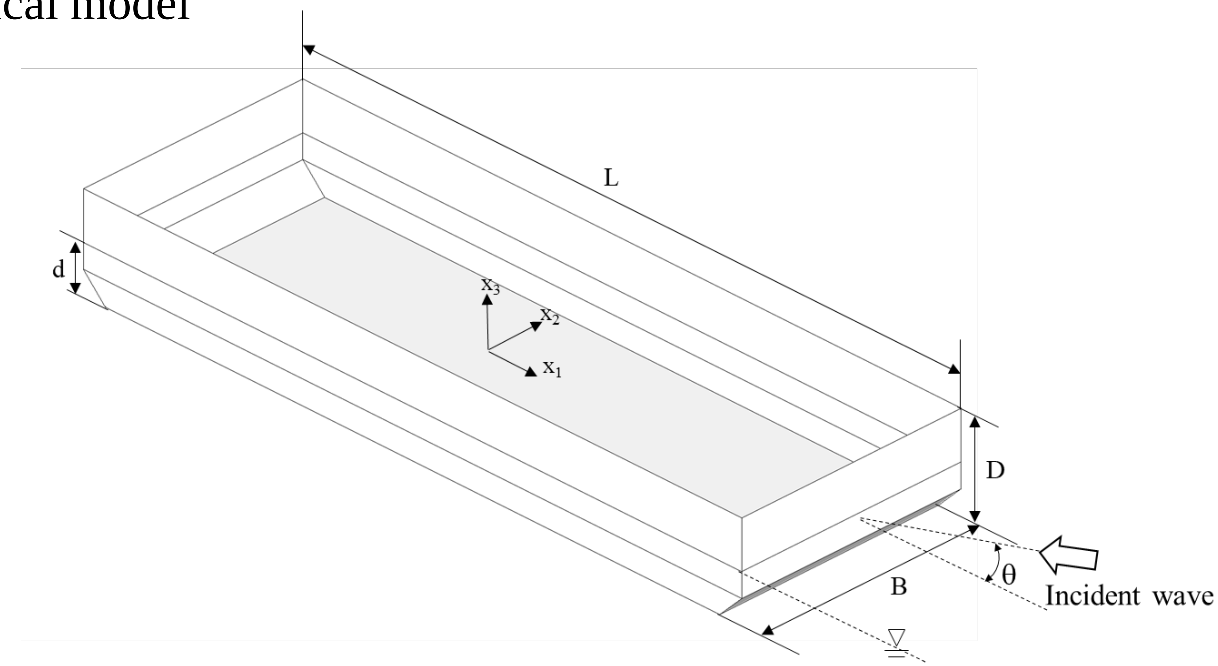
- **Non-matching mesh treatment(3 node)**



- CASE 3-2 shows a partially wet element with 2 wet nodes. The wet surface part of the element is divided into two subtriangles. Three-point Gaussian quadrature is performed in each subtriangular areas.

Numerical example

- **Simple barge problem : Rigid body**
  - ✓ Numerical model

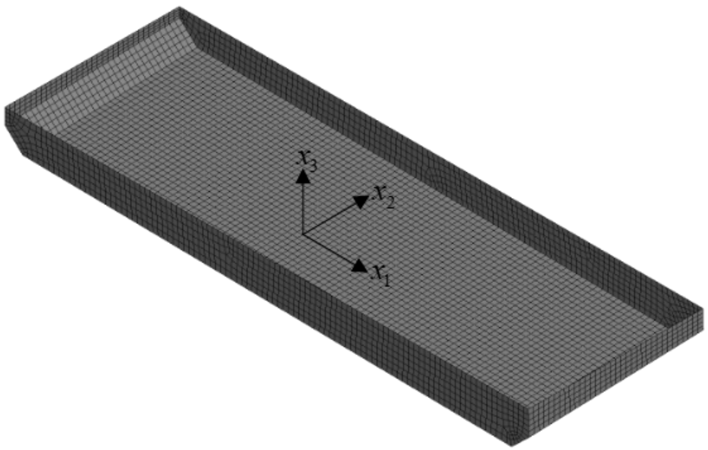


<table>
<tr><td colspan="2">Length [m]</td><td>150.0</td></tr>
<tr><td colspan="2">Breadth [m]</td><td>50</td></tr>
<tr><td colspan="2">Draft [m]</td><td>10</td></tr>
<tr><td colspan="2">Displacement [m^3]</td><td>73,750</td></tr>
<tr><td colspan="2">KG [m]</td><td>10</td></tr>
<tr><td rowspan="3">Radius of gyration [m]</td><td>Roll</td><td>20</td></tr>
<tr><td>Pitch</td><td>39</td></tr>
<tr><td>Yaw</td><td>39</td></tr>
</table>

- **Simple barge problem : Rigid body**

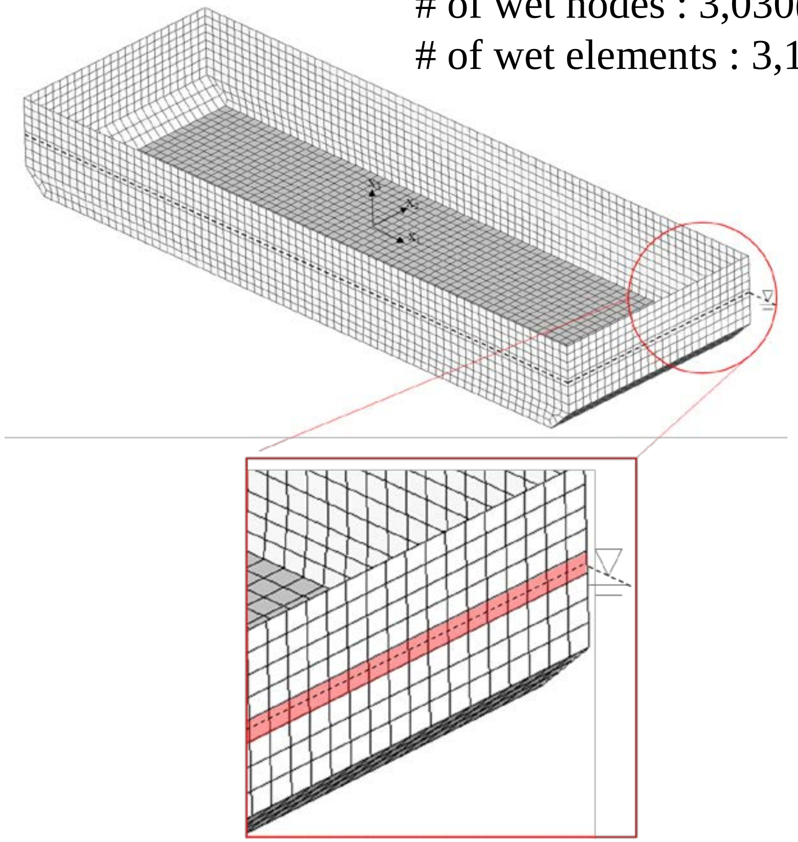
✓ Numerical model

# of wet nodes : 3,030(4,020)
# of wet elements : 3,128(3,920)



<Numerical model for AQWA>

# of nodes : 3,675
# of elements : 3,578

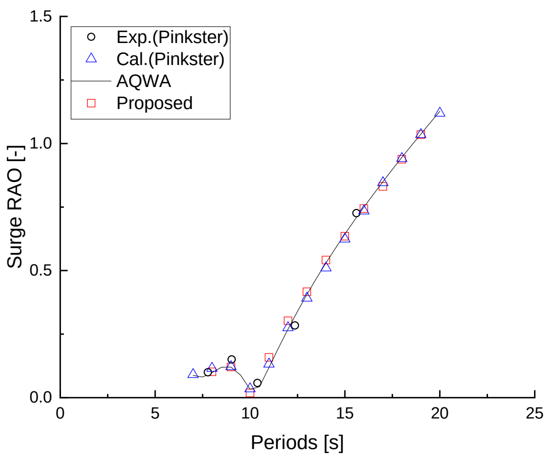


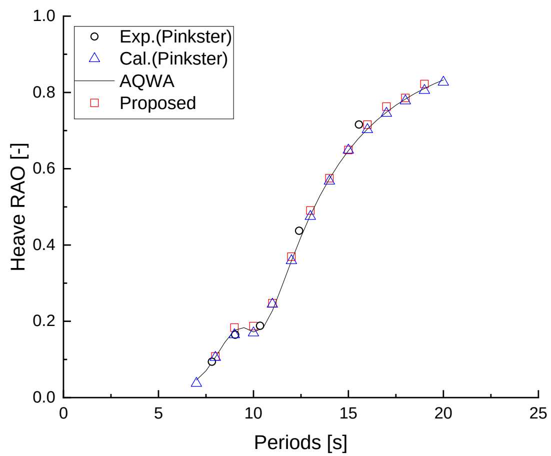
<Numerical model for proposed method>

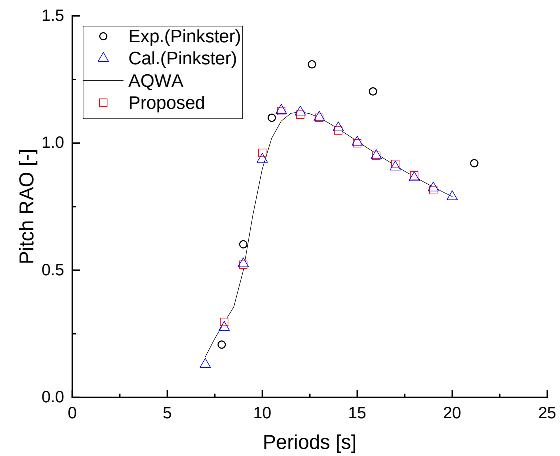
- The wet surface was discretized for the AQWA analysis.
- Non-matching mesh condition was intended to verified the proposed method.
- One angle of incident wave (θ =0°), and wave periods T from 7 to 20 s ($\triangle T$ = 1 s) are considered.

Numerical example

- **Simple barge problem : Rigid body**
  - ✓ Results







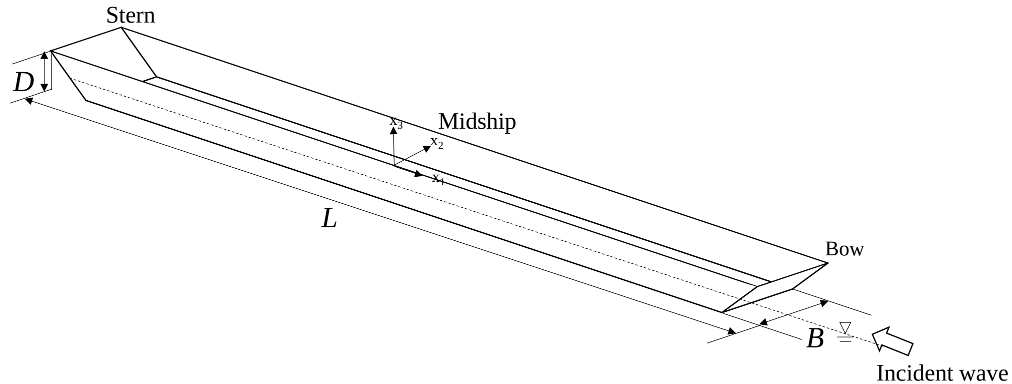
- The results from proposed method, AQWA results, and calculation and experiment of Pinkster(1977)* are compared.
- The surge and heave results from various methods are in good agreement.
- In the case of pitch motion, there was a slight difference in low frequency.
  - ✓ Joe et al.** inferred the difference between the test result of the pitching motion and the calculation as an error in the radius of gyration for the model test.

- Pinkster JA, Oortmerssen G van. Computation of the First and Second Order Wave Forces on Oscillating Bodies in Regular Waves. *Proc 2nd Int Conf Numer Sh Hydrodyn*. Published online 1977:136-159.

**Jo, H.J., et al., 1997. A study on the steady drift forces on barge. Bulletin of the Korean Society of Fisheries Technology, 33(1), pp. 38-45.

Numerical example

- **A floating hull : Rigid and flexible body**
  - ✓ Numerical model



Length [m]	100.0
Breadth [m]	10
Depth [m]	4
Displacement [m^3]	73,750

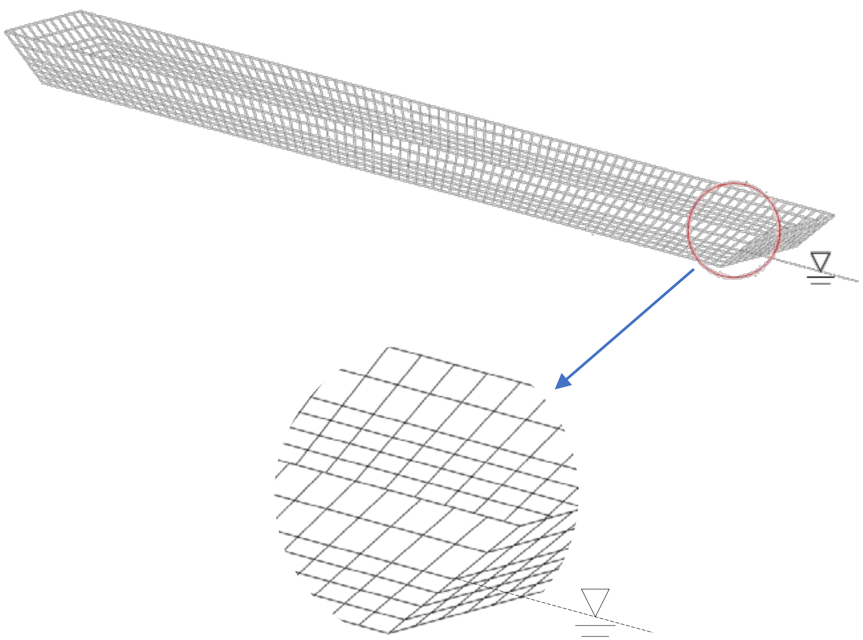
	Thickness [m]	Density [kg/m^3]	Young's modulus [Pa]	Poisson's ratio
Side	0.03	5.0e+5	2e+12	0.3
Bottom	0.03	6.3585e+4	2e+12	0.3

- To verify the proposed method in flexible structure, simple floating structure model* is used in this example.
- The initial configuration as the hydrostatic equilibrium state of the rigid body case and use this configuration for the reference configuration of hydrodynamic analysis.
- One angle of incident wave (θ =0°), and wave periods T from 3 to 12 s are considered.

* Kim K-T, Lee P-S, Park KC. A direct coupling method for 3D hydroelastic analysis of floating structures. *Int J Numer Methods Eng*. 2013;96(13)

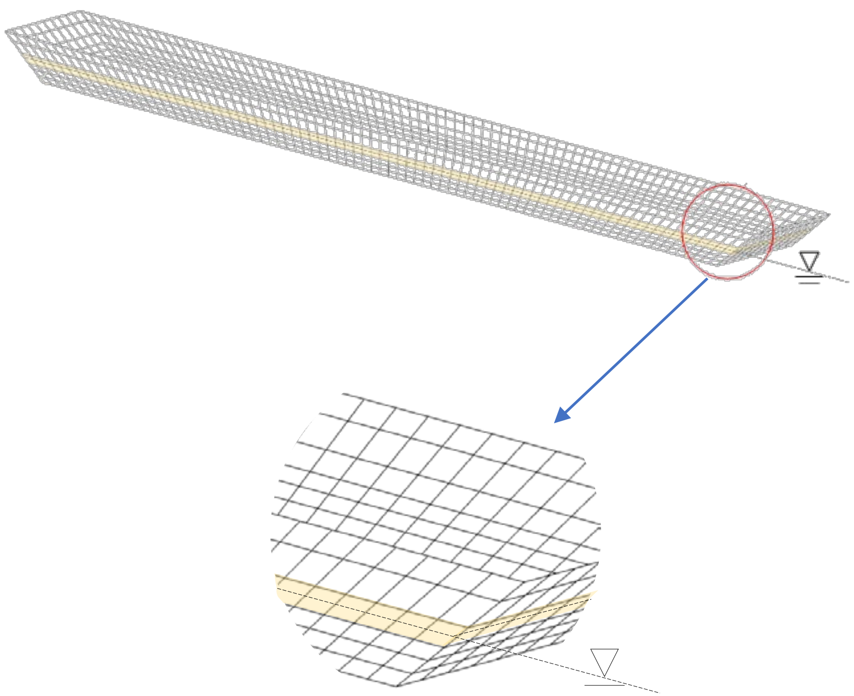
Numerical example

- **A floating hull : Matching mesh model vs. Non-matching mesh model**
  - ✓ Numerical model



<Matching mesh model>

# of wet nodes : 1,611(1,971)
# of wet elements : 1,520(1,880)

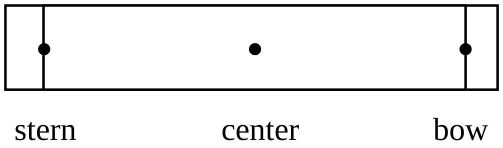


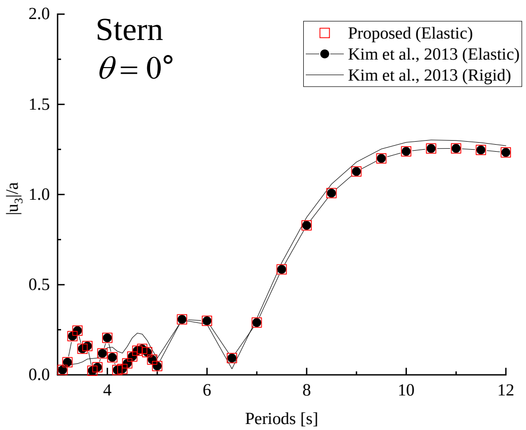
<Non-matching mesh model>

# of wet nodes : 1,431(1,971)
# of wet elements : 1,520(1,880)

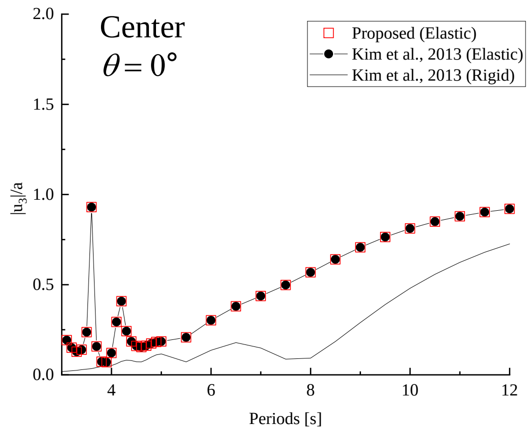
Numerical example

- **A floating hull: Rigid and flexible body**
  - ✓ Results

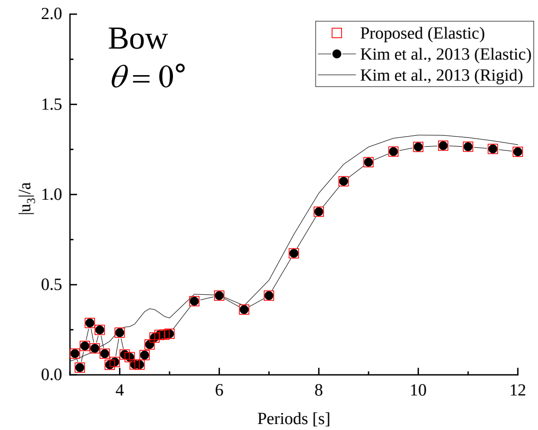




<Stern>



<Center>



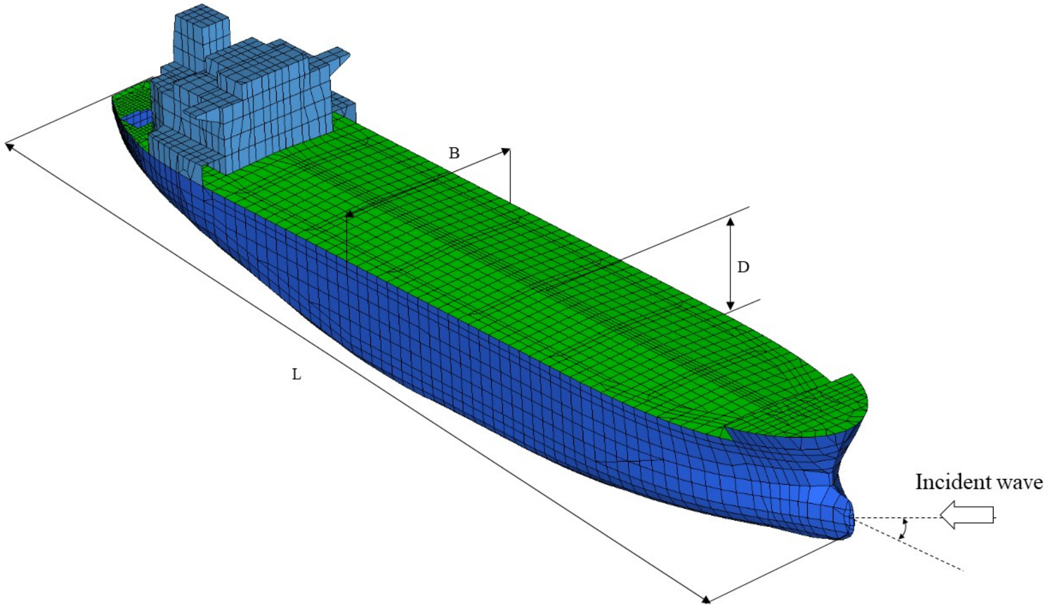
<Bow>

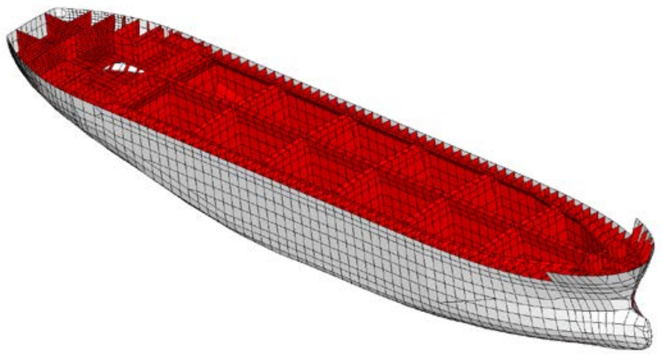
- Rigid results are calculated from the matching mesh model.
- Results are indicated at 3 points(stern, center, bow) on the bottom of the floating hull.
- It can be seen that the results for flexible body obtained from the proposed(non matching mesh model) and previous(matching mesh model) methods are similar.

Numerical example

- **Whole ship model**

  ✓ Numerical model



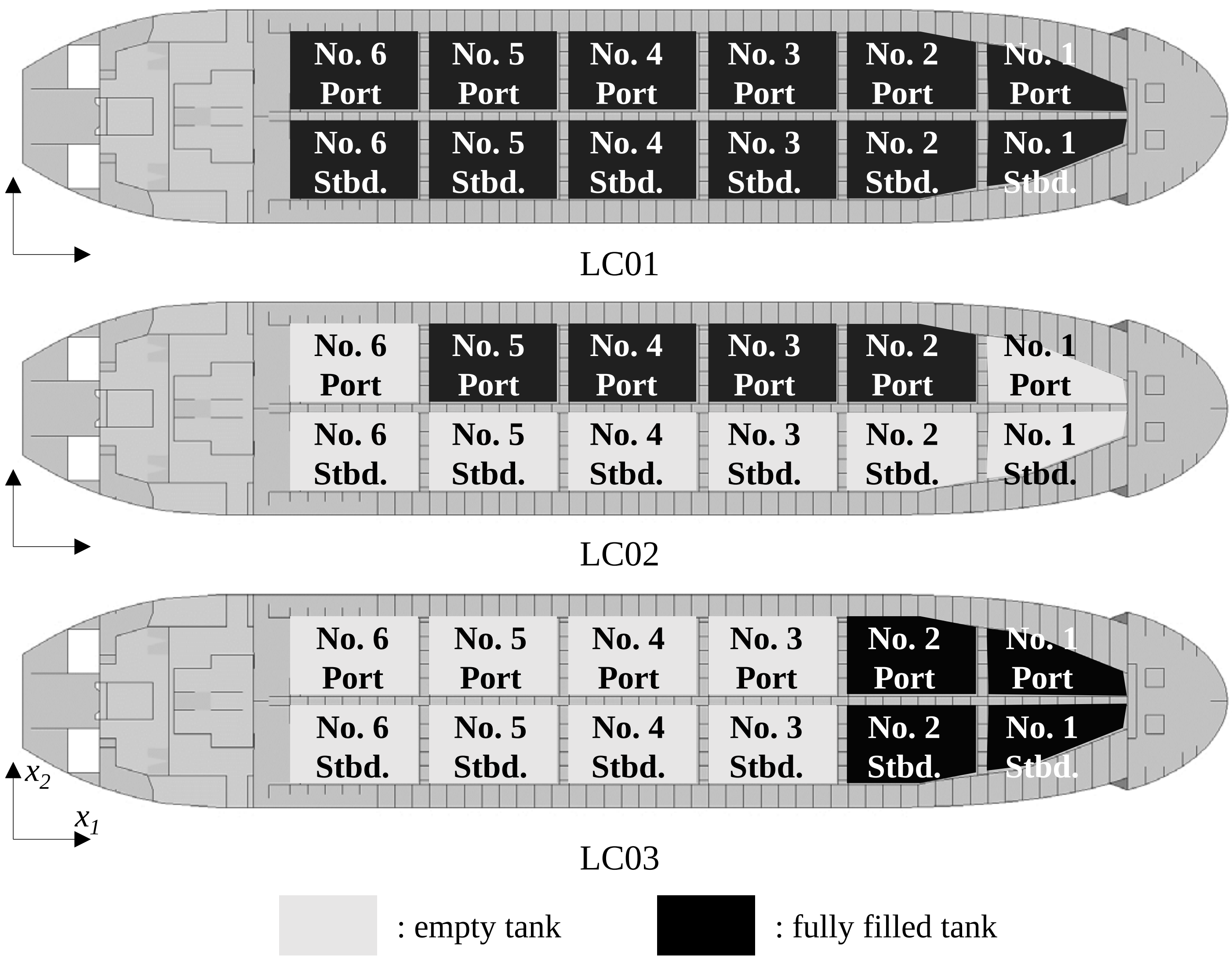


Length [m]	181.0
Breadth [m]	32.2
Depth [m]	19.5

- In order to confirm the applicability of the proposed method in the whole ship model, it was applied to the whole ship model.
- The total number of elements used is 17,029 and the total degree of freedom is 57,585.
- All the RAO results of rigid body case obtained from the proposed method and AQWA .
- 3 loading case are considered.
- One angle of incident wave (θ =45°), and wave periods T from 8 to 26 s ($\triangle T$ = 1 s) are considered.

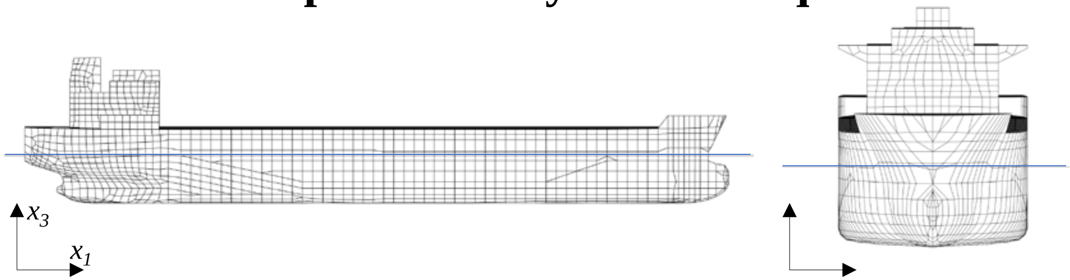
Numerical example

- **Whole ship model : 3 Loading cases**

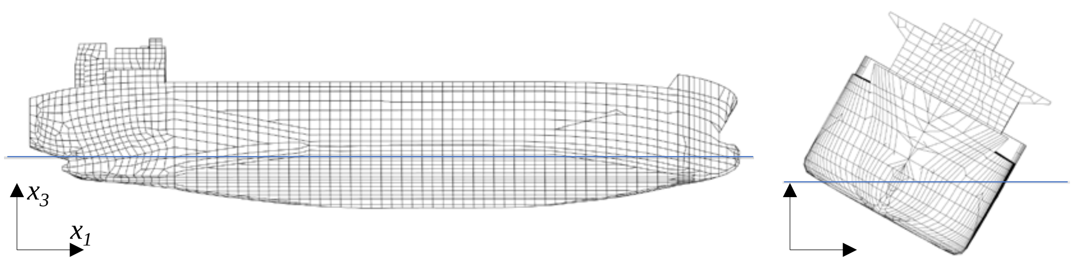


Numerical example

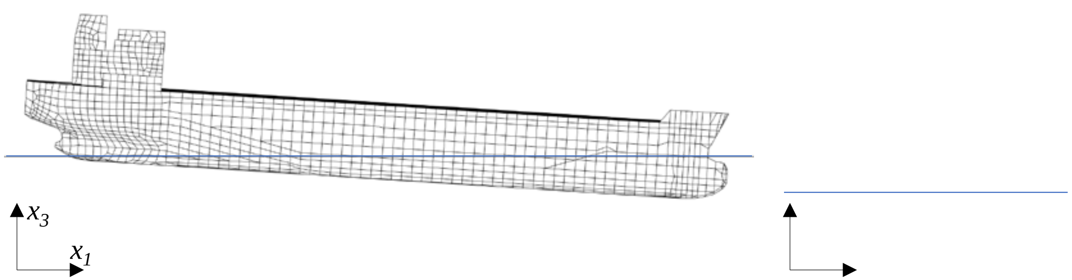
- **Whole ship model : Hydrostatic equilibrium state**



LC01



LC02

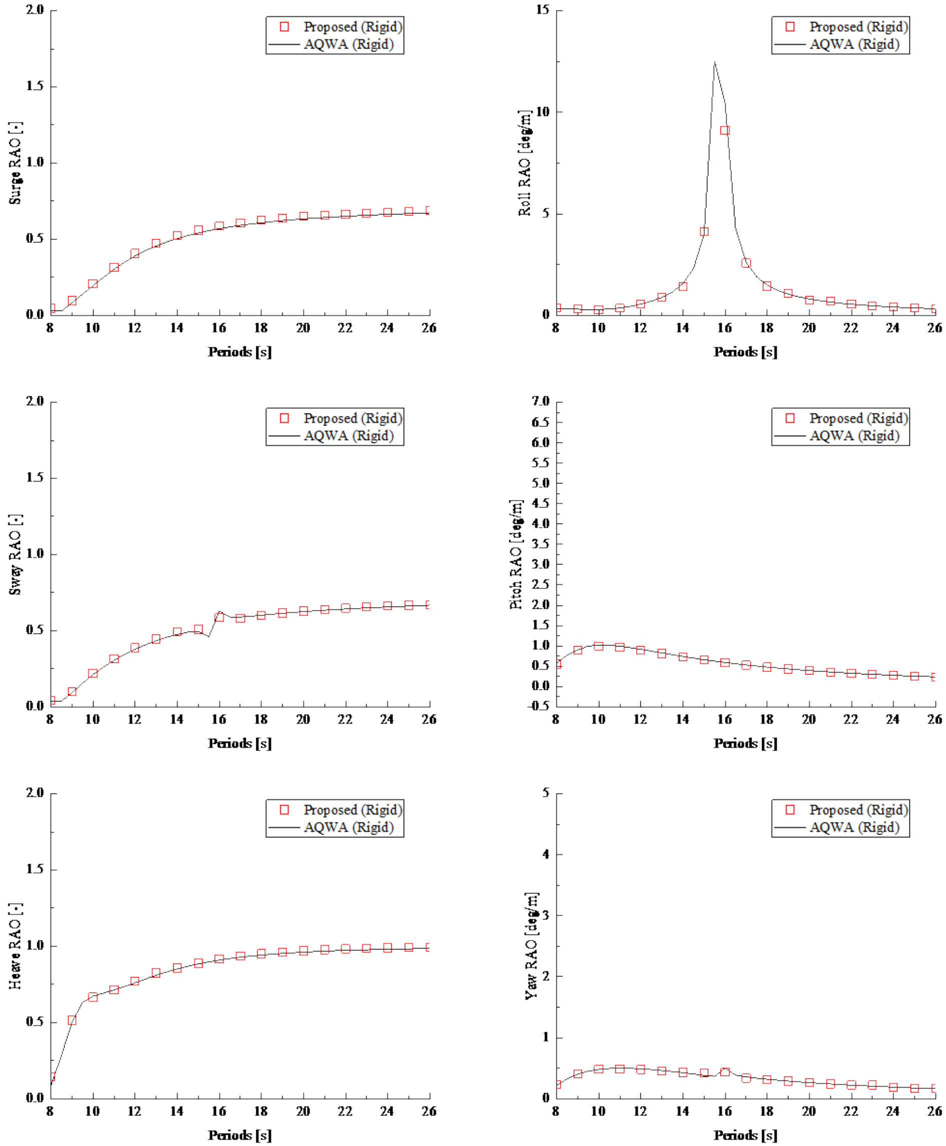


LC03

Loading cases		**LC01**	**LC02**	**LC03**
Number of iterations		10	9	9
Displacement [m^3]		52,525	27,025	25,750
Radius of gyration [m]	Roll	12.3	12.2	12.6
	Pitch	40.7	40.9	46.6
	Yaw	41.2	41.0	46.9
COG [m]	x	94.35	91.489	112.03
	y	0.000	2.573	-0.002
	z	-1.400	0.588	2.1915
COB [m]	x	94.35	91.489	112.03
	y	0.000	2.573	-0.002
	z	-5.385	-3.543	-3.518
Trim angle [◦]		0.0	0.0	3.6
Heel angle [◦]		0.0	29.9	0.0

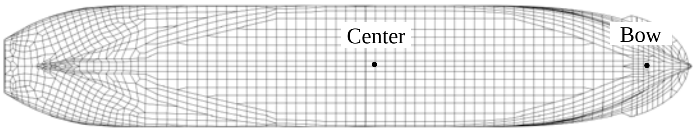
Numerical example

- **Whole ship model : rigid body results (LC01, θ = 45°)**

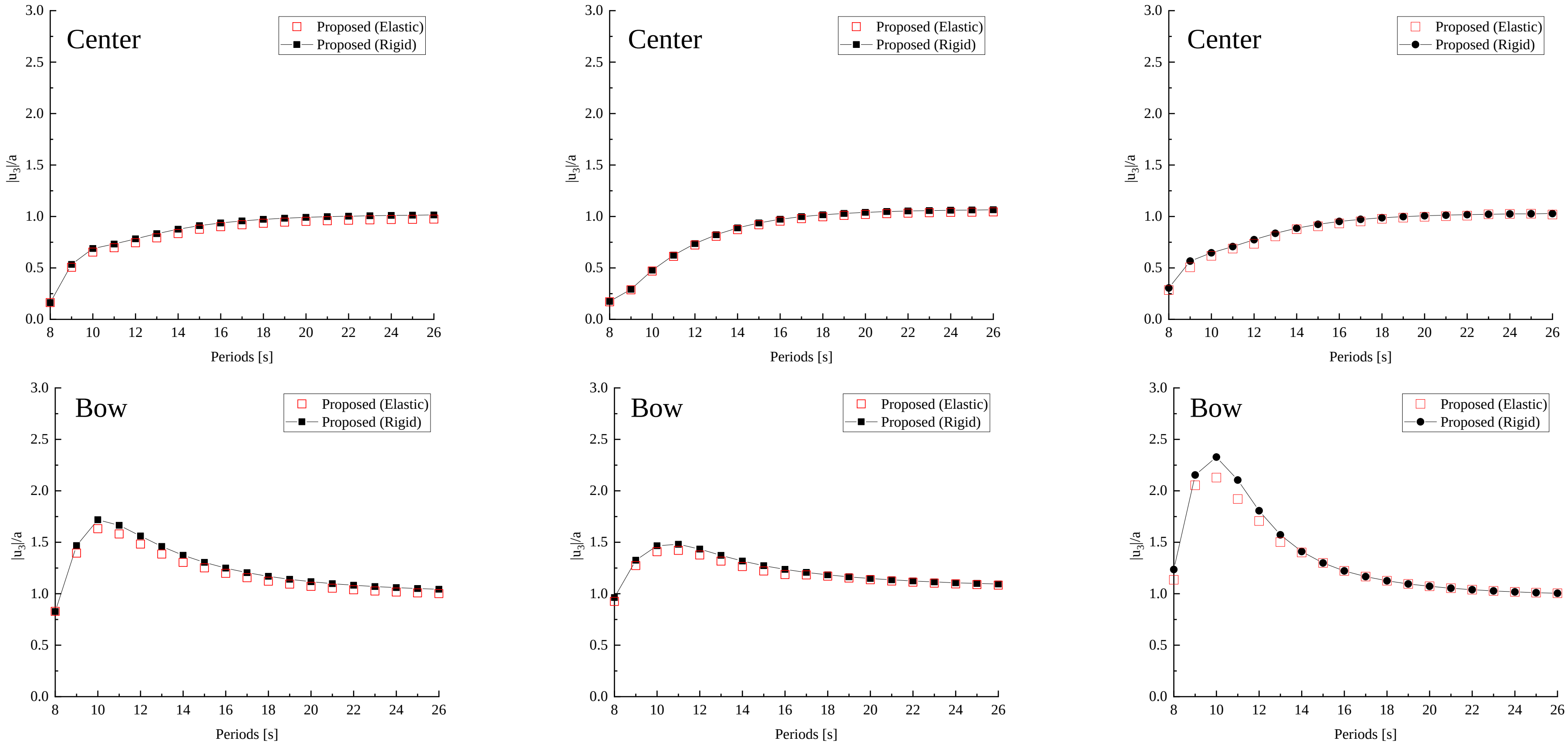


Numerical example

- **Whole ship model : flexible body results (θ = 45°)**



(a) Measuring points



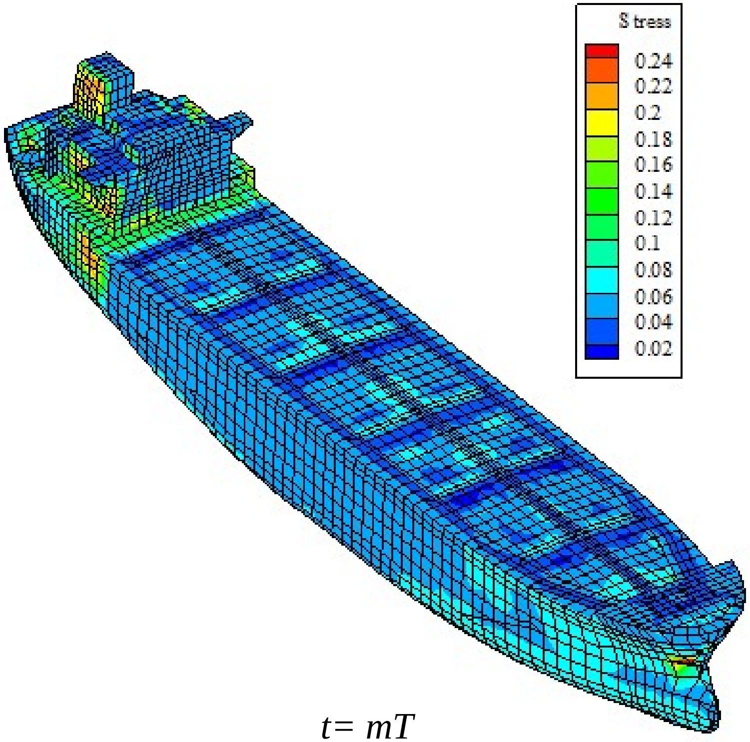
<LC01> <LC02> <LC03>

- For the flexible case, elastic modulus *E*= 210GPa, and Poisson's ratio *ν* = 0.3 are used.
- Results are indicated at two points on the bottom of the ship.

Numerical example

- **Whole ship model : flexible body results (LC01, θ = 45°)**

- The von-Mises stress distribution are represented.
- The stresses normalized by the yield stress (355 MPa) are represented.



t= mT

Numerical example

- **Whole ship model**

✓ Time required for each process

Conventional method (ORCA3D/AQWA/ANSYS)	Kim et. al., 2013	Proposed method
Hydrostatic panel modeling: 30 min *	Hydrostatic mesh modeling: 20 min *	Integrated mesh modeling: 20 min *
Hydrostatic analysis 3×3 min	Hydrostatic analysis: 3×15 min	Hydrostatic analysis: 3×15 min
Hydrodynamic panel modeling: 3×30 min *	Hydroelastic mesh modeling: 3×60 min *	
Hydrodynamic analysis: 3×4 min	Hydroelastic analysis: 3×12 min	Hydroelastic analysis: 3×12 min
Structural mesh modeling: 3×60 min*		
Structural FE analysis: 3×2 min		
Total time: 327 min (100 %)	Total time: 281 min (85.9 %)	**Total time: 101 min (30.9 %)**

* Manual operations are involved.

- The total time required is reduced by 30% compared to the conventional method.

Closure

✓ Conclusions

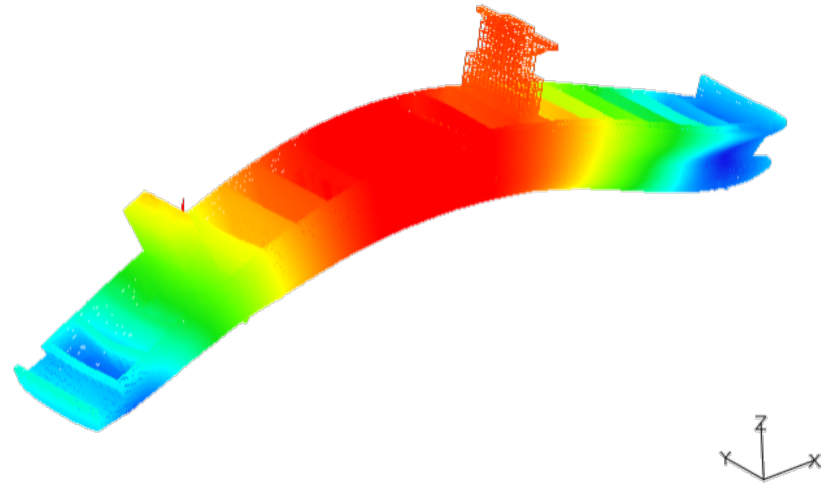
- An integrated hydro–static and dynamic analysis has been proposed
  - ➢ An integrated hydro–static and dynamic formulation has been proposed.
    - Hydrostatic analysis : Incremental nonlinear analysis
    - Hydrodynamic analysis : Frequency domain
  - ➢ An effective non-matching mesh treatment method for hydrodynamic analysis of flexible floating structures were developed.
    - Hydro static and dynamic analysis are performed using a single mesh model.

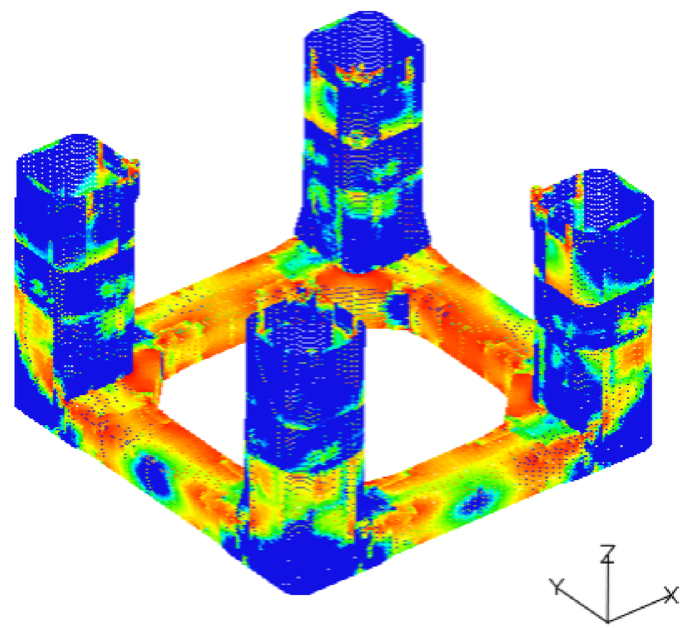
- Compared to conventional procedures, similar solution accuracy was obtained but total analysis time was significantly reduced.

3. Direct calculation of stress RAOs in hydroelastic analysis

Motivations

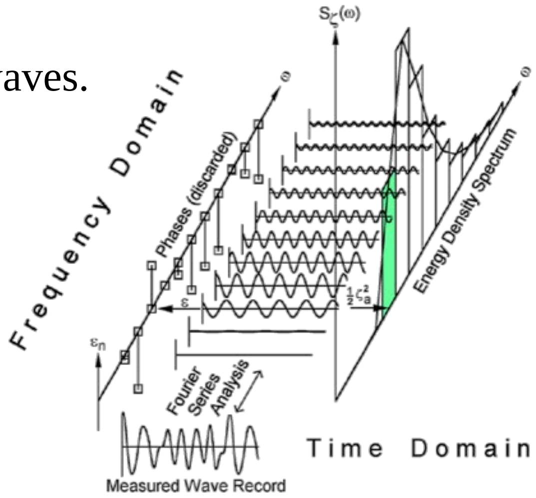
- **Hydroelastic analysis is performed for the strength evaluation of ships and floating structures**



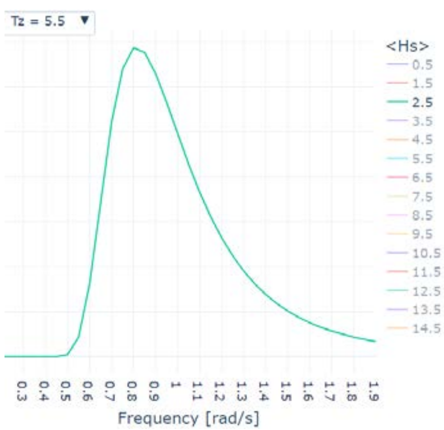


Motivations

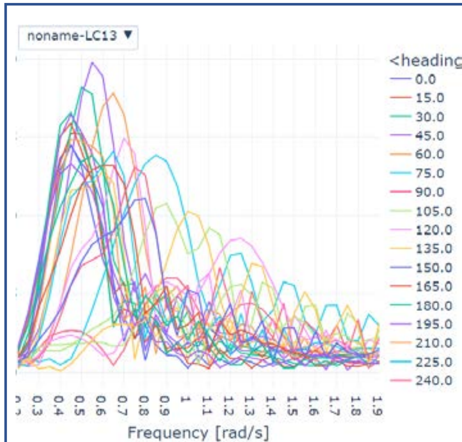
- **Wave spectrum :** $S(\omega)$
  - The waves at sea are irregular, but irregular waves can be represented as the linear superposition of regular waves.
  - Pierson-Moskowitz, JONSWAP, etc.



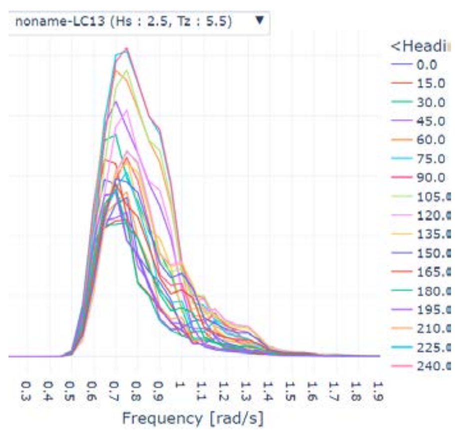
- **RAO (Response Amplitude Operator)** : $H(\omega,\theta)$
  - The magnitude of the response to regular waves with unit amplitude.
- **Response spectrum :** $R(\omega,\theta)$
  - $R(\omega,\theta) = H(\omega,\theta)^2 \times S(\omega)$



Wave Spectrum



RAO



Response Spectrum

Motivations

- **Displacement RAO**

$$[-\omega^2({}^0\mathbf{S}_M + {}^0\mathbf{S}_{MA}) + j\omega\,{}^0\mathbf{S}_{CW} + {}^0\mathbf{S}_K + {}^0\mathbf{S}_{CH}]\hat{\mathbf{U}} = {}^0\mathbf{R}_W$$

$$\hat{\mathbf{U}} = \hat{\mathbf{U}}^{\mathrm{Re}} + \hat{j}\hat{\mathbf{U}}^{\mathrm{Im}}$$

$$H(\omega,\theta) = \frac{|\hat{\mathbf{U}}|}{A}$$ A : wave amplitude

- **Stress(component) RAO**

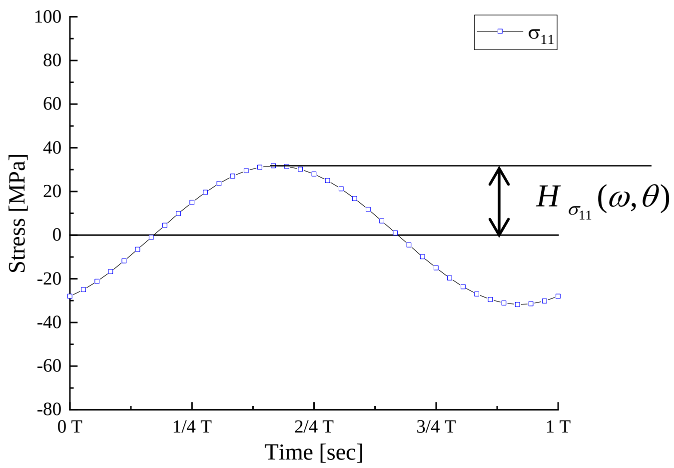
$$\hat{\boldsymbol{\sigma}} = \hat{\boldsymbol{\sigma}}^{\mathrm{Re}} + \hat{j}\hat{\boldsymbol{\sigma}}^{\mathrm{Im}}$$

$$\hat{\boldsymbol{\varepsilon}} = [\hat{\varepsilon}_{11} \quad \hat{\varepsilon}_{22} \quad \hat{\varepsilon}_{33} \quad \hat{\varepsilon}_{12} \quad \hat{\varepsilon}_{23} \quad \hat{\varepsilon}_{31}]^{\mathrm{T}} \qquad \hat{\varepsilon}_{ij} = \frac{1}{2}\left(\frac{\partial \hat{u}_i}{\partial x_j} + \frac{\partial \hat{u}_j}{\partial x_i}\right)$$

$$\hat{\boldsymbol{\sigma}}^{\mathrm{Re}} = \mathbf{C}\hat{\boldsymbol{\varepsilon}}^{\mathrm{Re}}, \quad \hat{\boldsymbol{\sigma}}^{\mathrm{Im}} = \mathbf{C}\hat{\boldsymbol{\varepsilon}}^{\mathrm{Im}}$$

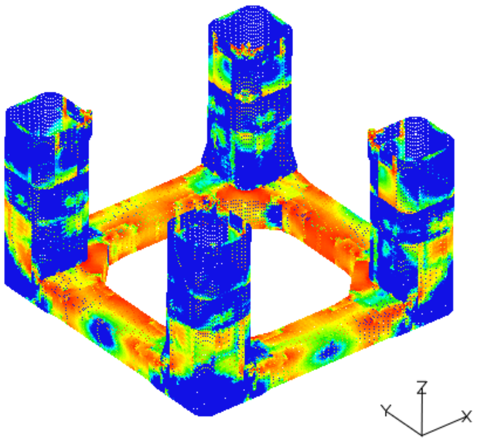
$\mathbf{C}$: stress-strain relation(material) tensor

$$H_{\sigma}(\omega,\theta) = \frac{|\hat{\boldsymbol{\sigma}}|}{A}$$ A : wave amplitude

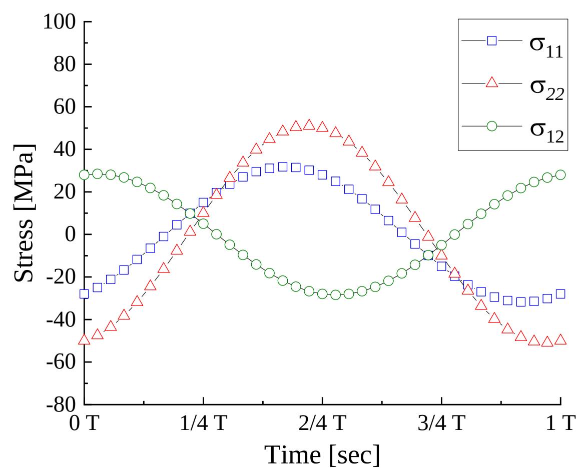


Motivations

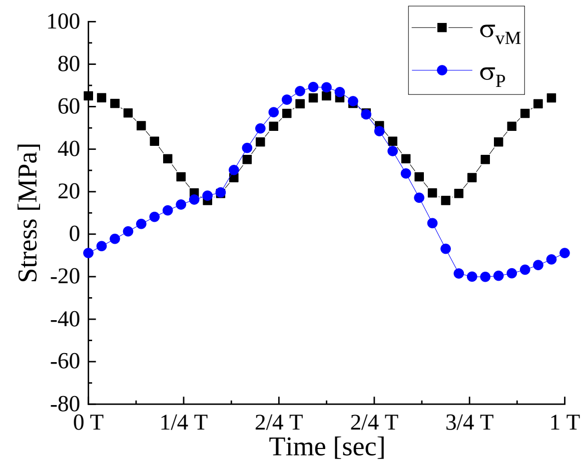
- **Combined stress RAO**



<Combined stress distribution>



<Stress component>



<Combined stress>

- Component stresses are harmonic responses : $\sigma_{ij}(t) = \hat{\sigma}_{ij}^{\mathrm{Re}} \cos \omega t - \hat{j}\hat{\sigma}_{ij}^{\mathrm{Im}} \sin \omega t$
- Various types of stress are used to evaluate the strength of structures.

$$\sigma_{vM}(t) = \sqrt{\sigma_{11}^2(t) + \sigma_{11}(t)\sigma_{22}(t) + \sigma_{22}^2(t) + 3\sigma_{12}^2(t)}$$

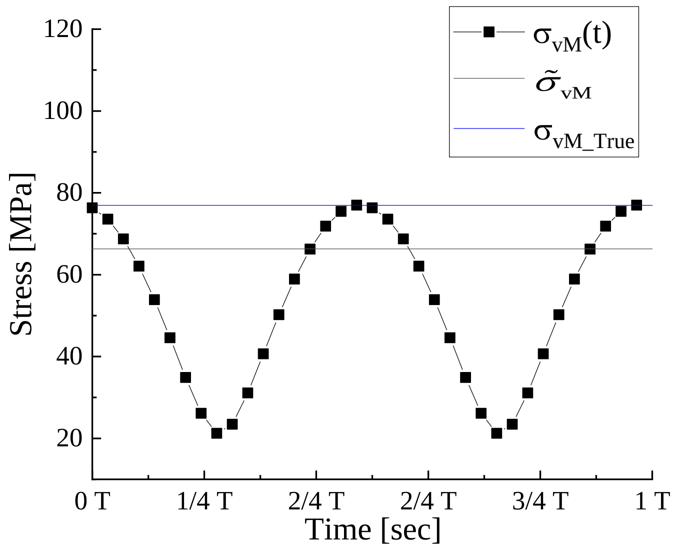
$$P_{1,2}(t) = \frac{\sigma_{11}(t) + \sigma_{22}(t)}{2} \pm \frac{1}{2}\sqrt{\left(\sigma_{11}(t) - \sigma_{22}(t)\right)^2 + 4\sigma_{12}^2(t)}$$

- It is no longer a harmonic response. (non-harmonic function)

→ It is not easy to find the maximum value for design.

Related works

- **Combined stress RAO calculation method**
  - ✓ Calculate stress RAO using the maximum stress for each component($\tilde{\sigma}_{vM}$)
    - A. Preumont, V. Pie´fort, 1994. Predicting random high-cycle fatigue life with finite elements, Journal of Vibration and Acoustics 116, 245–248.
    - T. Lagoda, E. Macha, A. Nieslony, 2005. Fatigue life calculation by means of the cycle counting and spectral methods under multiaxial random loading, Fatigue & Fracture of Engineering Materials & Structures 28, 409–420.
    - HEXAGON, 2021. MSC Apex User Manual.



$$\sigma_{ij}(t) = \hat{\sigma}_{ij}^{\mathrm{Re}} \cos \omega t - \hat{j}\hat{\sigma}_{ij}^{\mathrm{Im}} \sin \omega t$$

$$\tilde{\sigma}_{vM} = \sqrt{\hat{\sigma}_{11}^2 - \hat{\sigma}_{11}\hat{\sigma}_{22} + \hat{\sigma}_{22}^2 + 3\hat{\sigma}_{12}^2}$$

$$\text{where} \quad \hat{\sigma}_{ij} = \sqrt{\hat{\sigma}_{ij}^{\mathrm{Re}} + \hat{\sigma}_{ij}^{\mathrm{Im}}}$$

✓ Phase differences between component stresses are not considered.

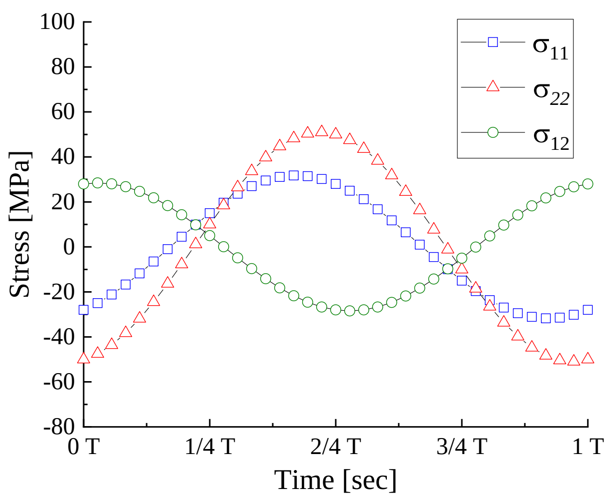
✓ The obtained $\tilde{\sigma}_{vM}$ may differ from the σ_{vM_True}.

Related works

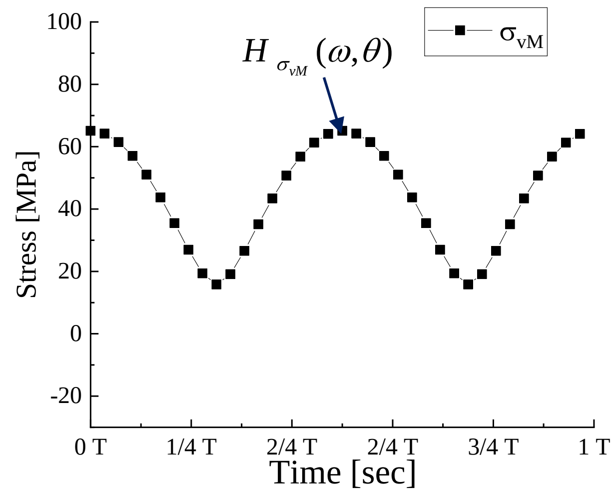
▪ Combined stress RAO calculation method

✓ The maximum stress found by stepping through the whole cycle $H_{\sigma_{vM}}(\omega,\theta) = \max\left(\sigma_{vM}(t)\right)$

- DNV, 2021. Sesam User Manual Xtract.
- ANSYS, 2009. Theory reference for the Mechanical APDL and Mechanical applications.



Stress component



Combined stress

- The stress at a given time (t) of the incoming wave is expressed as a harmonic function.

$$\sigma_{ij}(t) = \hat{\sigma}_{ij}^{\mathrm{Re}} \cos \omega t - \hat{j}\hat{\sigma}_{ij}^{\mathrm{Im}} \sin \omega t$$

- The stresses are calculated by stepping through the whole cycle $(t = \frac{T}{36}, \frac{2T}{36}, \dots, T)$.

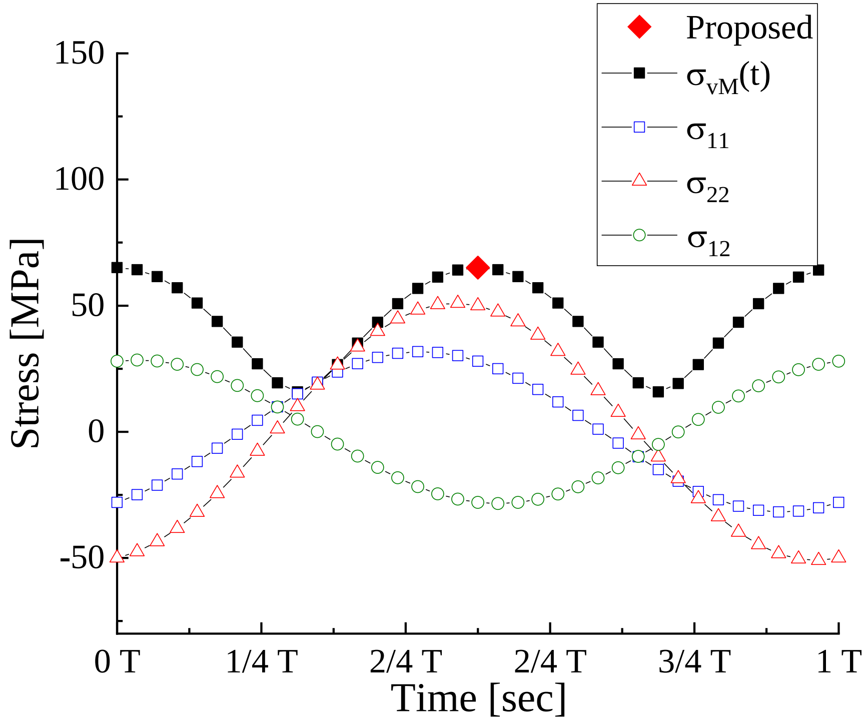
$$\sigma_{vM}(t) = \sqrt{\sigma_{11}^2(t) + \sigma_{11}(t)\sigma_{22}(t) + \sigma_{22}^2(t) + 3\sigma_{12}^2(t)} \qquad \sigma_P(t) = \frac{\sigma_{11}(t) + \sigma_{22}(t)}{2} + \frac{1}{2}\sqrt{\left(\sigma_{11}(t) - \sigma_{22}(t)\right)^2 + 4\sigma_{12}^2(t)}$$

- RAO is determined as the maximum value obtained by calculating the above equation.

$$H_{\sigma_{vM}}(\omega,\theta) = \max\left(\sigma_{vM}(t)\right) \qquad H_{\sigma_P}(\omega,\theta) = \max\left(\sigma_P(t)\right)$$

Research purpose(proposed)

- **Direct calculation method of stress RAO in hydroelastic analysis**



The maximum stress is found by direct calculation without stepping through the whole cycle.

Direct calculation of stress RAOs(proposed)

- **von-Mises stress**
  - ✓ von-Mises stress in time-domain

$$\sigma_{vM}(t)=\sqrt{\frac{3}{2}\left(\sigma_{ij}(t)-\frac{1}{3}\delta_{ij}\sigma_{kk}(t)\right)^2} \qquad \sigma_{ij}(t)=\hat{\sigma}_{ij}^{\mathrm{Re}}\cos\omega t-\hat{\sigma}_{ij}^{\mathrm{Im}}\sin\omega t$$

$$\sigma_{vM}(t)=\sqrt{\frac{\sqrt{(A-B)^2+C^2}}{2}\sin(2\omega t+\phi_1)+\frac{A+B}{2}}$$

$$A=\frac{3}{2}\left(\hat{\sigma}_{ij}^{\mathrm{Re}}-\frac{1}{3}\delta_{ij}\hat{\sigma}_{kk}^{\mathrm{Re}}\right)\left(\hat{\sigma}_{ij}^{\mathrm{Re}}-\frac{1}{3}\delta_{ij}\hat{\sigma}_{kk}^{\mathrm{Re}}\right) \qquad B=\frac{3}{2}\left(\hat{\sigma}_{ij}^{\mathrm{Im}}-\frac{1}{3}\delta_{ij}\hat{\sigma}_{kk}^{\mathrm{Im}}\right)\left(\hat{\sigma}_{ij}^{\mathrm{Im}}-\frac{1}{3}\delta_{ij}\hat{\sigma}_{kk}^{\mathrm{Im}}\right)$$

$$C=3\left(\hat{\sigma}_{ij}^{\mathrm{Re}}-\frac{1}{3}\delta_{ij}\hat{\sigma}_{kk}^{\mathrm{Re}}\right)\left(\hat{\sigma}_{ij}^{\mathrm{Im}}-\frac{1}{3}\delta_{ij}\hat{\sigma}_{kk}^{\mathrm{Im}}\right) \qquad \sin\phi_1=\frac{(A-B)}{\sqrt{(A-B)^2+C^2}},\ \cos\phi_1=\frac{-C}{\sqrt{(A-B)^2+C^2}}$$

  - ✓ The RAO of von-Mises stress

$$H_{\sigma_{vM}}(\omega,\theta)=\sqrt{\frac{\sqrt{(A-B)^2+C^2}}{2}+\frac{A+B}{2}}$$

$$0\le\omega t=n\pi+\frac{\pi}{4}-\frac{\phi_1}{2}\le 2\pi$$

Direct calculation of stress RAOs(proposed)

- **Principal stress**
  - ✓ Principal stresses are represent using stress invariants(I_1, I_2, I_3).

$$P_1 = \frac{I_1(t)}{3} + \frac{2}{3}\left(\sqrt{I_1^2(t) - 3I_2(t)}\right)\cos\phi \qquad P_2 = \frac{I_1(t)}{3} + \frac{2}{3}\left(\sqrt{I_1^2(t) - 3I_2(t)}\right)\cos\left(\phi(t) - \frac{2\pi}{3}\right)$$

$$P_3(t) = \frac{I_1(t)}{3} + \frac{2}{3}\left(\sqrt{I_1^2(t) - 3I_2(t)}\right)\cos\left(\phi(t) - \frac{4\pi}{3}\right)$$

$$I_1(t) = \sigma_{ii}(t) = \hat{\sigma}_{ii}^{\mathrm{Re}}\cos\omega t - \hat{\sigma}_{ii}^{\mathrm{Im}}\sin\omega t$$

$$I_2(t) = \frac{1}{2}\left\{\left(\hat{\sigma}_{ii}^{\mathrm{Re}}\hat{\sigma}_{jj}^{\mathrm{Re}} - \hat{\sigma}_{ij}^{\mathrm{Re}}\hat{\sigma}_{ij}^{\mathrm{Re}}\right)\cos^2\omega t + \left(\hat{\sigma}_{ii}^{\mathrm{Im}}\hat{\sigma}_{jj}^{\mathrm{Im}} - \hat{\sigma}_{ij}^{\mathrm{Im}}\hat{\sigma}_{ij}^{\mathrm{Im}}\right)\sin^2\omega t\right\} - \frac{1}{2}\left\{\left(\hat{\sigma}_{ii}^{\mathrm{Re}}\hat{\sigma}_{jj}^{\mathrm{Im}} + \hat{\sigma}_{jj}^{\mathrm{Re}}\hat{\sigma}_{ii}^{\mathrm{Im}} - \hat{\sigma}_{ij}^{\mathrm{Im}}\hat{\sigma}_{ij}^{\mathrm{Re}} - \hat{\sigma}_{ij}^{\mathrm{Re}}\hat{\sigma}_{ij}^{\mathrm{Im}}\right)\sin\omega t\cos\omega t\right\}$$

$$I_3(t) = \varepsilon_{ijk}\left\{\hat{\sigma}_{1i}^{\mathrm{Re}}\hat{\sigma}_{2j}^{\mathrm{Re}}\hat{\sigma}_{3k}^{\mathrm{Re}}\cos^3\omega t - \left(\hat{\sigma}_{1i}^{\mathrm{Re}}\hat{\sigma}_{2j}^{\mathrm{Im}}\hat{\sigma}_{3k}^{\mathrm{Re}} + \hat{\sigma}_{1i}^{\mathrm{Re}}\hat{\sigma}_{2j}^{\mathrm{Re}}\hat{\sigma}_{3k}^{\mathrm{Im}} + \hat{\sigma}_{1i}^{\mathrm{Im}}\hat{\sigma}_{2j}^{\mathrm{Re}}\hat{\sigma}_{3k}^{\mathrm{Re}}\right)\sin\omega t\cos^2\omega t\right\}$$

$$+\varepsilon_{ijk}\left\{\left(\hat{\sigma}_{1i}^{\mathrm{Im}}\hat{\sigma}_{2j}^{\mathrm{Im}}\hat{\sigma}_{3k}^{\mathrm{Re}} + \hat{\sigma}_{1i}^{\mathrm{Im}}\hat{\sigma}_{2j}^{\mathrm{Re}}\hat{\sigma}_{3k}^{\mathrm{Im}} + \hat{\sigma}_{1i}^{\mathrm{Re}}\hat{\sigma}_{2j}^{\mathrm{Im}}\hat{\sigma}_{3k}^{\mathrm{Im}}\right)\sin^2\omega t\cos\omega t - \hat{\sigma}_{1i}^{\mathrm{Im}}\hat{\sigma}_{2j}^{\mathrm{Im}}\hat{\sigma}_{3k}^{\mathrm{Im}}\sin^3\omega t\right\}$$

$$\phi(t) = \frac{1}{3}\cos^{-1}\left(\frac{2I_1^3(t) - 9I_1(t)I_2(t) + 27I_3(t)}{2(I_1^2(t) - 3I_2(t))^{3/2}}\right)$$

  - ✓ To find a maximum value in one cycle, the Newton-Raphson method is used.

Numerical example

- **von-Mises stress calculation**

✓ Stress component

$$\begin{bmatrix}\hat{\sigma}_{11}^{Re}\\ \hat{\sigma}_{22}^{Re}\\ \hat{\sigma}_{33}^{Re}\\ \hat{\sigma}_{12}^{Re}\\ \hat{\sigma}_{23}^{Re}\\ \hat{\sigma}_{31}^{Re}\end{bmatrix} = \begin{bmatrix}-28\\ -50\\ 0\\ 28\\ 0\\ 0\end{bmatrix}[\text{MPa}] \qquad \begin{bmatrix}\hat{\sigma}_{11}^{Im}\\ \hat{\sigma}_{22}^{Im}\\ \hat{\sigma}_{33}^{Im}\\ \hat{\sigma}_{12}^{Im}\\ \hat{\sigma}_{23}^{Im}\\ \hat{\sigma}_{31}^{Im}\end{bmatrix} = \begin{bmatrix}-15\\ -10\\ 0\\ -5\\ 0\\ 0\end{bmatrix}[\text{MPa}]$$

✓ Coefficients for von-Mises stress

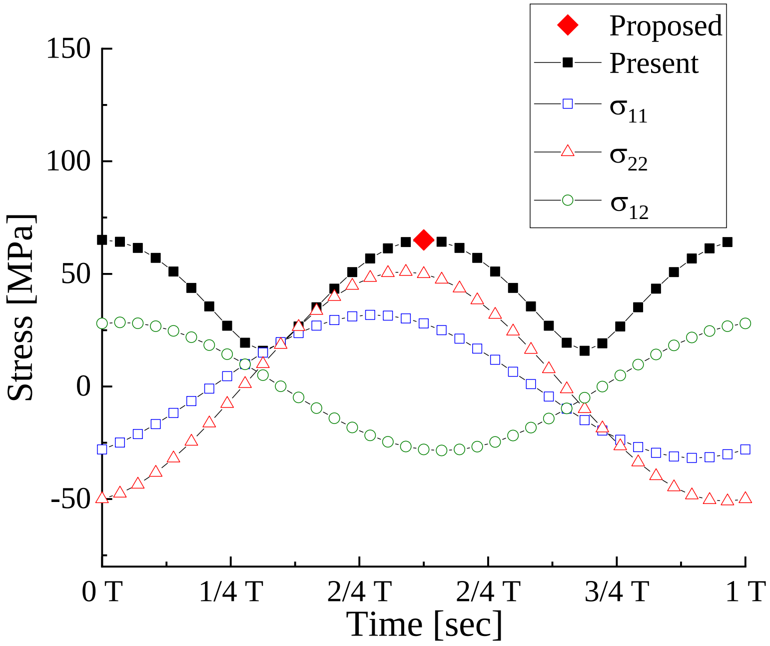
$$\sigma_{vM}(t) = \sqrt{\frac{\sqrt{(A-B)^2 + C^2}}{2}\sin(2\omega t + \phi_1) + \frac{A+B}{2}}$$

$$A = \frac{3}{2}\left(\hat{\sigma}_{ij}^{Re} - \frac{1}{3}\delta_{ij}\hat{\sigma}_{kk}^{Re}\right)\left(\hat{\sigma}_{ij}^{Re} - \frac{1}{3}\delta_{ij}\hat{\sigma}_{kk}^{Re}\right) = 4236$$

$$B = \frac{3}{2}\left(\hat{\sigma}_{ij}^{Im} - \frac{1}{3}\delta_{ij}\hat{\sigma}_{kk}^{Im}\right)\left(\hat{\sigma}_{ij}^{Im} - \frac{1}{3}\delta_{ij}\hat{\sigma}_{kk}^{Im}\right) = 250$$

$$C = 3\left(\hat{\sigma}_{ij}^{Re} - \frac{1}{3}\delta_{ij}\hat{\sigma}_{kk}^{Re}\right)\left(\hat{\sigma}_{ij}^{Im} - \frac{1}{3}\delta_{ij}\hat{\sigma}_{kk}^{Im}\right) = -30$$

$$\sin\phi_1 = \frac{(A-B)}{\sqrt{(A-B)^2 + C^2}}, \quad \cos\phi_1 = \frac{-C}{\sqrt{(A-B)^2 + C^2}}$$



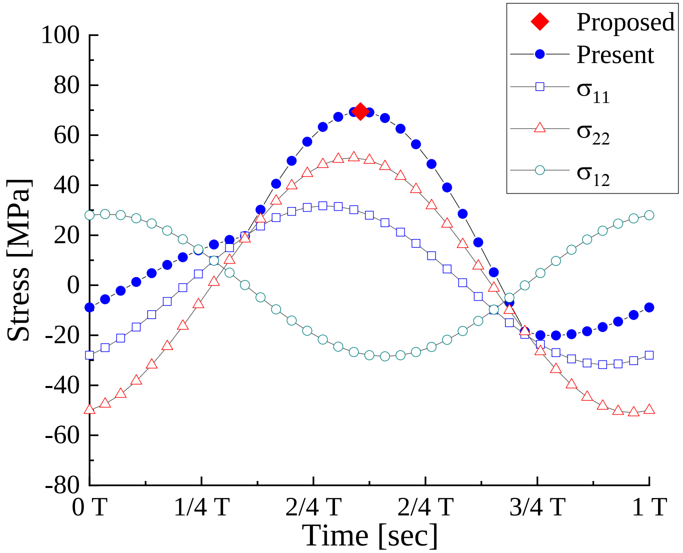
✓ Results

	$H_{\sigma_{vM}}(\omega,\theta)$	Time [sec]	# of calculations
Proposed	**65.0880 (100%)**	**0.5001 T (179.07/360)**	**1**
Previous	65.0545 (99.95%)	0.5 T (180/360)	36

Numerical example

- **Principal stress calculation**
  - ✓ Stress component

$$\begin{bmatrix}\hat{\sigma}_{11}^{Re}\\ \hat{\sigma}_{22}^{Re}\\ \hat{\sigma}_{33}^{Re}\\ \hat{\sigma}_{12}^{Re}\\ \hat{\sigma}_{23}^{Re}\\ \hat{\sigma}_{31}^{Re}\end{bmatrix}=\begin{bmatrix}-28\\-50\\0\\28\\0\\0\end{bmatrix}[\mathrm{MPa}] \qquad \begin{bmatrix}\hat{\sigma}_{11}^{Im}\\ \hat{\sigma}_{22}^{Im}\\ \hat{\sigma}_{33}^{Im}\\ \hat{\sigma}_{12}^{Im}\\ \hat{\sigma}_{23}^{Im}\\ \hat{\sigma}_{31}^{Im}\end{bmatrix}=\begin{bmatrix}-15\\-10\\0\\-5\\0\\0\end{bmatrix}[\mathrm{MPa}]$$



  - ✓ Coefficients for principal stress

$$P_1=\sqrt{D^2+E^2}\cos(\omega t+\phi_1)+\sqrt{\sqrt{\left(\left(\frac{F-G}{2}\right)^2+\left(\frac{H}{2}\right)^2\right)}\cos(2\omega t+\phi_2)+\frac{F+G}{2}}$$

$$D=\frac{1}{2}\delta_{ij}\hat{\sigma}_{kk}^{Re}=-39 \qquad E=\frac{1}{2}\delta_{ij}\hat{\sigma}_{kk}^{Im}=-12.5 \qquad F=\frac{1}{4}\left(\hat{\sigma}_{ij}^{Re}-\frac{1}{2}\delta_{ij}\hat{\sigma}_{kk}^{Re}\right)\left(\hat{\sigma}_{ij}^{Re}-\frac{1}{2}\delta_{ij}\hat{\sigma}_{kk}^{Re}\right)=905$$

$$G=\frac{1}{4}\left(\hat{\sigma}_{ij}^{Im}-\frac{1}{2}\delta_{ij}\hat{\sigma}_{kk}^{Im}\right)\left(\hat{\sigma}_{ij}^{Im}-\frac{1}{2}\delta_{ij}\hat{\sigma}_{kk}^{Im}\right)=31.25 \qquad H=\frac{1}{2}\left(\hat{\sigma}_{ij}^{Re}-\frac{1}{2}\delta_{ij}\hat{\sigma}_{kk}^{Re}\right)\left(\hat{\sigma}_{ij}^{Im}-\frac{1}{2}\delta_{ij}\hat{\sigma}_{kk}^{Im}\right)=-335$$

$$\cos\phi_1=\frac{D}{\sqrt{D^2+E^2}}=-0.9523 \qquad \sin\phi_1=\frac{E}{\sqrt{D^2+E^2}}=-0.3052 \qquad \cos\phi_2=\frac{F-G}{\sqrt{(F-G)^2+(H)^2}}=0.9337 \qquad \sin\phi_2=\frac{H}{\sqrt{(F-G)^2+(H)^2}}=-0.3580$$

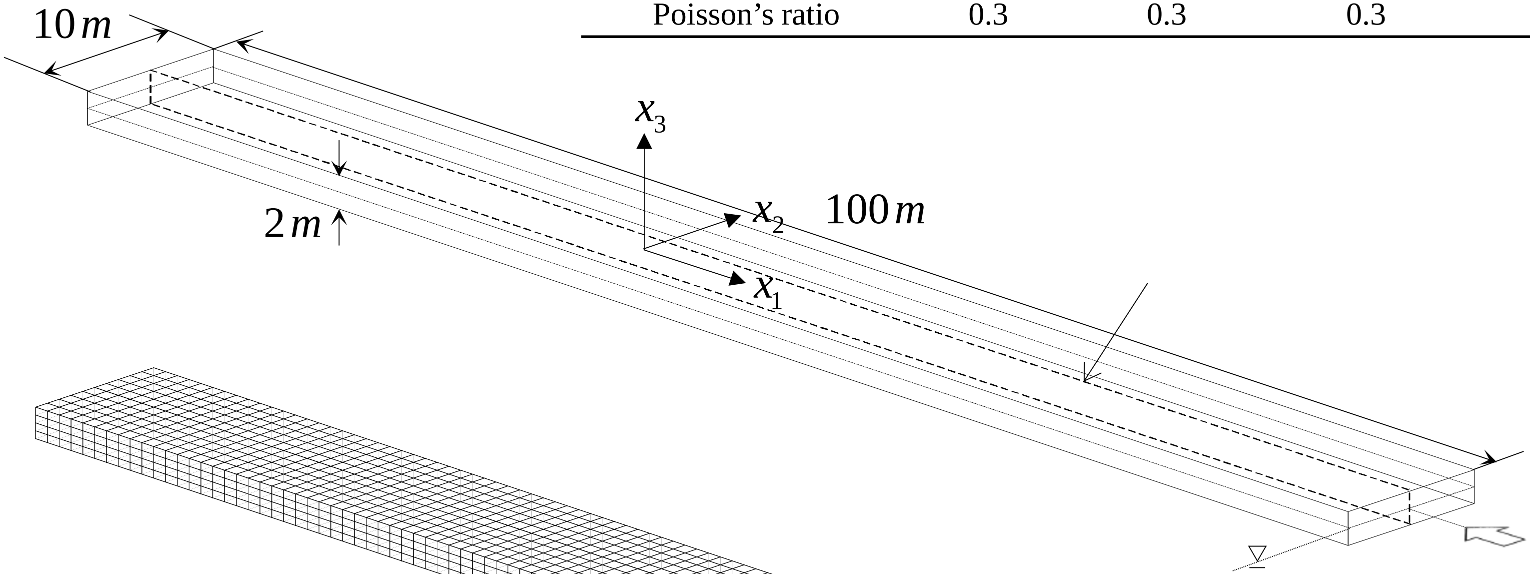
  - ✓ Max. value of P_1 using Newton-Raphson method(# of iteration : 2)
  - ✓ Results

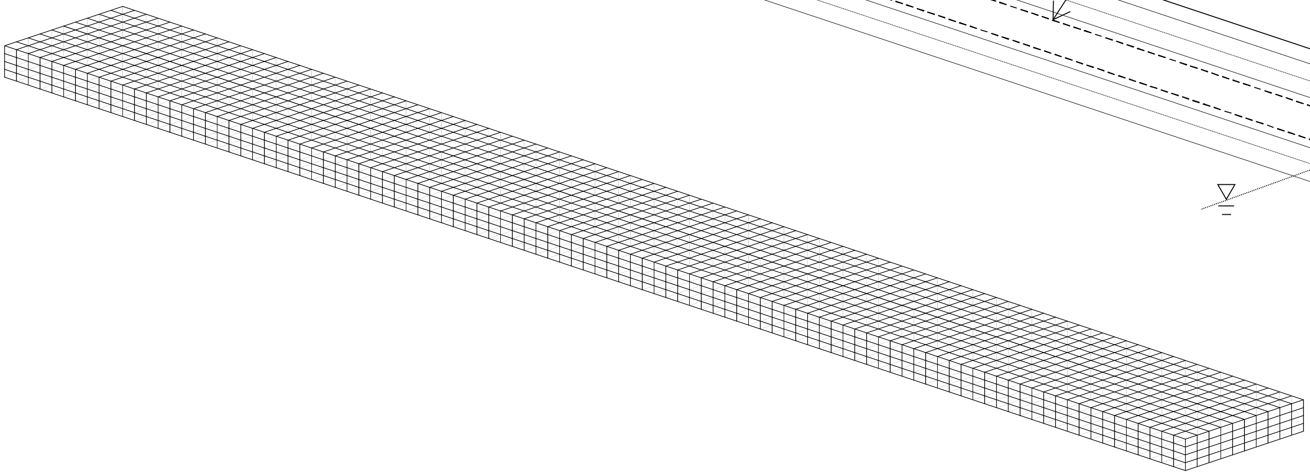
	$H_{\sigma_{P1}}(\omega,\theta)$	Time [sec]	# of calculations
Proposed	**69.4301(100%)**	**0.4841** T **(174.27/360)**	**2**
Previous	69.2376(99.72%)	0.4722 T (170/360)	36

Numerical example

- **Floating barge problem**
  - ✓ Numerical model

	Top deck	Bottom deck	Side hull	Bulk head
Thickness [m]	0.005	0.02	0.02	0.005
Density [kg/m^3]	1.0 x 10^3	3.3389 x 10^4	3.3389 x 10^4	3.3389 x 10^4
Young' modulus [GPa]	100	100	100	100
Poisson's ratio	0.3	0.3	0.3	0.3



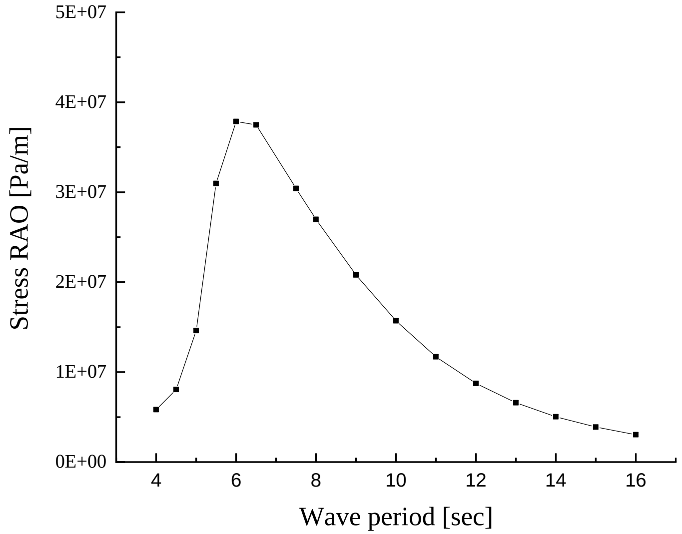


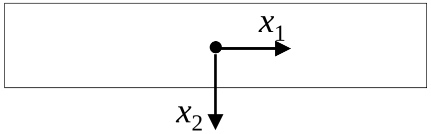
- One angle of incident wave (θ =0°), and wave periods T are 4-16 sec considered.

Numerical example

- **Floating barge problem: results**

✓ von-Mises stress RAO(T=4.0 ~16.0 sec)



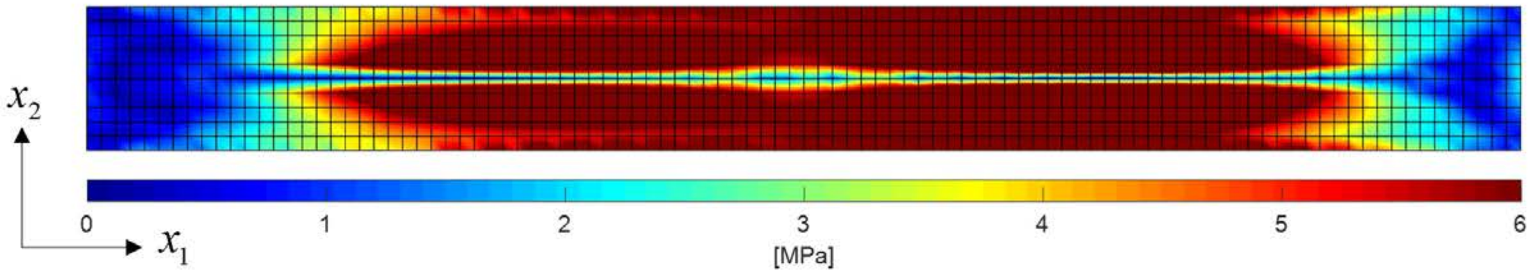


<Measuring point>

	Previous	Proposed
Computational time [sec]	19.36	1.47 (7.6%)

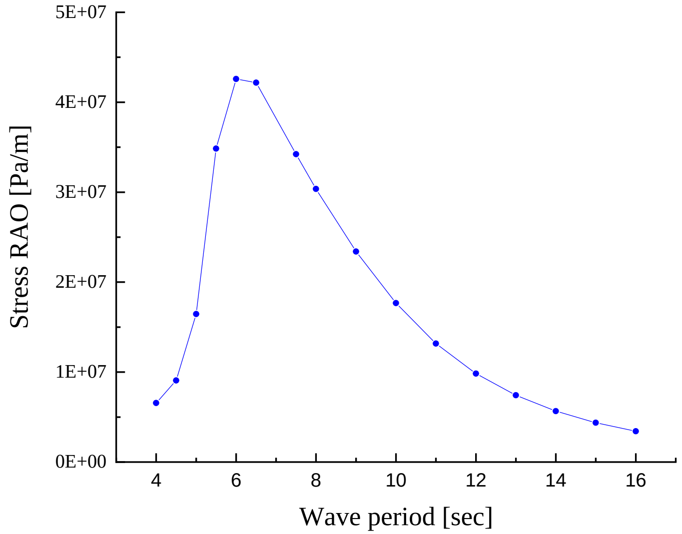
<Computational time>

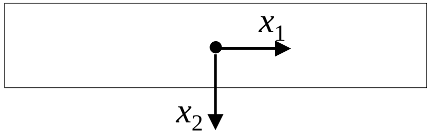
✓ von-Mises stress RAO distribution(T=4.0 sec)



- **Floating barge problem: results**

✓ Principal stress RAO(T=4.0 ~16.0 sec)



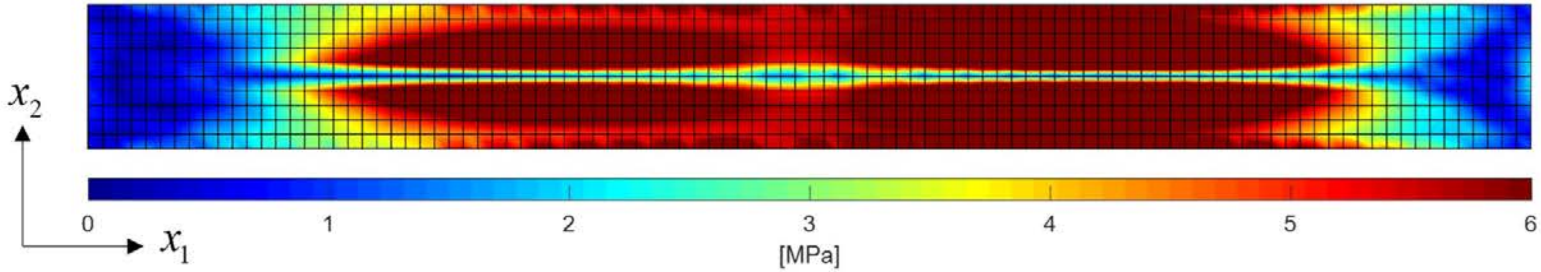


<Measuring point>

	Previous	Proposed
Computational time [sec]	19.78	5.42 (27.4%)

<Computational time>

✓ Principal stress RAO distribution(T=4.0 sec)



Closure

✓ Conclusions

- Direct Calculation Method of Stress RAO in Frequency Domain were developed.
  - A direct calculation method was developed using the periodic function relationship.
  - Method for von-Mises stress and Principal stress were developed.
  - A more accurate solution can be obtained with less calculation.
  - Calculation time can be reduced up to 7.6%.

4. Conclusions & future works

Conclusions & future works

✓ Conclusions

- An integrated hydro–static and dynamic analysis has been proposed
  - ➢ An integrated hydro–static and dynamic formulation has been proposed.
  - ➢ An effective non-matching mesh treatment method for hydrodynamic analysis of flexible floating structures were developed.
    - Hydro static and dynamic analysis are performed using a single mesh model.
  - ➢ Compared to conventional procedures, similar solution accuracy was obtained but total analysis time was significantly reduced.

- Direct Calculation Method of Stress RAO in Frequency Domain were developed.
  - ➢ A direct calculation method was developed using the periodic function relationship.
  - ➢ Method for von-Mises stress and Principal stress were developed.
  - ➢ Calculation time can be reduced up to 7.6%.

- The proposed method will contribute to practical applications in the field of structural design.

Conclusions & future works

✓ Future works

- It will be valuable to extend the present direct coupled formulation to nonlinear hydroelastic analyses (large motions of floating structures and fluid).

- The present formulation can be extended to the transient analysis of flexible floating structures.

- Hydrodynamic analysis considering loading information application method.

 - Internal fluid, cargo loading(container, oar, grains, etc)

Thank you for your attention