KAIST Department of Mechanical Engineering

A practical method for evaluating stability of local structures using strain energy variation

(변형에너지 변분을 이용한 국부 구조의 실용적인 안정성 평가 방법)

KAIST 기계공학과 오민한

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Ph.D. degree dissertation presentation

Contents

INTRODUCTIONS

Introductions

Mississippi bridge collapse case

Bowed gusset plates (June 2003)

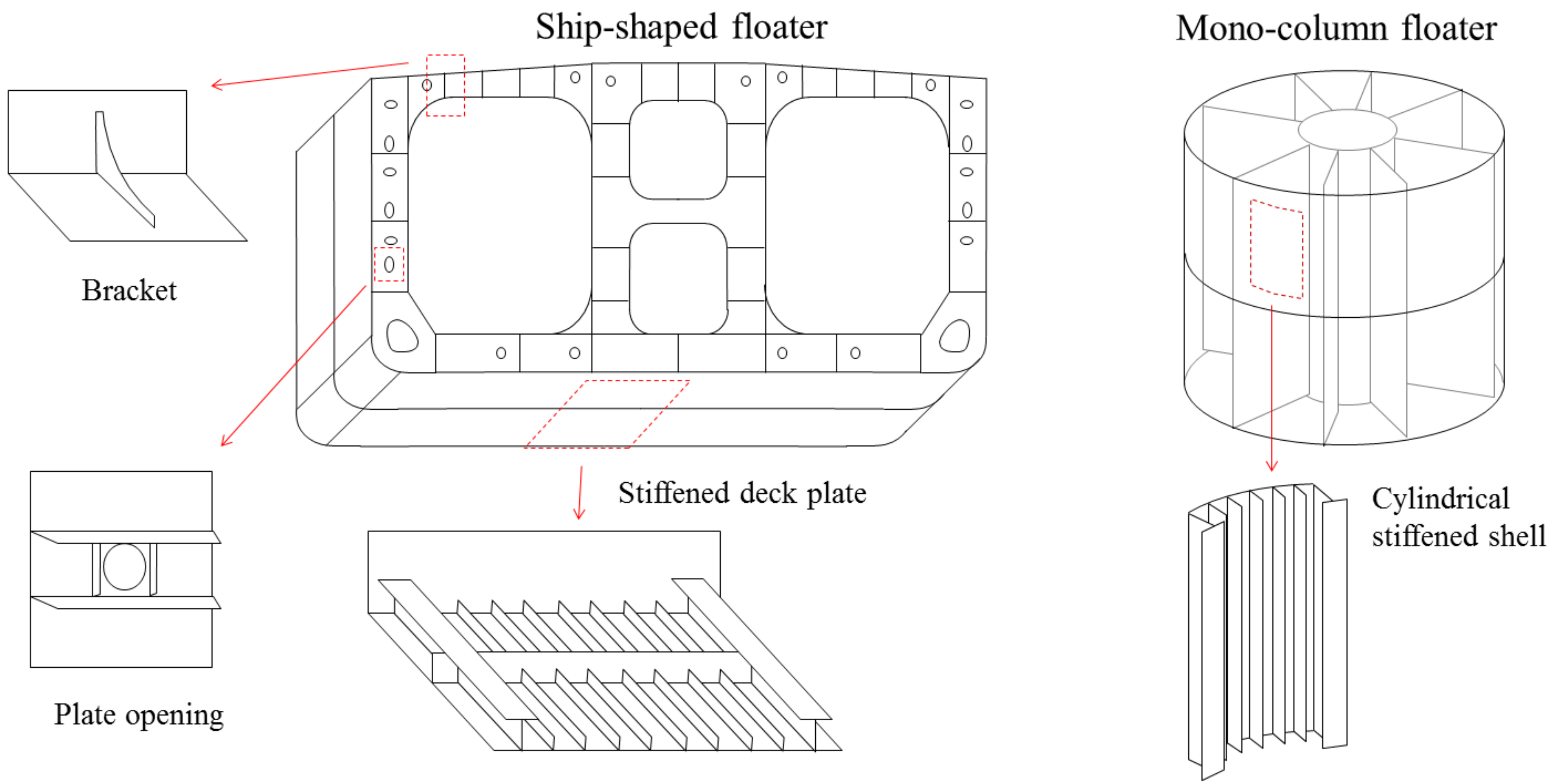
Overall collapse (August 1, 2007)

Cause of collapse

Misprediction of resistance capacity of local gusset structure subject to bridge pay loads

Introductions

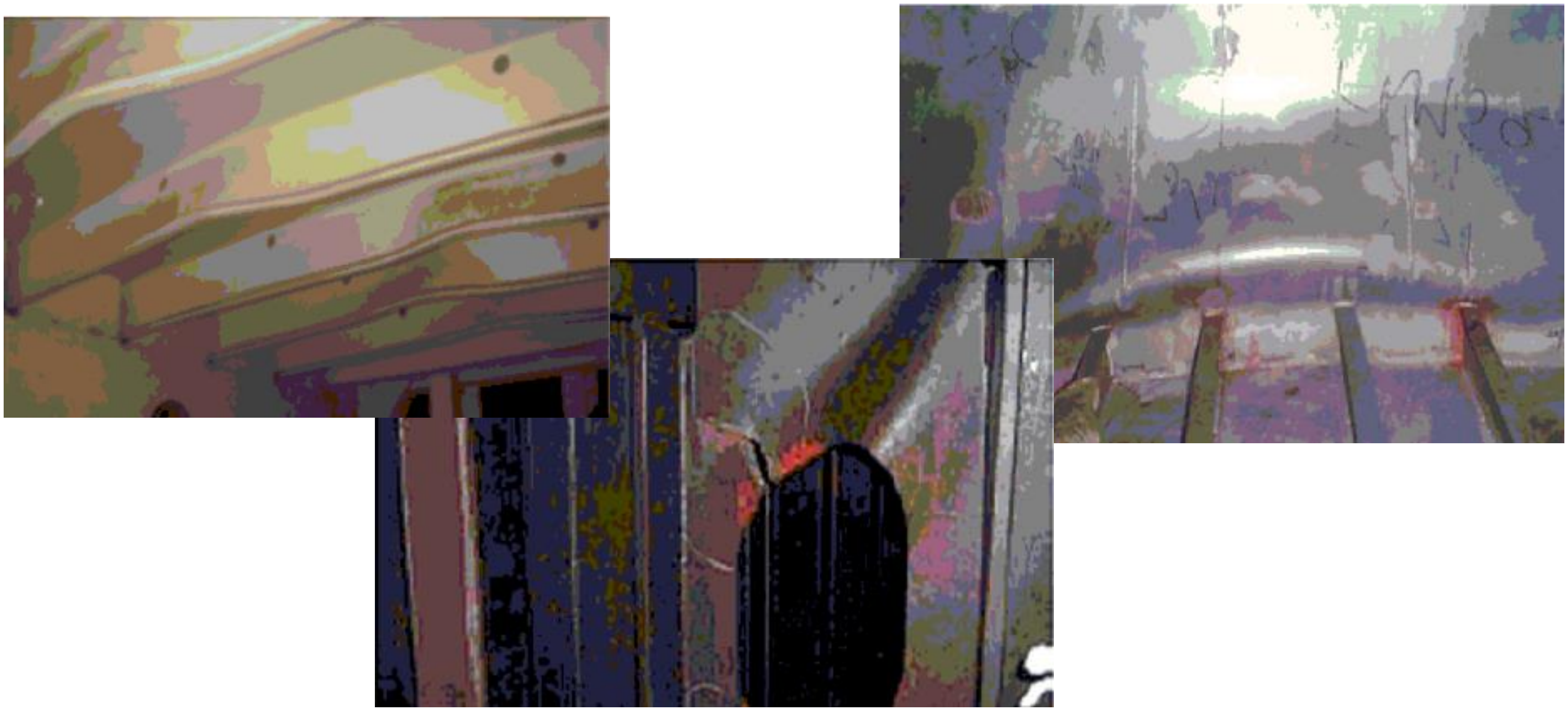
Typical local structures in thin-walled ship and offshore structures



Trends in weight reduction of hull and offshore structures in design optimization

Introductions

How to remedy buckling failures : Practical and reasonable action plan



Photos of buckling failure patterns in ship and offshore steel structures

Introductions

Code and Rule based limit state evaluation for regular geometry


ANSI/AISC 360-16
An American National Standard

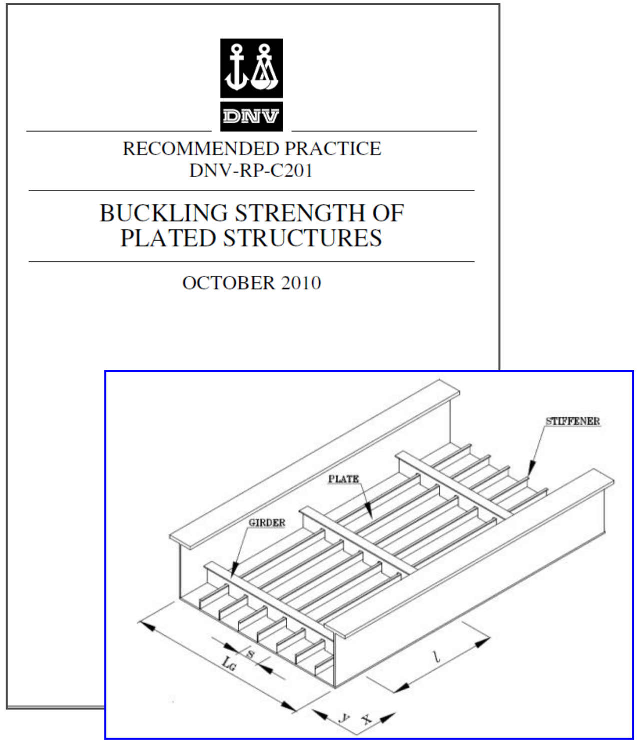
Specification
for Structural Steel Buildings

July 7, 2016

Supersedes the *Specification for Structural Steel Buildings* dated June 22, 2010 and all previous versions

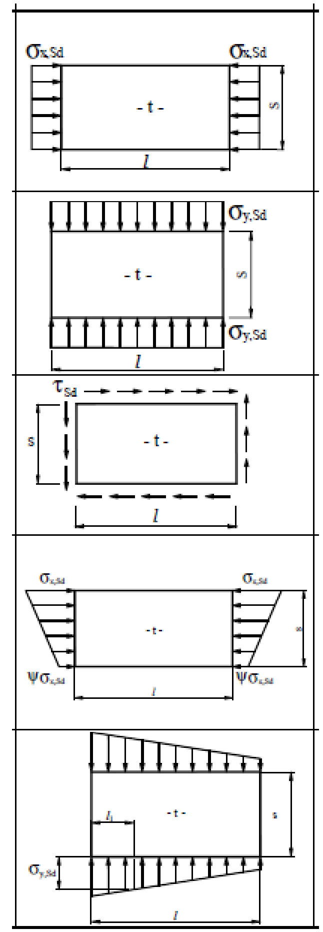
Approved by the Committee on Specifications

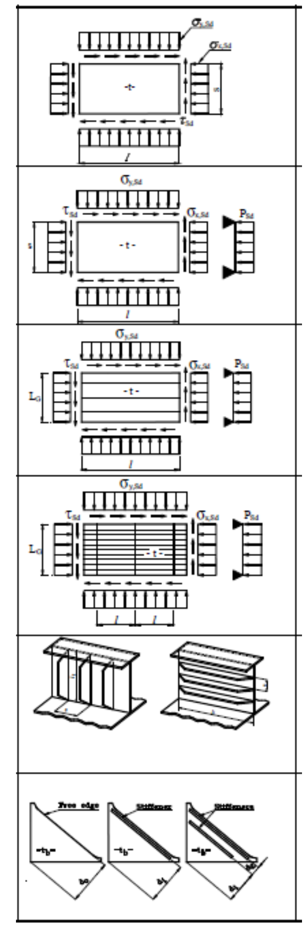
AMERICAN INSTITUTE OF STEEL CONSTRUCTION
130 East Randolph Street, Suite 2000, Chicago, Illinois 60601
www.aisc.org


DNV
RECOMMENDED PRACTICE
DNV-RP-C201

BUCKLING STRENGTH OF
PLATED STRUCTURES

OCTOBER 2010

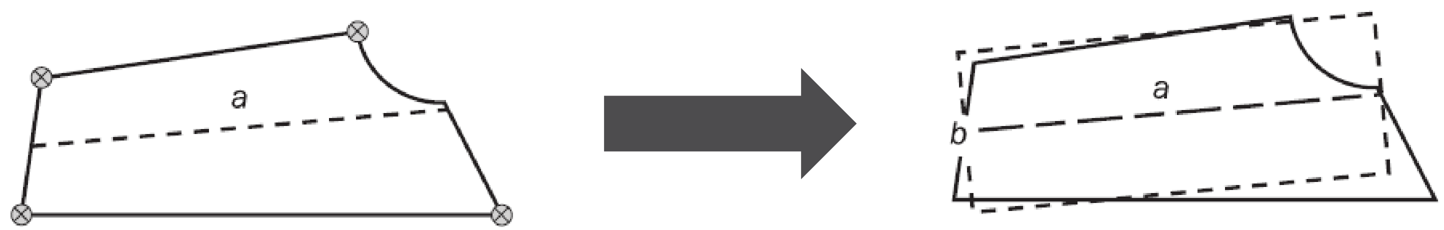




- As per the guidance issued by AISC, API and Classification Societies.
- Typical types of structural geometry and load distribution are pre-defined.

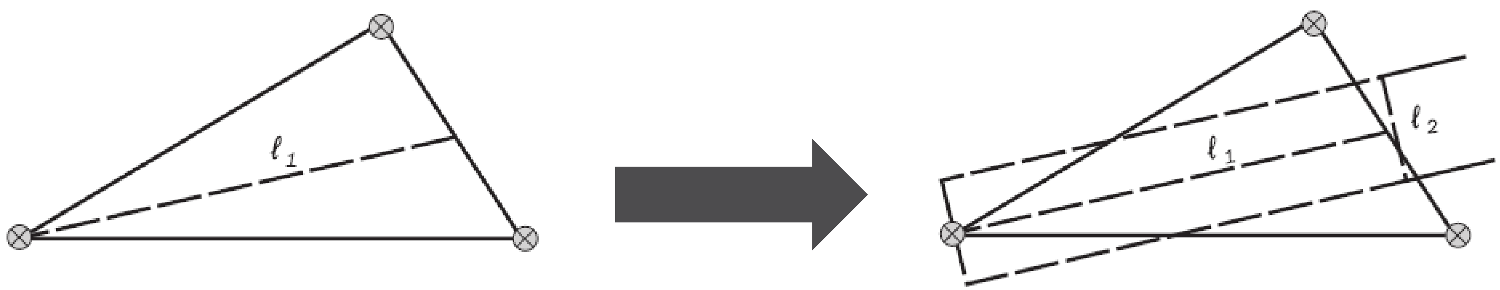
Introductions

Idealization of irregular geometry : Very limited to apply



b = Plate Area / a

where, a, b = characterized lengths



l_2 = Plate Area / l_1

where, l_1, l_2 = characterized lengths

Needs for the method for evaluating stability of local structures

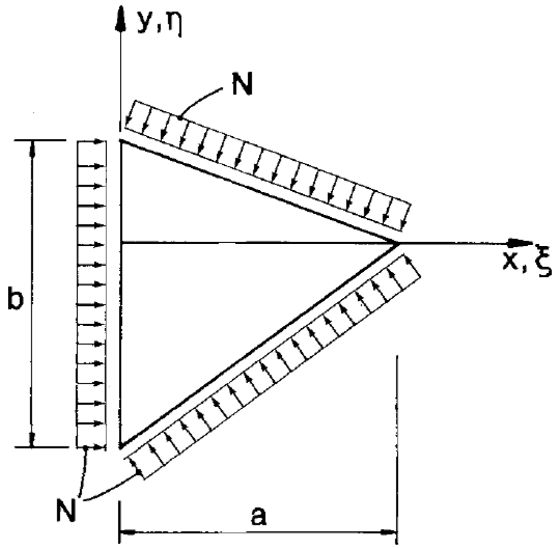
- Simple and applicable to arbitrary parts of structures
- Precise failure decision of the evaluated area
- Quantitative evaluation of safety margin
- Identification of damaged areas
- Practical and cost-effective with FE analysis

Research Objectives

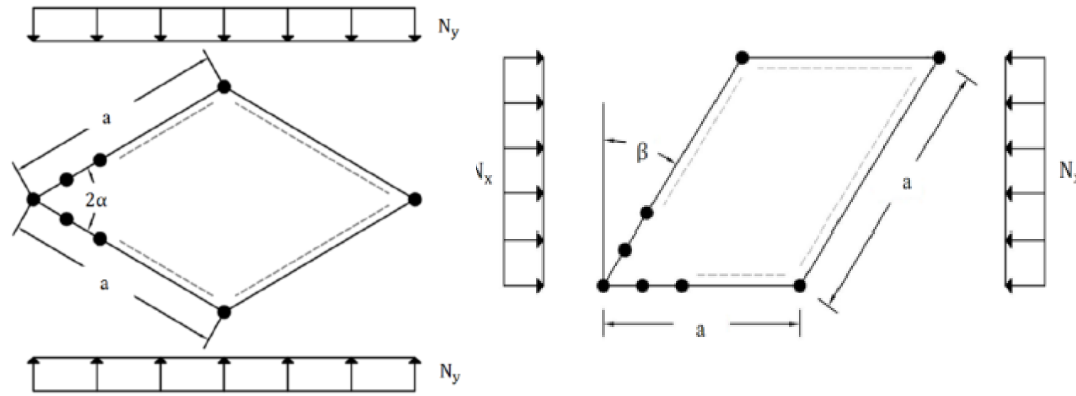
- Develop a method using nonlinear FE analysis to evaluate the stability of arbitrary shape of structures.
- Suggest a method to identify damaged areas that lost stability without performing an iterative FE analysis.
- Reduce the computing time required for extensive nonlinear FE analysis to evaluate large degrees of freedom models using the model reduction method.

PREVIOUS STUDIES

Previous Studies

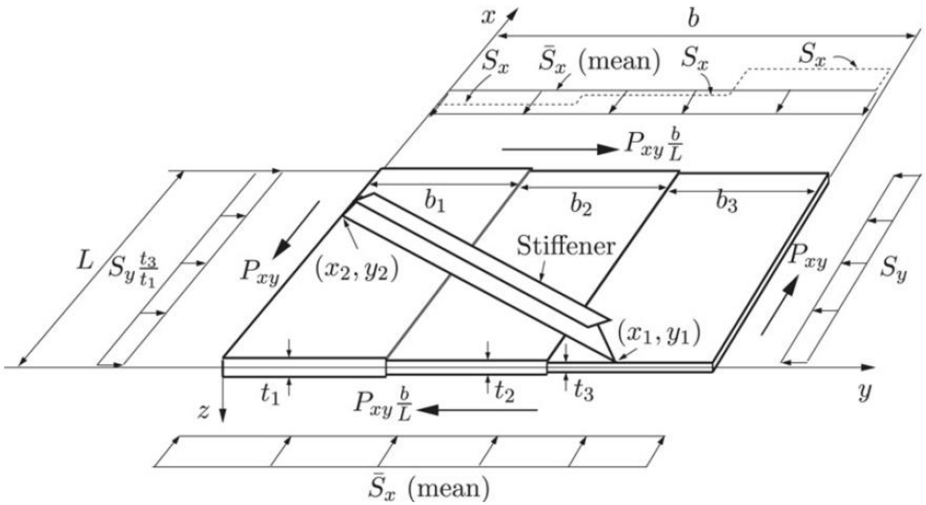


Triangular plate
(Aung 2006, Jaunky et al. 1995a, Wang and Liew 1994, Xiang 2002, Xiang et al. 1994)

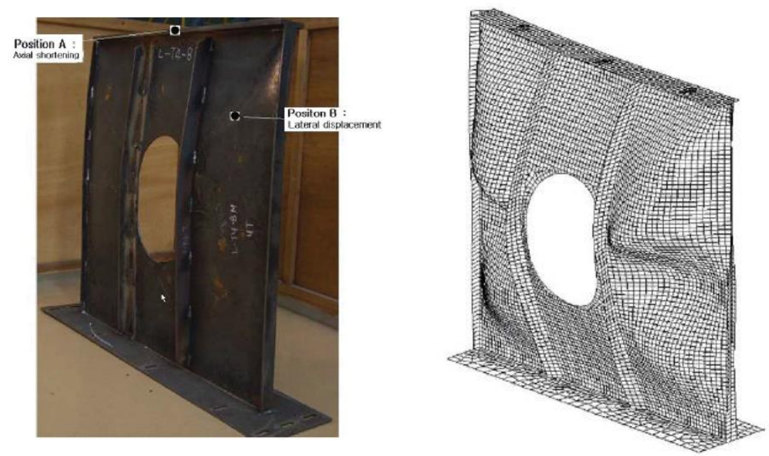


Various plate and girder shapes
(Jamshidi and Fallah 2019, Jaunky et al. 1995b, Saadatpour et al. 1998, Tham and Szeto 1990, Wang et al. 1994, Wu and Feng 2003)

Previous Studies

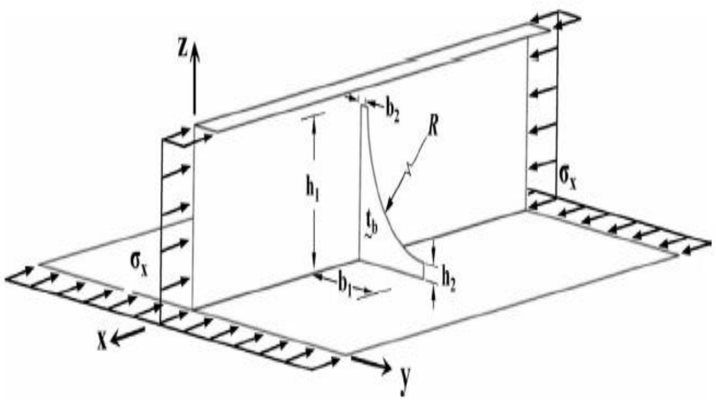


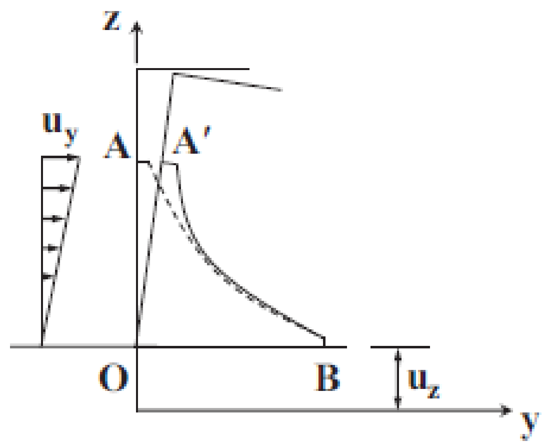
Arbitrarily stiffened plate
(Brubak et al. 2007a, Brubak et al. 2007b, Kim et al. 2018, Kim et al. 2019)

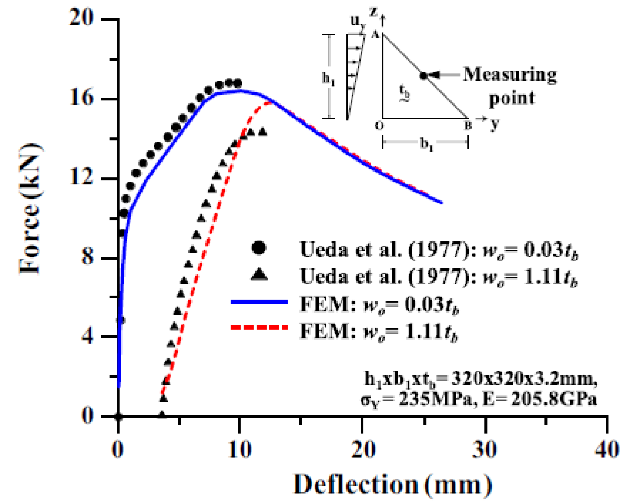


Perforated plates
(Kim et al. 2009, Kim et al. 2015, Komur and Sonmez 2008, Mohammadzadeh et al. 2018, Muhammad and Singh 2005, Saad-Eldeen et al. 2016

Previous Studies



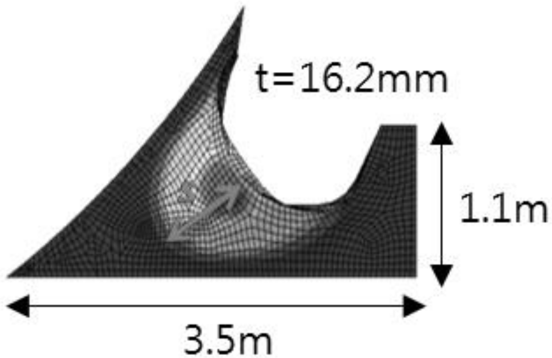


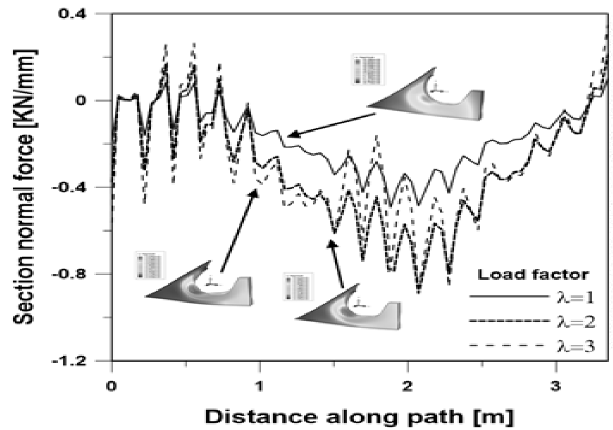


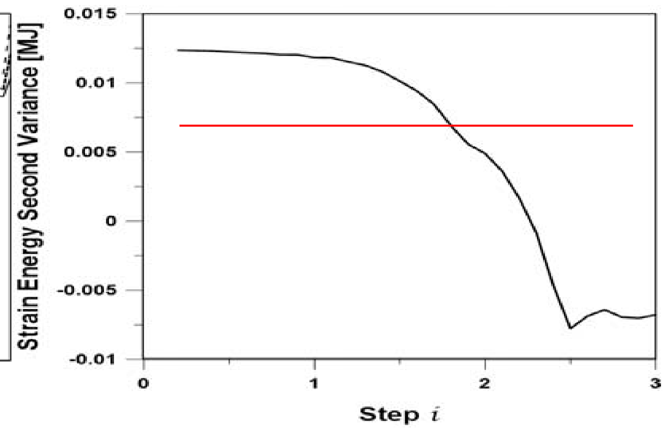
Lee et al. (2015)

- Ultimate strength characteristics was investigated for the ship brackets through nonlinear FEA, and developed a simple design formula to predict ultimate strength.
- One side of the bracket was subjected to prescribed rotational displacement while maintaining a straight line and the other side was fixed.
- In this way, the displacement of the bracket boundary is idealized and the ultimate reaction moment is extracted.

Previous Studies







Zi et al. (2017)

- Zi et al. performed the ultimate strength analysis for non-typical local structures applied in Tension Leg Platform (TLP), a type of offshore structure.
- Nonlinear FEA of the bracket girder itself was performed with linear increment of prescribed displacement vector from the global FEA at reference load.
- The critical load, which changes the structure from stable to unstable, was judged by using the strain energy criterion.
- The safety factor is proposed to be the square root of the ratio of the strain energy at critical load to the strain energy at reference load.

Practical contributions

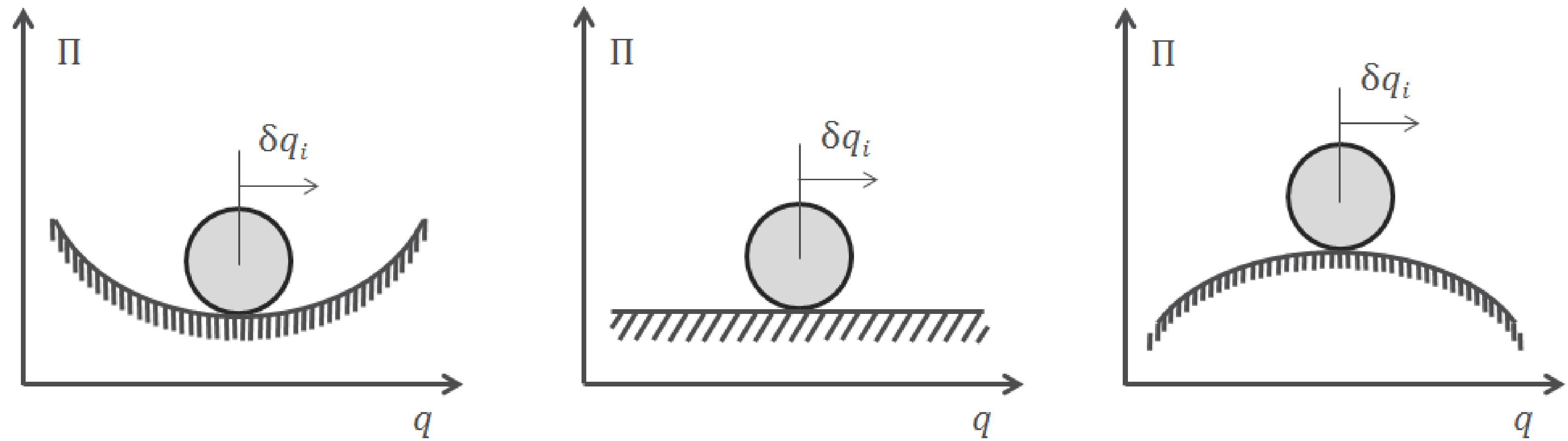
The proposed method can be used for following practical purposes:

- Identify whether the target local structure is buckled and find the magnitude of the external force acting on the global structure.
- Identify areas where local buckling occurs in the global structure. That is, damaged areas can be identified.
- Calculate residual strength of the damaged areas and evaluate the effectiveness of the local reinforcement.
- Reduce analysis time economically through the model-reduction technique for large FE models.

PROPOSED METHOD

Stability

Stability of a ball in different gravity field conditions



Stable Unstable (neutral equilibrium) Unstable

Variation criterion for stable state

Law of conservation of energy

$$\Pi = U - W$$

Small Variation of Equilibrium state at constant load

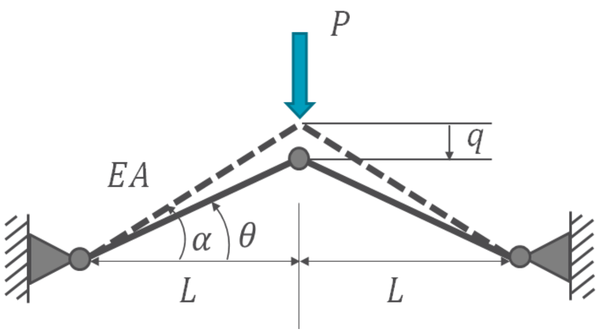
$$\Delta\Pi = \Pi\left(q_1 + \delta q_1, \ldots, q_n + \delta q_n\right) - \Pi\left(q_1, \ldots., q_n\right) \approx \delta\Pi + \delta^2\Pi$$

$$\delta\Pi = \frac{1}{1!}\sum_{i=1}^{n}\frac{\partial\Pi}{\partial q_i}\delta q_i \qquad \delta^2\Pi = \frac{1}{2!}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial^2\Pi}{\partial q_i \partial q_j}\delta q_i \delta q_j$$

$\delta\Pi = 0$ at equilibrium state

$\delta^2\Pi > 0$ if equilibrium state is stable (Lagrange-Dirichlet theorem)

Variation criterion for stable state

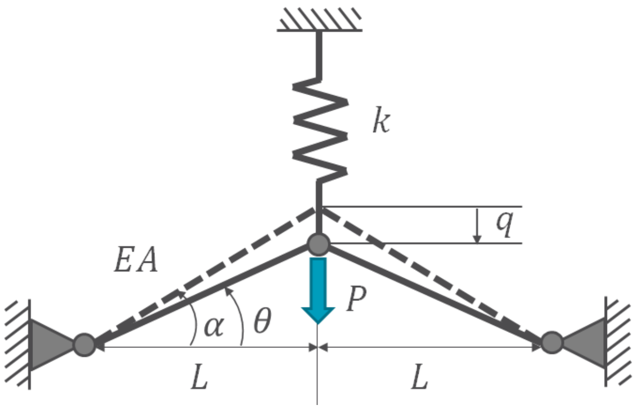


von Mises truss

$$\Pi = U - W = 2\int \frac{1}{2} E\varepsilon^2 dV - Pq = \frac{EAL}{\cos\alpha}\left(\frac{\cos\alpha}{\cos\theta} - 1\right)^2 - Pq$$

$$\frac{\partial^2 \Pi}{\partial q^2} = \frac{2EA}{L}\left(\cos\alpha - \cos^3\theta\right) = \frac{2EA}{L}\left(\cos\alpha - \cos^3\left(\tan^{-1}\left(\tan\alpha - \frac{q}{L}\right)\right)\right)$$

$$\frac{\partial^2 \Pi}{\partial q^2} > 0 \quad \Rightarrow \quad \cos\alpha - \cos^3\theta > 0$$



von Mises truss with spring

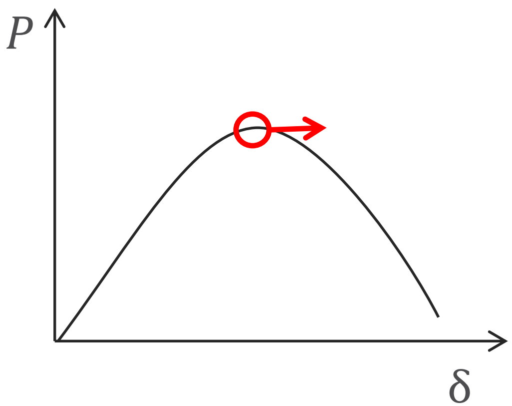
$$\Pi = 2\int \frac{1}{2} E\varepsilon^2 dV + \frac{1}{2}Kq^2 - Pq = \frac{EAL}{\cos\alpha}\left(\frac{\cos\alpha}{\cos\theta} - 1\right)^2 + \frac{1}{2}Kq^2 - Pq$$

$$\frac{\partial^2 \Pi}{\partial q^2} = \frac{2EA}{L}\left(\cos\alpha - \cos^3\theta\right) + K = \frac{2EA}{L}\left(\cos\alpha - \cos^3\left(\tan^{-1}\left(\tan\alpha - \frac{q}{L}\right)\right)\right) + K$$

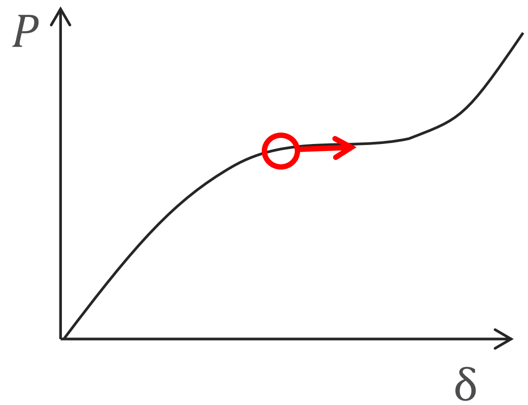
$$\frac{\partial^2 \Pi}{\partial q^2} > 0 \quad \Rightarrow \quad \frac{2EA}{L}\left(\cos\alpha - \cos^3\theta\right) + K > 0$$

Structural Stability

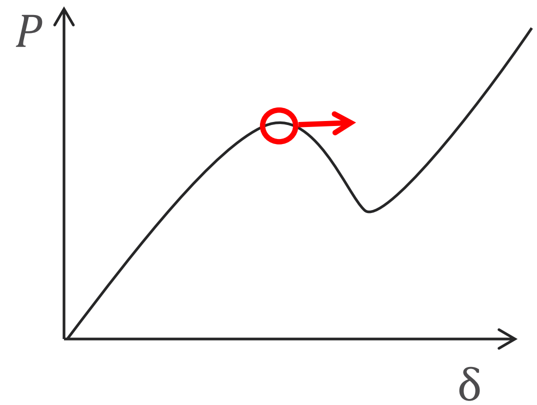
Stability evaluation by *P*- δ curve



Globally unstable



Globally stable

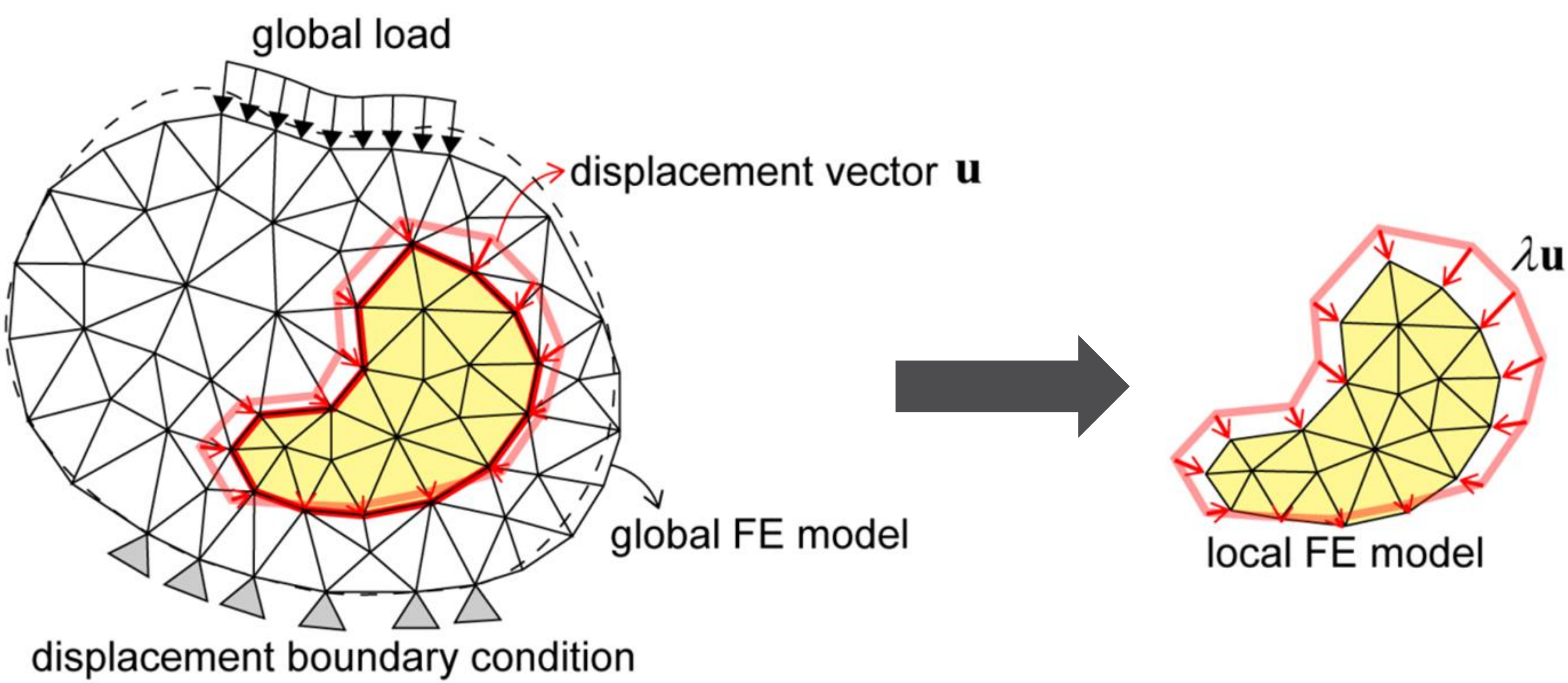


Globally unstable

Locally Stable or Unstable?

Evaluation through local FE analyses (Zi. et al. 2017)

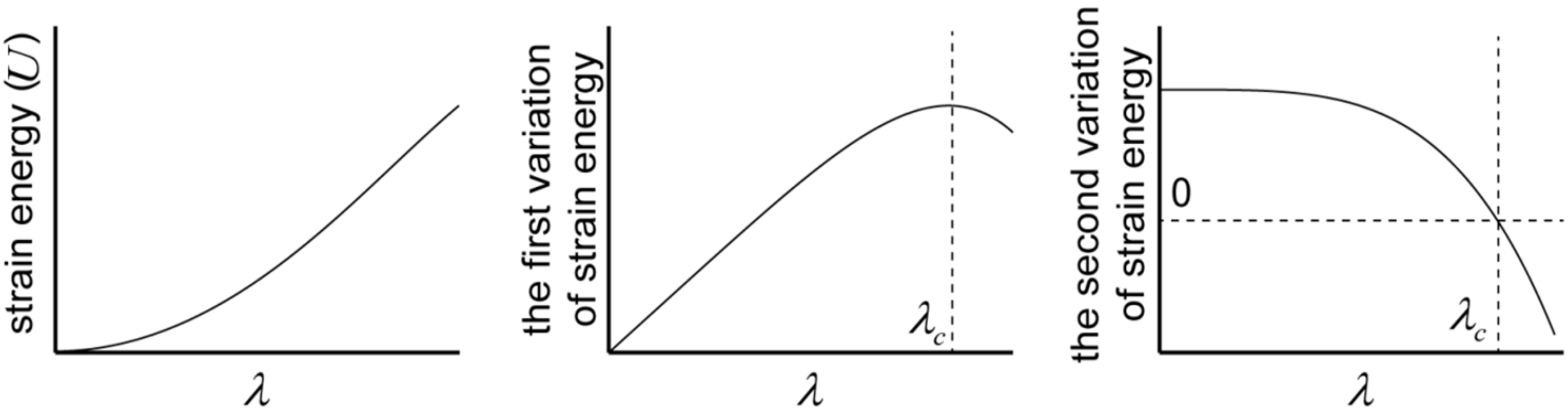
Evaluation through local FE analyses (Zi. et al. 2017)



1. Linear FE analysis for the global structure

2. Nonlinear FE analysis only for the local structure with the prescribed displacement

Evaluation through local FE analyses (Zi. et al. 2017)



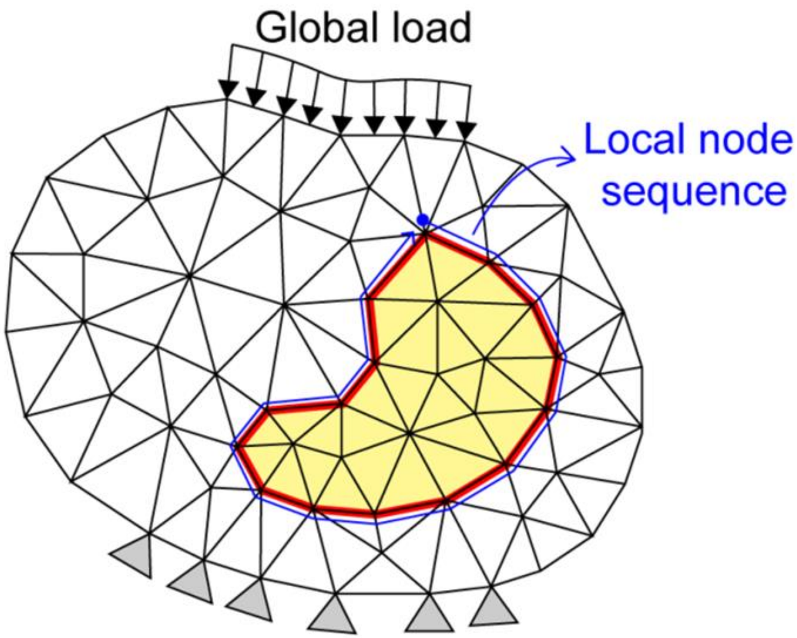
3. Buckling occurs when the second variation becomes zero first at λ_c on the plots of the variations of the strain energy (U) with respect to λ,

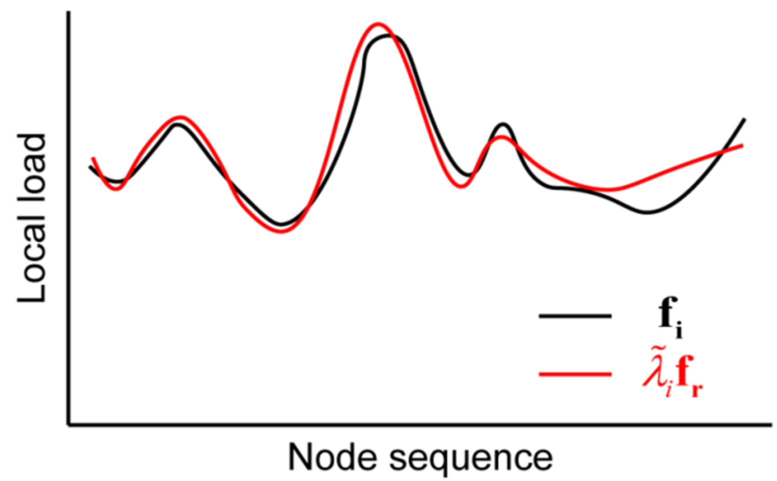
4. Usage factor is calculated with the displacement factor λ_s corresponding to the service load:

$$\eta = \sqrt{\frac{U(\lambda_s)}{U(\lambda_c)}}$$

Direct evaluation through global FE analysis (Proposed evaluation method)

Direct evaluation through global FE analysis





1. Nonlinear incremental FE analysis for the global structure

2. Local load is approximated to with a local load ratio

$\mathbf{f}_i \simeq \tilde{\lambda}_i \mathbf{f}_r$, $\mathbf{f}_r$ is reference local load

Direct evaluation through global FE analysis

Local load approximation

Approximated load method

Sum squared error function : $\varphi_i = \left(\mathbf{f}_i - \tilde{\lambda}_i \mathbf{f}_r\right) \cdot \left(\mathbf{f}_i - \tilde{\lambda}_i \mathbf{f}_r\right)$

Local load ratio ($\tilde{\lambda}_i$) is derived by the least squares fitting method

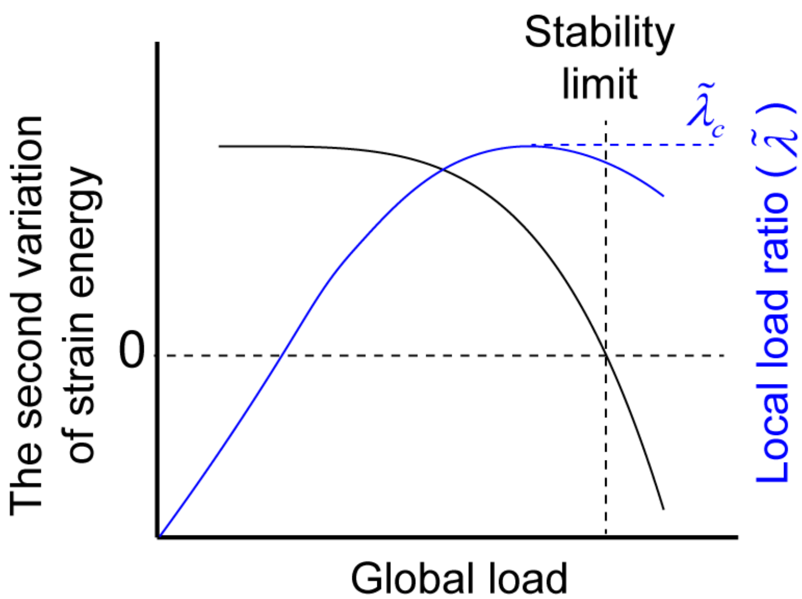
$$\tilde{\lambda}_i = \frac{\mathbf{f}_i \cdot \mathbf{f}_r}{\mathbf{f}_r \cdot \mathbf{f}_r}$$

Norm value method

Local load ratio ($\bar{\lambda}_i$) defined as a ratio to the norm values (magnitudes of local load vectors)

$$\bar{\lambda}_i = \sqrt{\frac{\mathbf{f}_i \cdot \mathbf{f}_i}{\mathbf{f}_r \cdot \mathbf{f}_r}}$$

Direct evaluation through global FE analysis

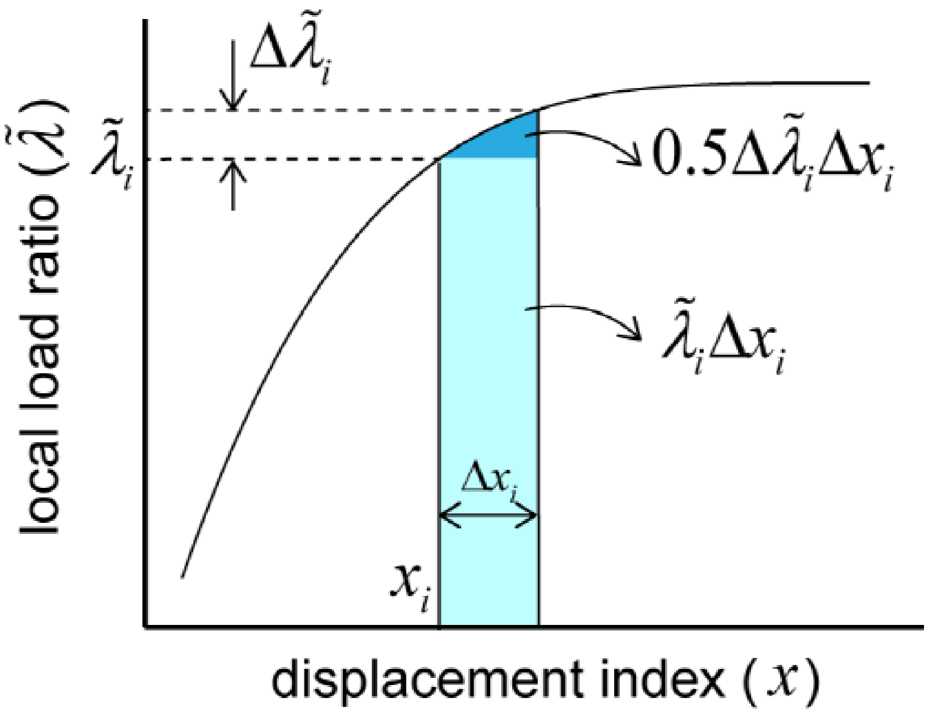


The second variation of the strain energy ≈ second-order term of the incremental external work

$$\Delta U_i = \Delta W_i = \mathbf{f}_i \cdot \Delta \mathbf{u}_i + 0.5 \Delta \mathbf{f}_i \cdot \Delta \mathbf{u}_i$$

3. Stability limit: Load level at which the sign of the second variation of the strain energy shifts from positive to negative $(0.5 \Delta \mathbf{f}_i \cdot \Delta \mathbf{u}_i = 0)$

Direct evaluation through global FE analysis



Displacement index is introduced to investigate tendencies such as the severity of deformation at a specific load step

$$\frac{\Delta W_i}{|\mathbf{f}_r|} = \tilde{\lambda}_i \Delta x_i + 0.5\Delta\tilde{\lambda}_i \Delta x_i$$

$$\Delta x_i = \frac{\Delta W_i}{|\mathbf{f}_r|\left(\tilde{\lambda}_i + 0.5\Delta\tilde{\lambda}_i\right)}$$

Direct evaluation through global FE analysis

4. local safety factor (LSF) is proposed as the measure of the safety in the structural design

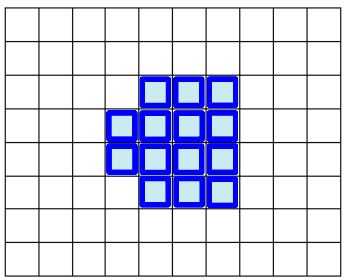
$$LSF = \frac{1}{\tilde{\eta}} = \frac{\tilde{\lambda}_c}{\tilde{\lambda}_s}$$

Challenge:

Quantitative measure of the local safety factor is very important because the local load distribution is varying depending on load level in the global structure.

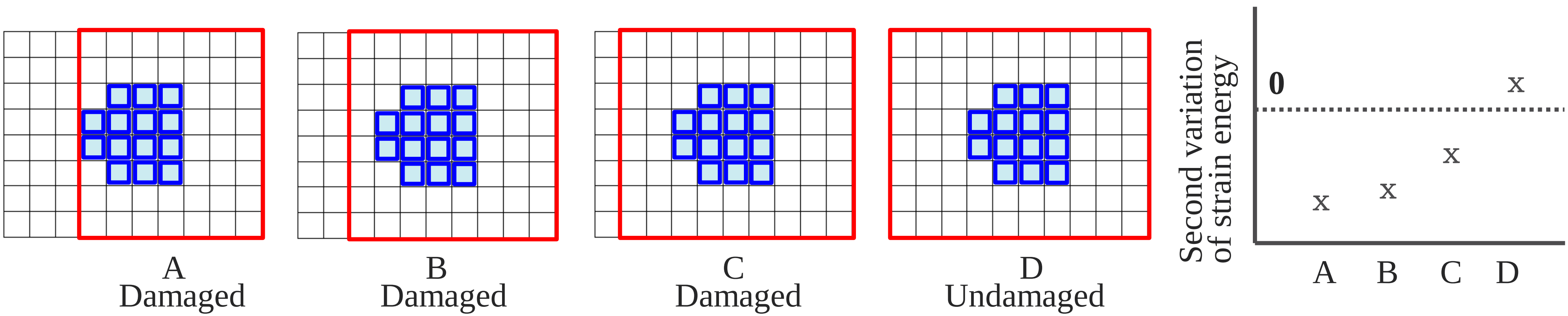
Direct evaluation through global FE analysis

Identification of damaged areas



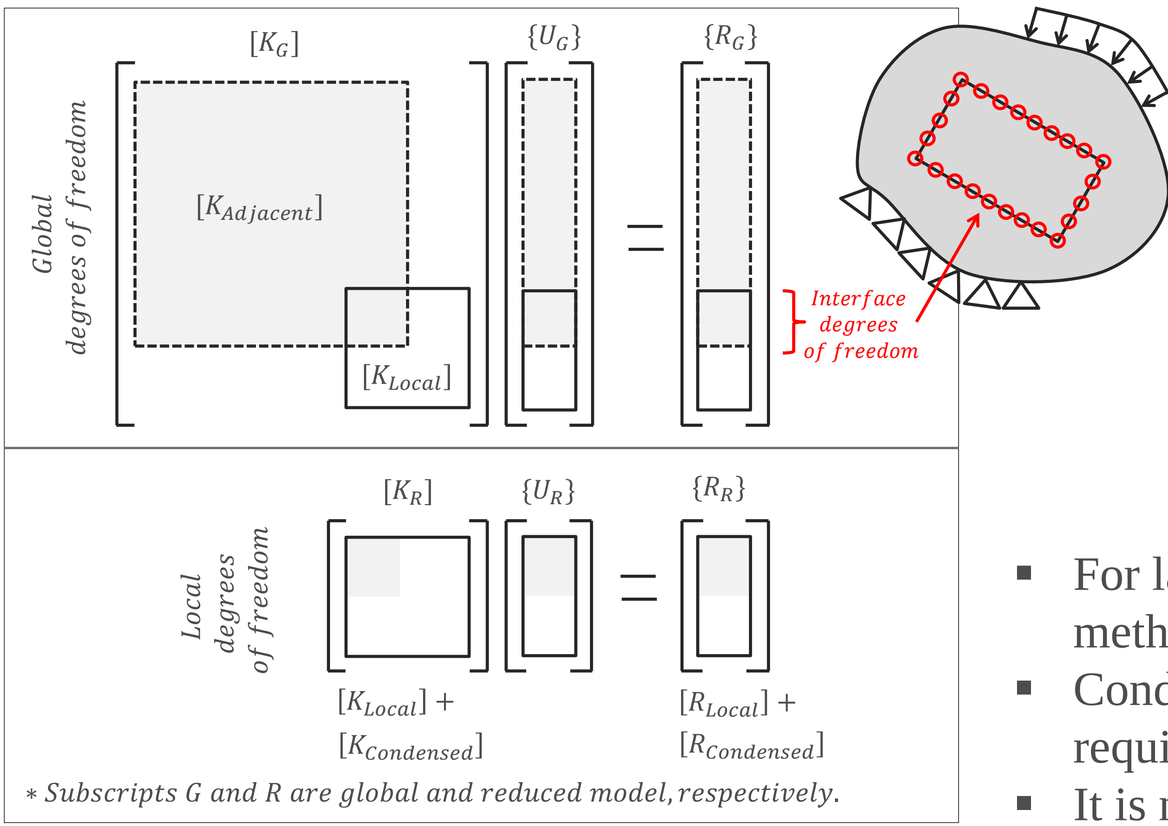
Unstable element
: negative second variation of the strain energy in a element

Considered damage evaluation area



Direct evaluation through global FE analysis

Computational efficiency improvement by static condensation



$$\begin{bmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} \\ \mathbf{K}_{21} & \mathbf{K}_{22} \end{bmatrix} \begin{Bmatrix} \mathbf{U}_1 \\ \mathbf{U}_2 \end{Bmatrix} = \begin{Bmatrix} \mathbf{R}_1 \\ \mathbf{R}_2 \end{Bmatrix}$$

$$\left[\mathbf{K}_{\text{condensed}}\right] = \left[-\mathbf{K}_{21}\mathbf{K}_{11}^{-1}\mathbf{K}_{12} + \mathbf{K}_{22}\right]$$

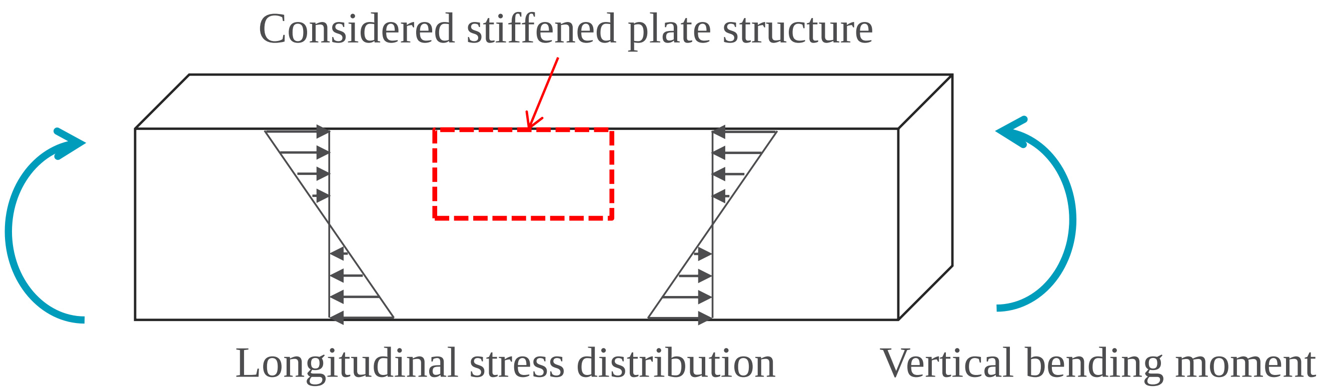
$$\left\{\mathbf{R}_{\text{condensed}}\right\} = \left\{-\mathbf{K}_{21}\mathbf{K}_{11}^{-1}\mathbf{R}_{1} + \mathbf{R}_{2}\right\}$$

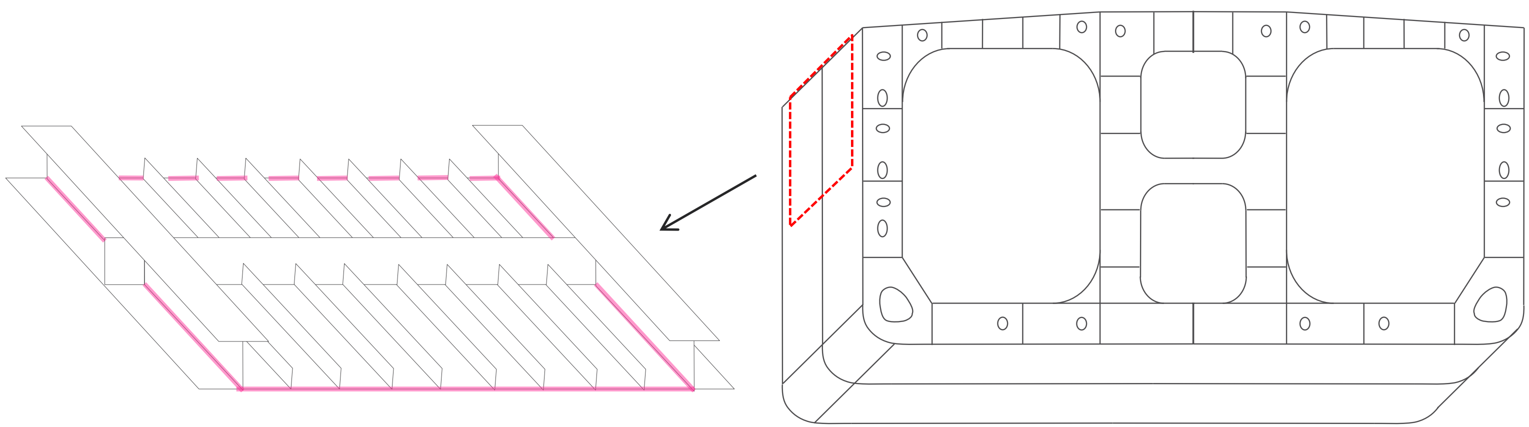
1: slave (removed) DOF
2: master (residual) DOF

- For large FE model, the static condensation method is cost-effective.
- Condensed stiffness is linear and does not require nonlinear iterations every time.
- It is necessary to select the area considering the analysis cost and the analysis error

NUMERICAL EXAMPLES

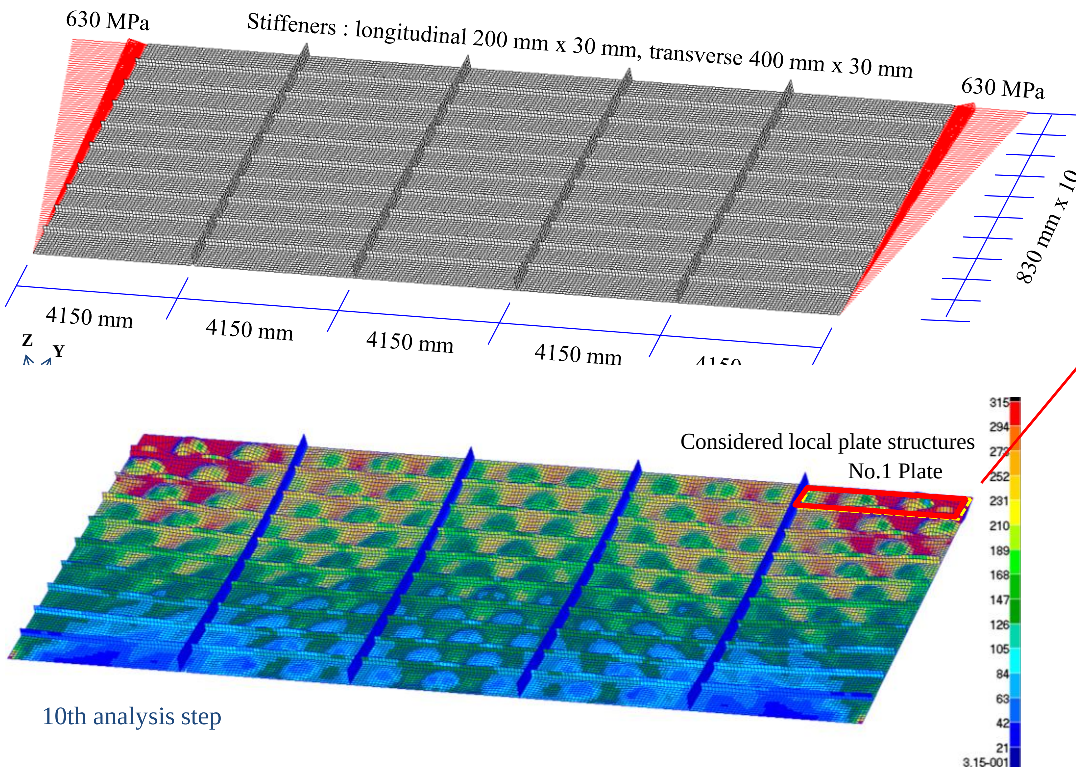
Numerical Examples: Stiffened plate



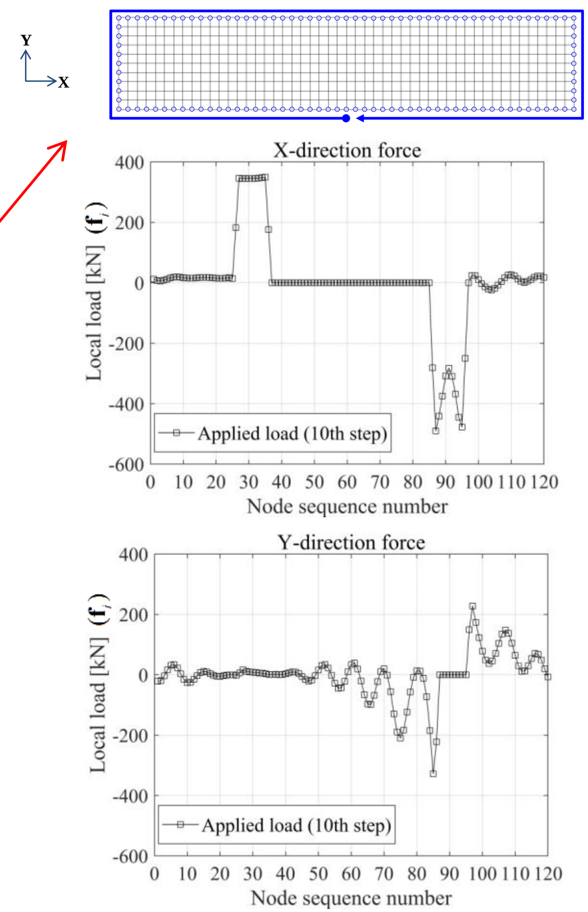


Numerical Examples: Stiffened plate

(Step 1) Local load distribution



Local load extraction node sequence

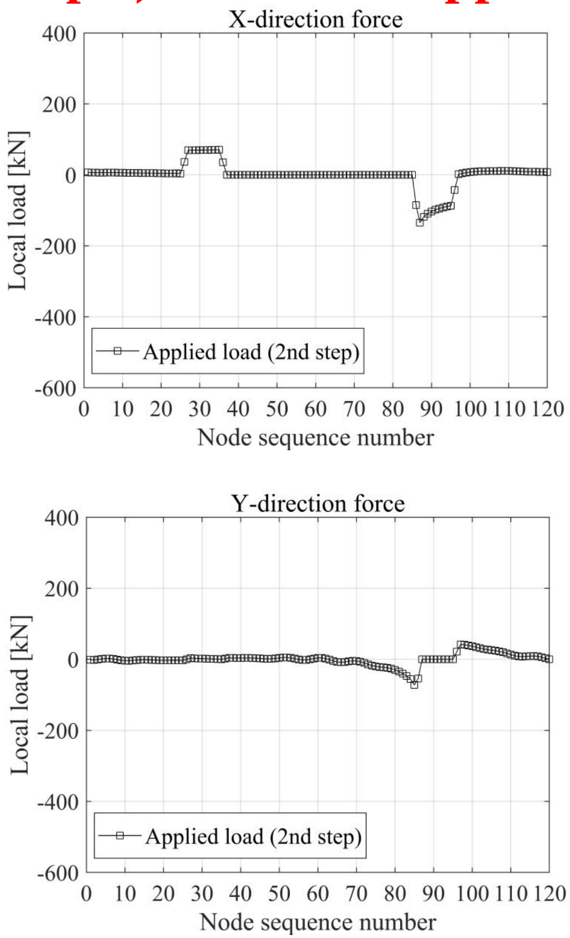


- Incremental FE analysis with 20 intervals
- Simply supported and straight edge
- Initial imperfection: 5.73 mm

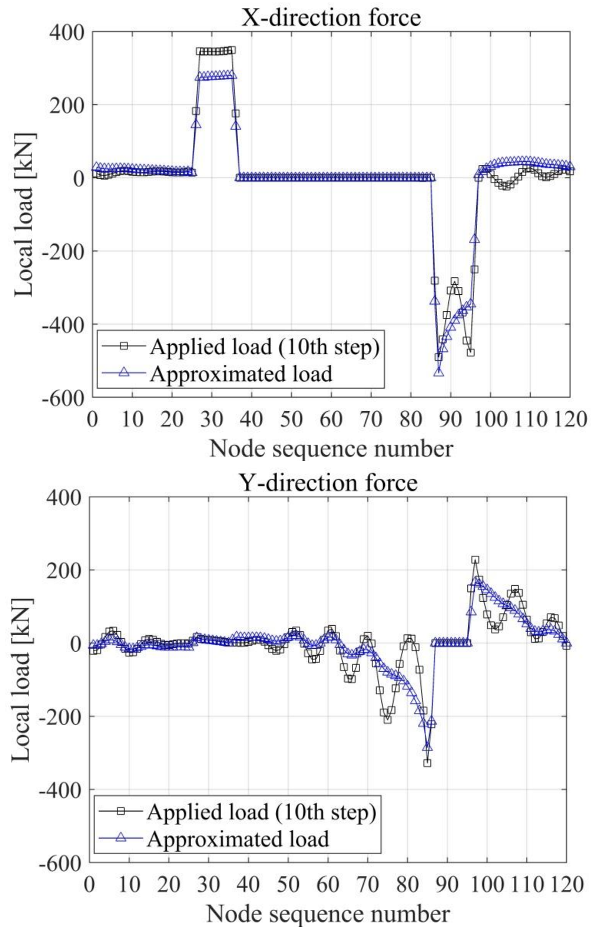
Local load distribution

Numerical Examples: Stiffened plate

(Step 2) Local load approximation

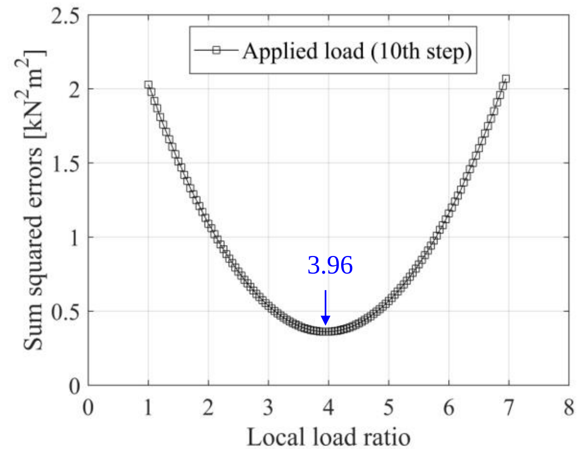


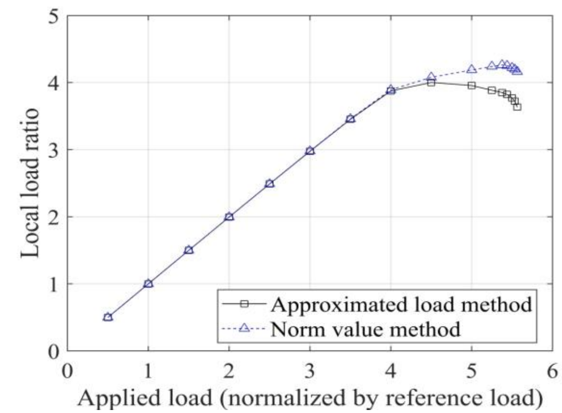
Ref. local load



Approximated local load

Sum squared error

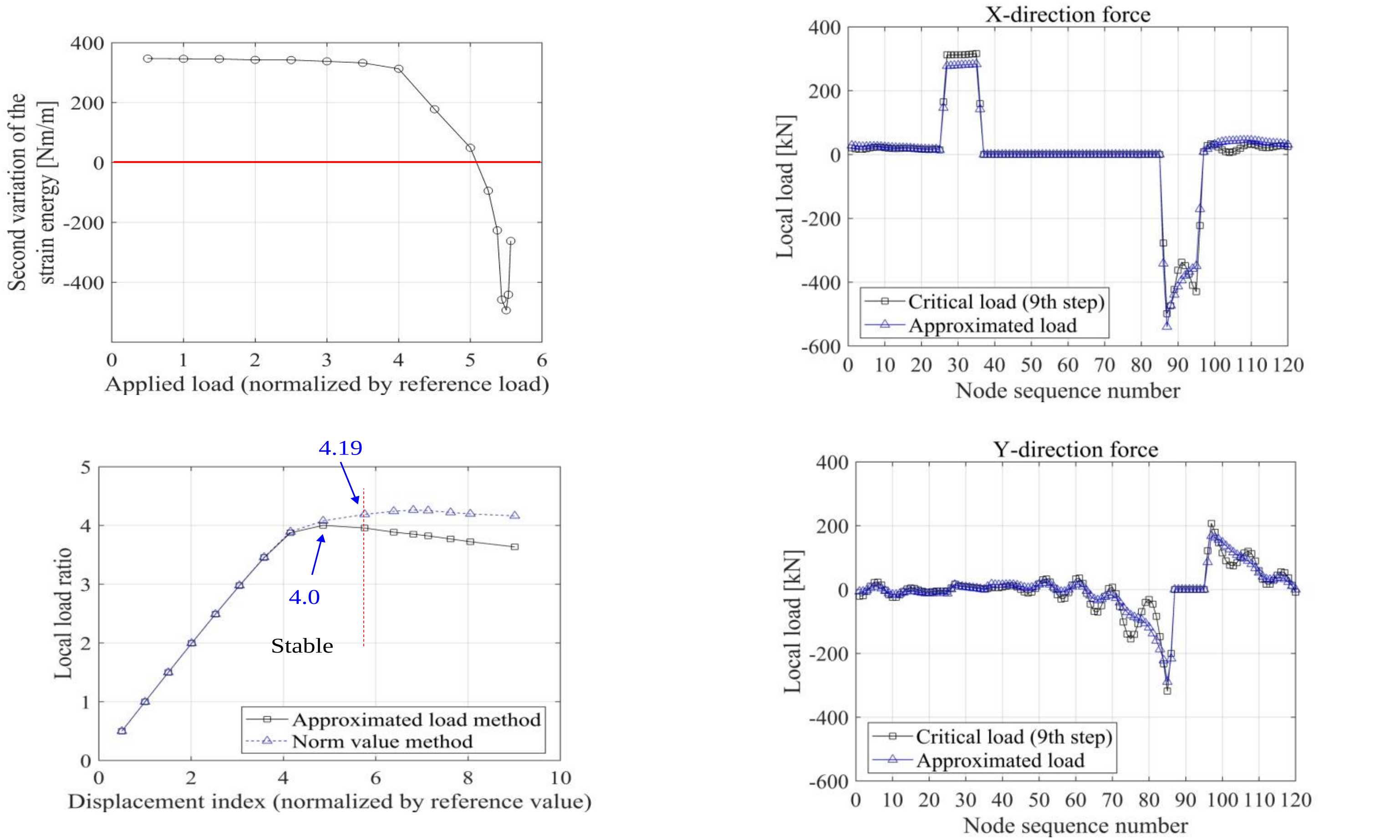




Local load ratios

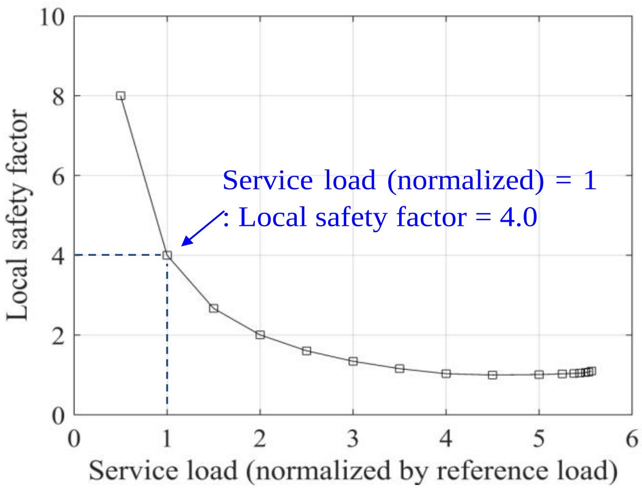
Numerical Examples: Stiffened plate

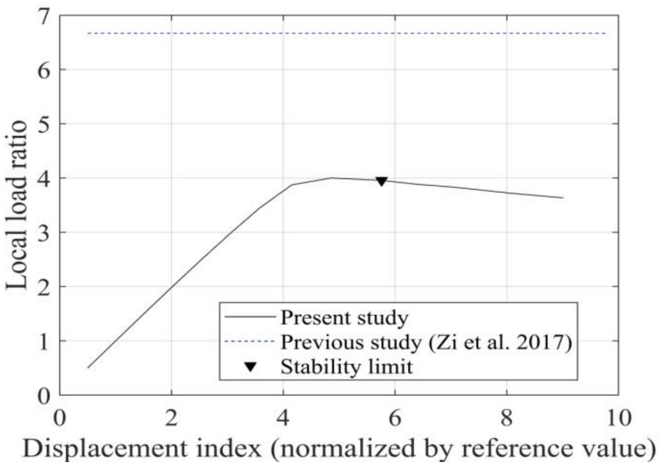
(Step 3) Stability evaluation



Numerical Examples: Stiffened plate

(Step 4) Measurement of LSF (Local Safety Factor)

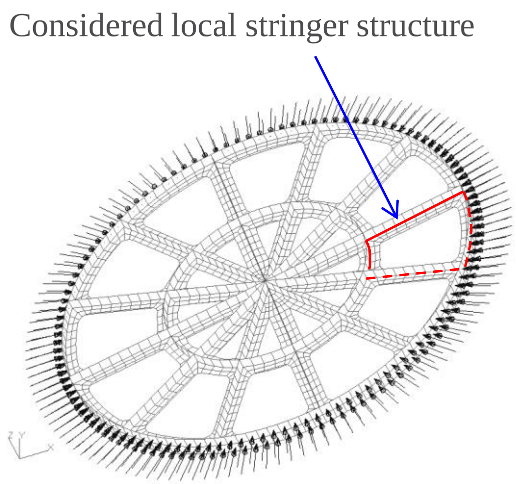




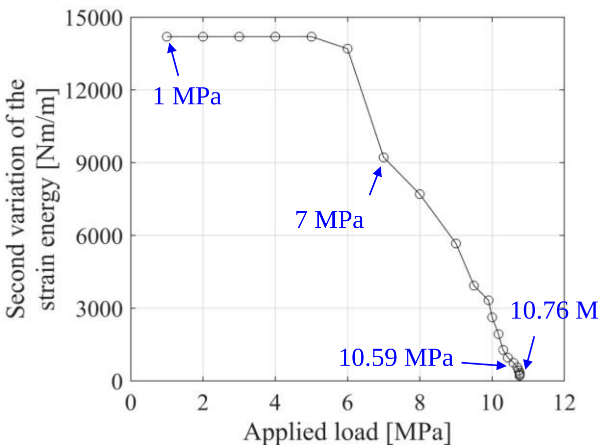
Local safety factors for a stiffened rectangular plate structure

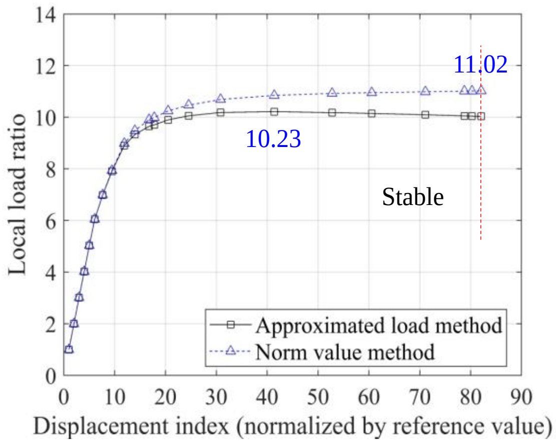
	Direct evaluation through global FE analysis (Proposed)	Evaluation method through local FE analysis (Zi et al. 2017)	Design formula (DNV, 2010)
No. 1 plate	4.00	6.67	4.26

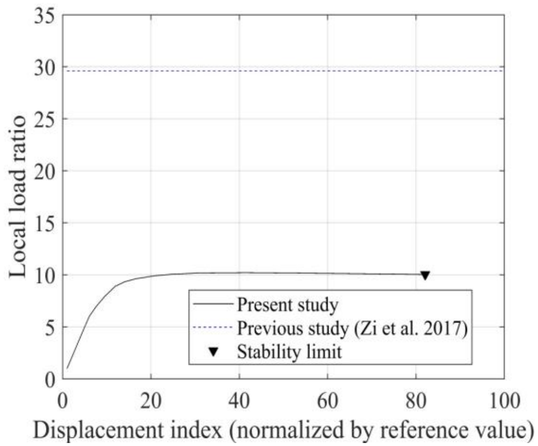
Numerical Examples: Stringer in cylindrical structure



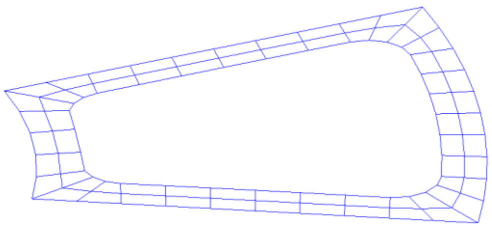
- Max. initial imperfection : 2.5mm (1st buckling mode shape)
- Applied load = 100 Mpa
- Reference load = 1 MPa wth load increment = 1 MPa



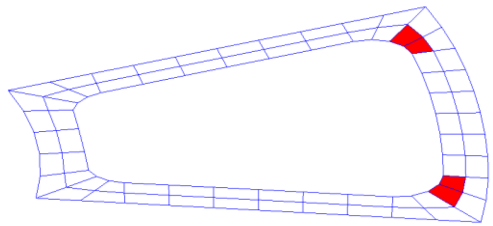




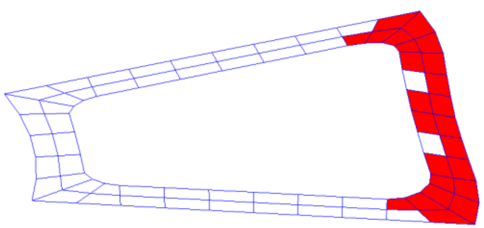
Numerical Examples: Stringer in cylindrical structure



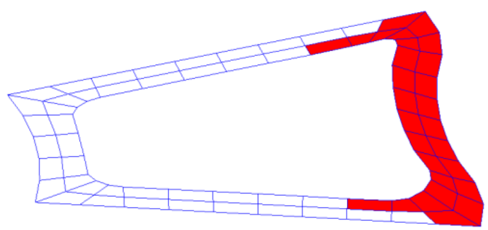
(a) 1 MPa



(b) 7 MPa

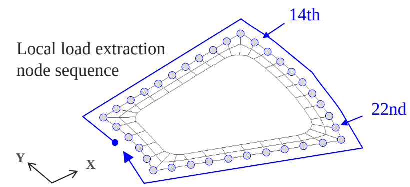


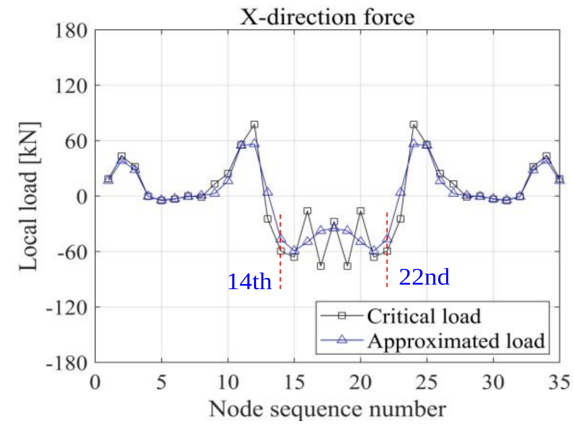
(c) 10.59 MPa

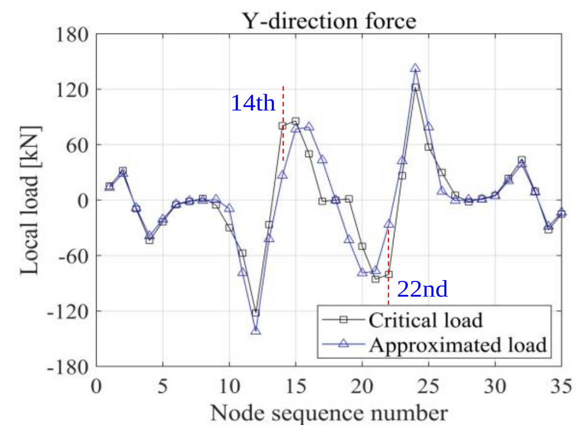


(d) 10.76 MPa

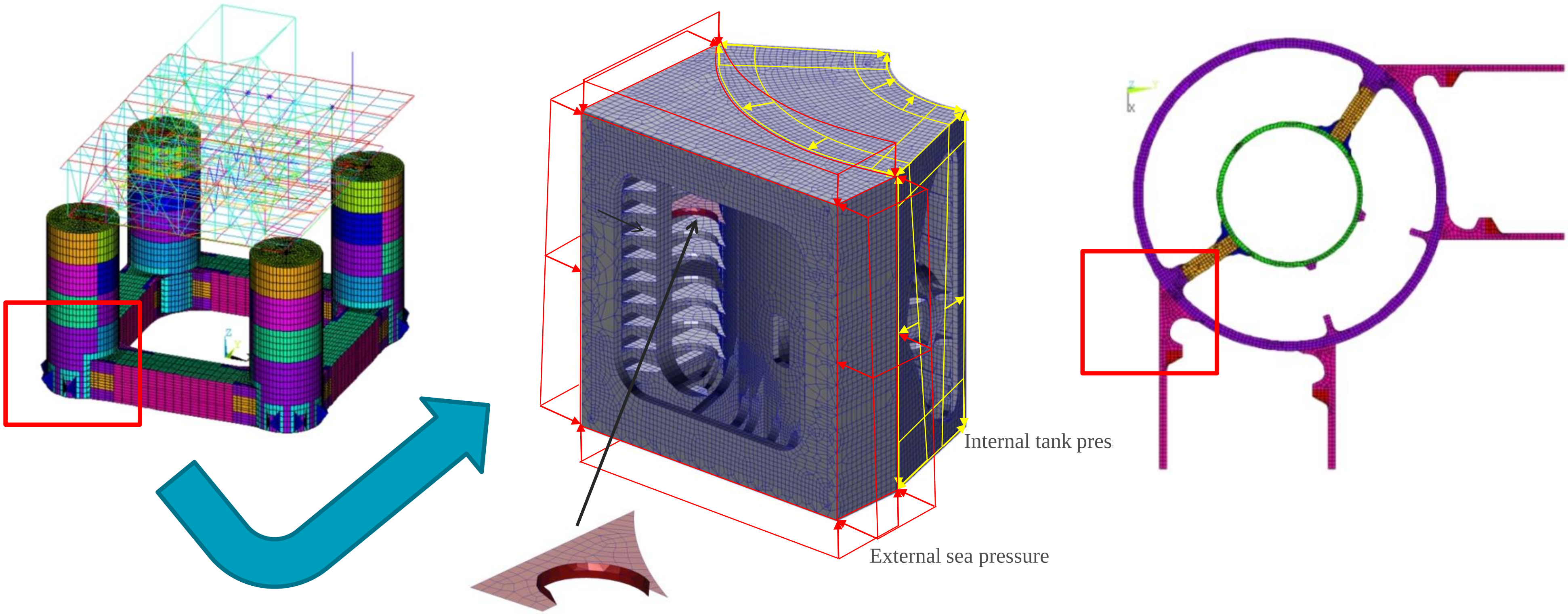
- Elastic and plastic deformations (five-time scale)
- Red color indicates material plasticity







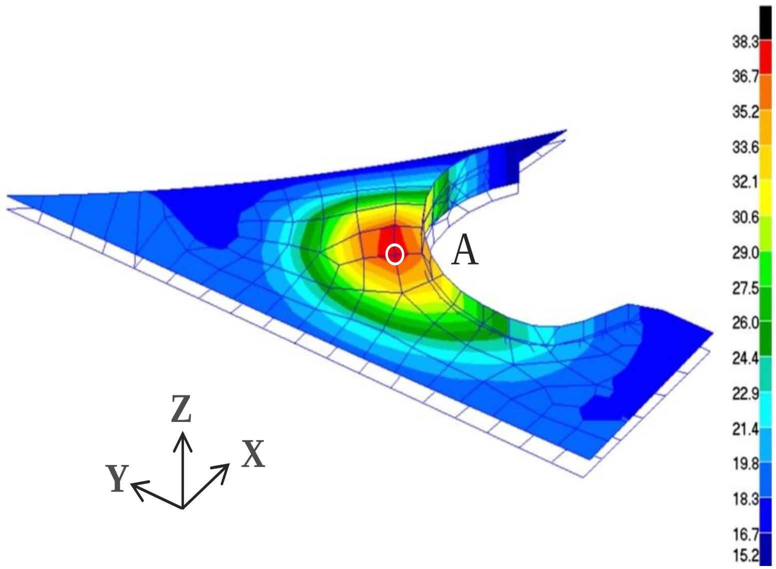
Numerical Examples: Bracket girder in TLP

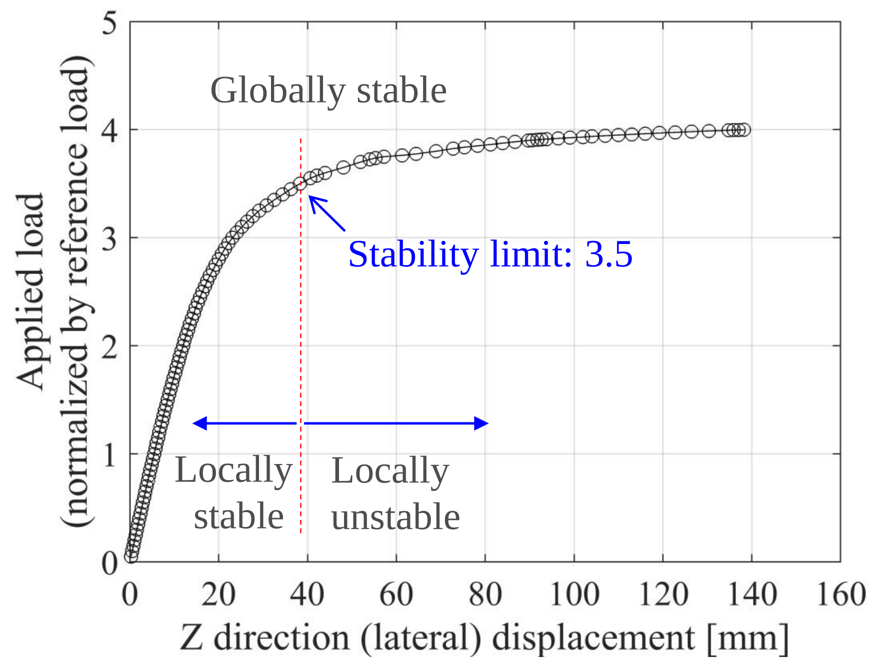


Considered local bracket girder structure

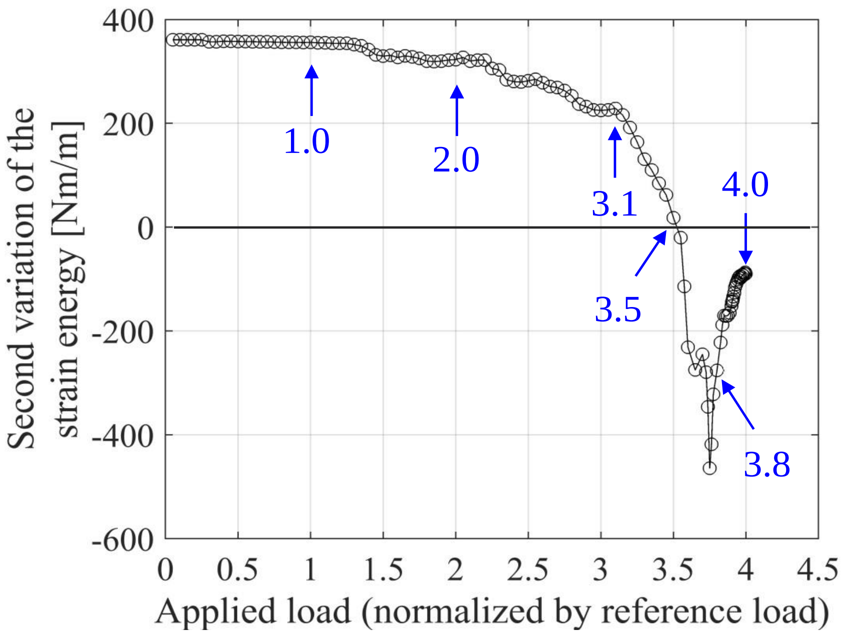
- Max. initial imperfection = 3.5mm (major buckling mode shape)
- Applied load = 100 x hydrostatic pressure
- Reference load = hydrostatic pressure

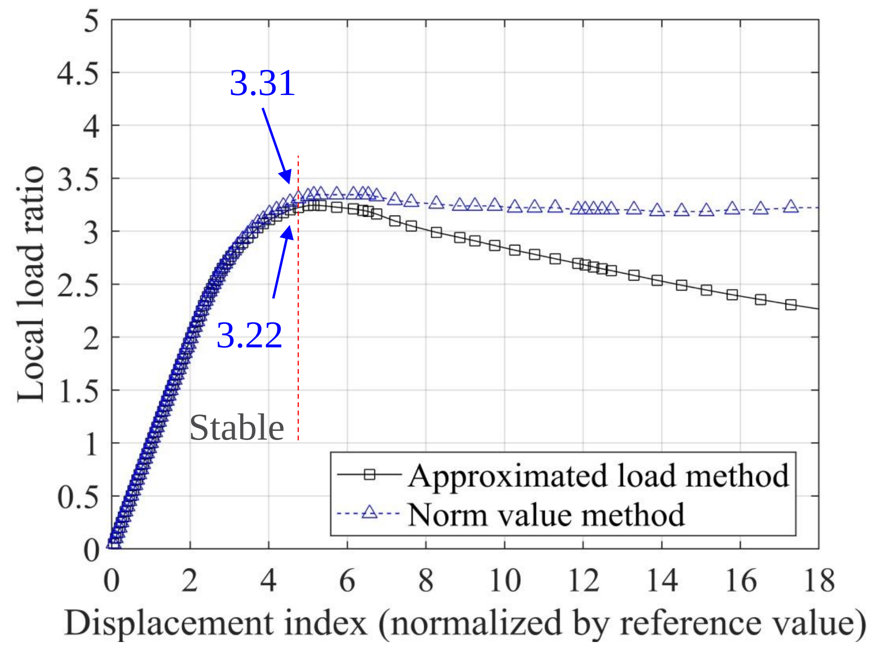
Numerical Examples: Bracket girder in TLP



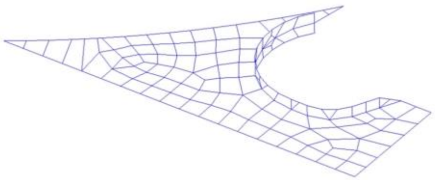


Numerical Examples: Bracket girder in TLP

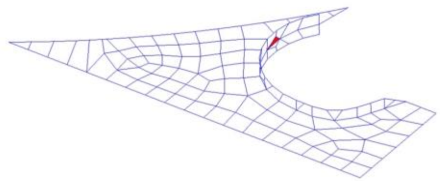




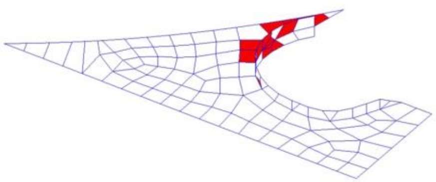
Numerical Examples: Bracket girder in TLP



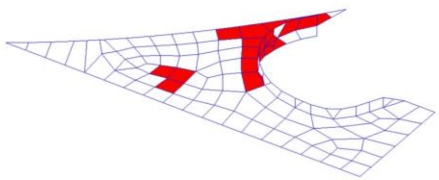
(a) 1.0



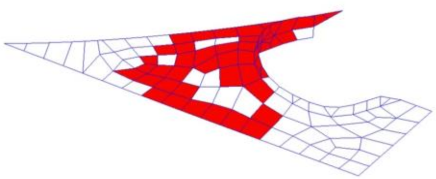
(b) 2.0



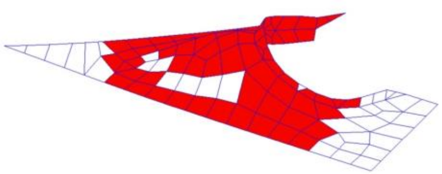
(c) 3.1



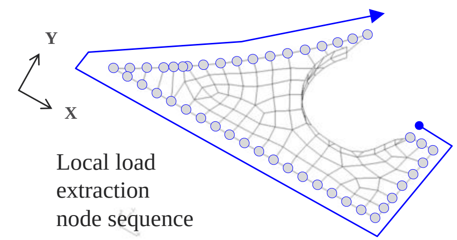
(d) 3.5

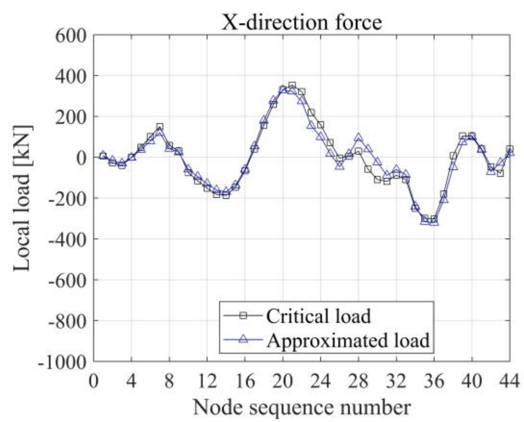


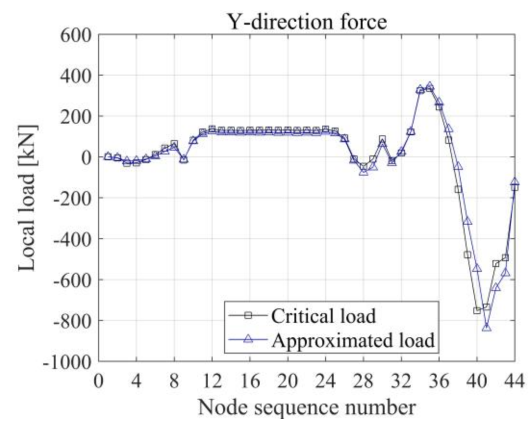
(e) 3.8



(f) 4.0



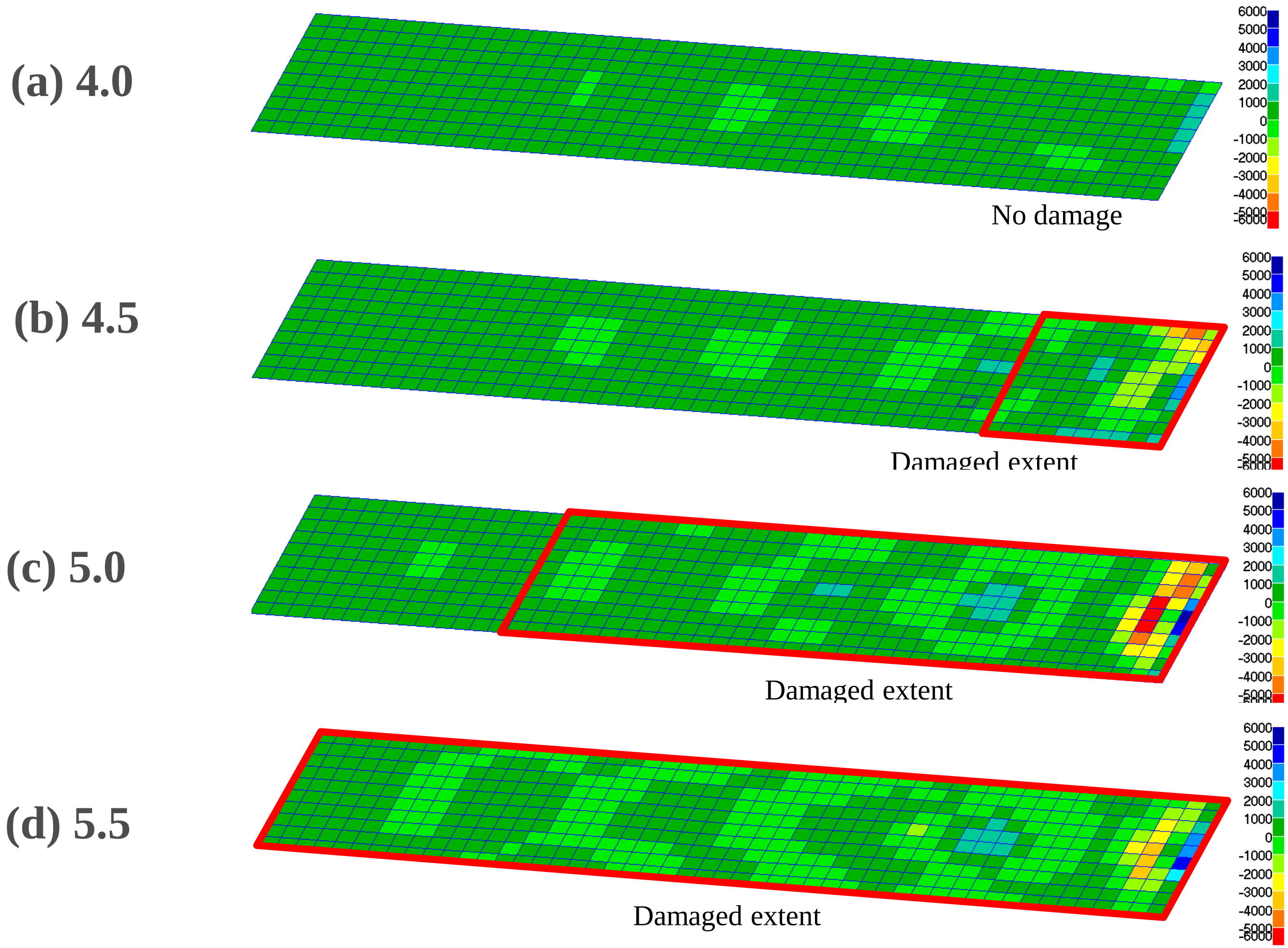




- Elastic and plastic deformations
- Red color indicates material plasticity

Damaged area identification

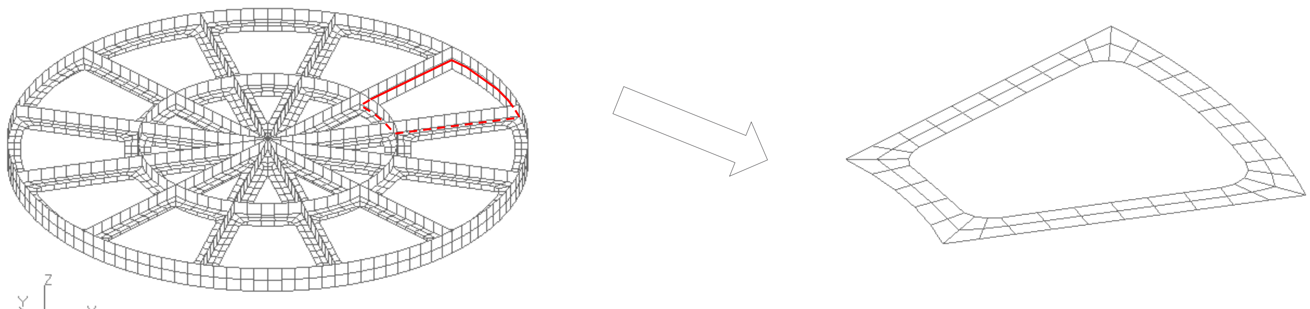
Damaged area identification : Stiffened plate



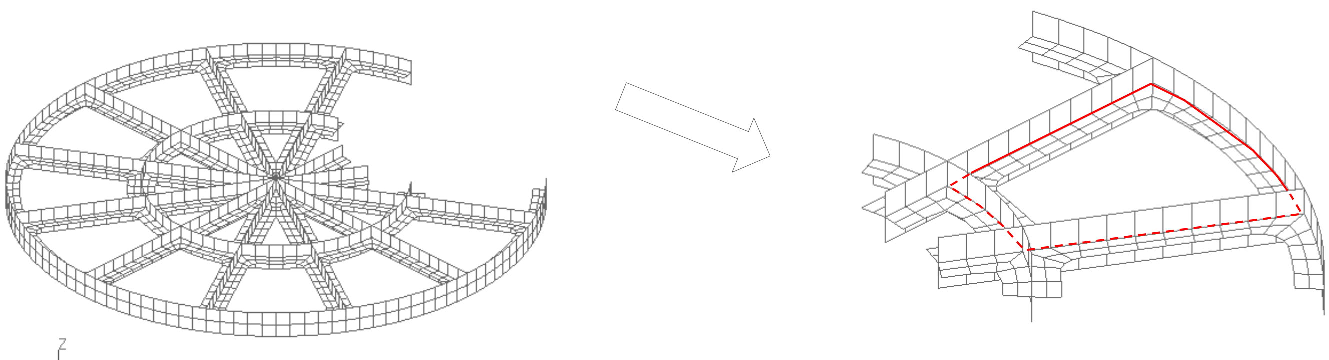
Model reduction

Numerical Examples: Stringer in cylindrical structure

Reduced domain structural analysis



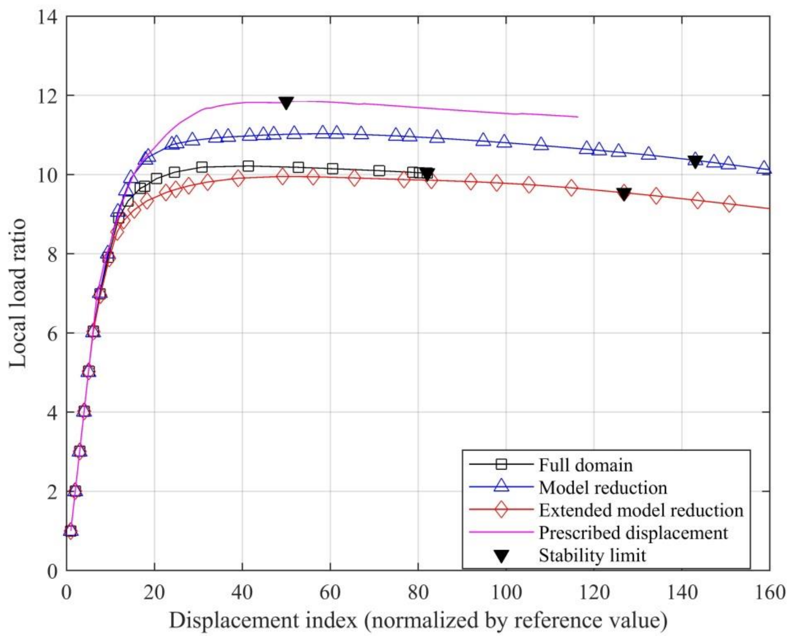
Nonlinear analysis domain of stringer unit for superelement analysis and prescribed displacement analysis

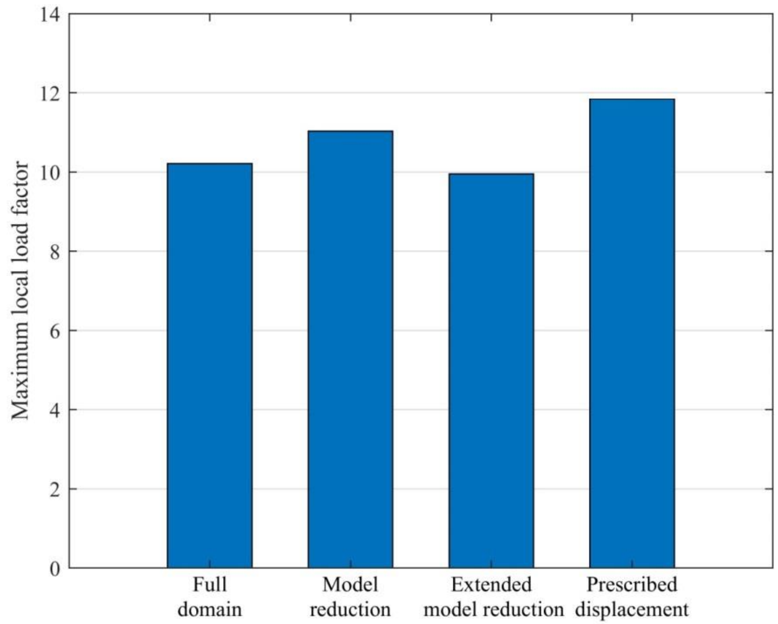


Nonlinear analysis domain of extended stringer for superelement analysis

Numerical Examples: Stringer in cylindrical structure

Comparisons with reduced domain structural analysis

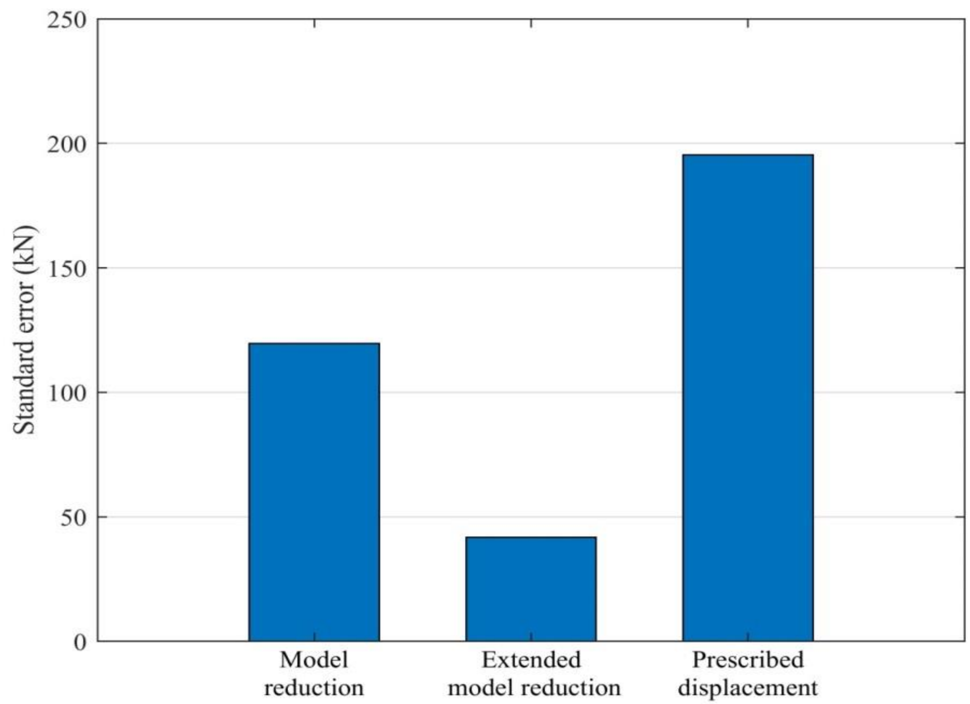




	Full domain	Model reduction	Extended model reduction	Prescribed displacement
Local safety factor	10.22	11.03	9.95	11.84
(Normalized)	1.00	1.08	0.97	1.16

Numerical Examples: Stringer in cylindrical structure

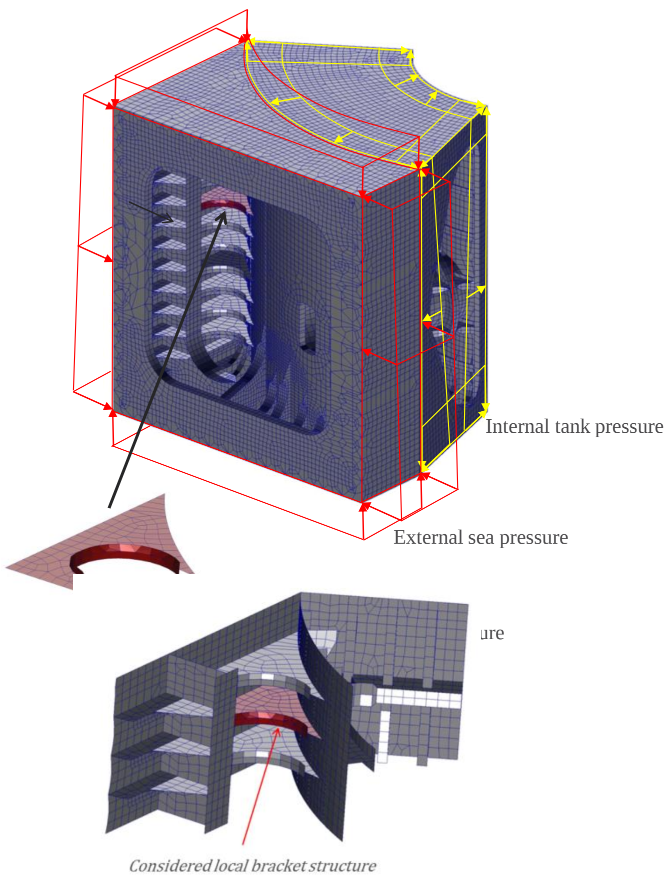
Standard Errors of local load

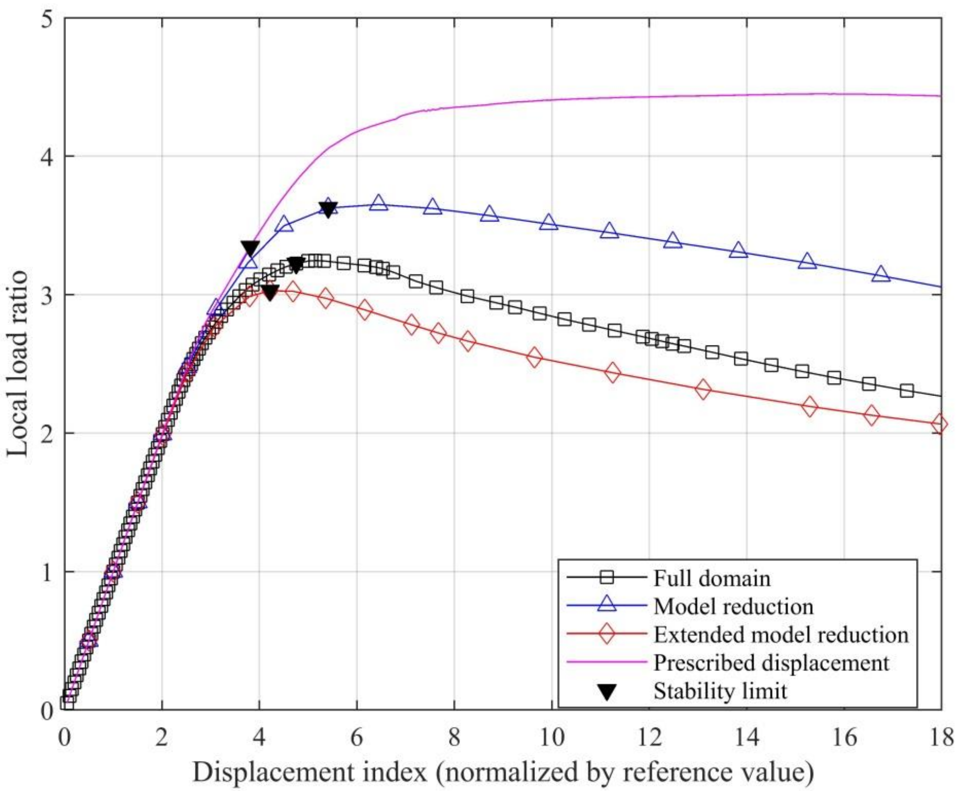


Standard error of local load	Model reduction	Extended model reduction	Prescribed displacement
	120	42	195

Numerical Examples: Bracket girder in TLP

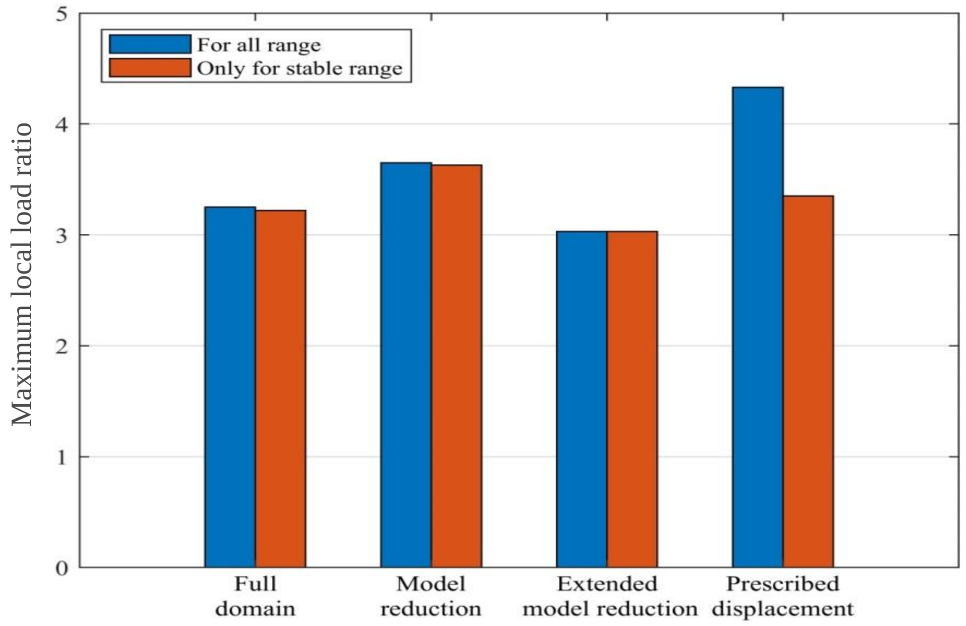
Comparisons with reduced domain structural analysis





Numerical Examples: Bracket girder in TLP

Comparisons with reduced domain structural analysis



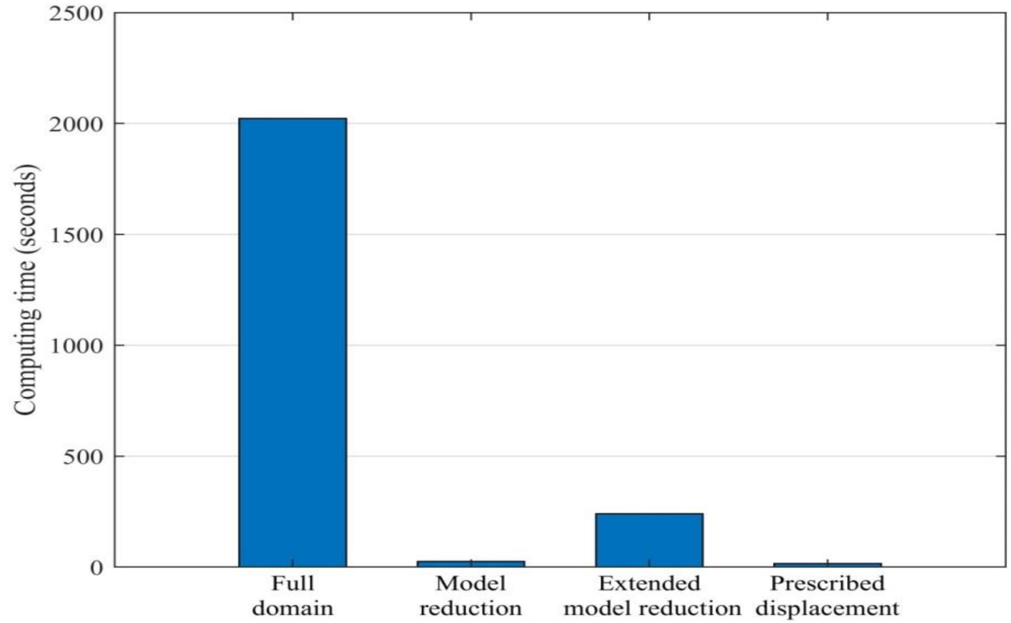
	Full domain	Model reduction	Extended model reduction	Prescribed displacement
Local safety factor	3.22	3.63	3.03	3.35
(Normalized)	1.00	1.12	0.94	1.04

Numerical Examples: Bracket girder in TLP

<u>Computing Resource</u>

1) System: GenuineIntel / 2600 MHz / RAM 251GB

2) Platform: Intel linux 3.10.0-693.el7.x86_64



Item		Full domain	Model reduction	Extended model reduction	Prescribed displacement
Number of Nodes	Residual model	37,809	159	2,423	159
	Condensed interface		44	263	
Time (seconds)	Condensation		16	54	
	Iterative run	2,023	9	186	16
	Total	2,023	25	240	16

CONCLUSIONS

Conclusions

- The proposed method was applicable to arbitrary parts of general structures and the local resistance capacity could be precisely quantified by the local safety factor.

- Through the global FE analysis, complicated interactions between local and global structures were directly considered.

- The method could be easily used to define vulnerable areas by easily changing the local extents without additional finite element analysis.

- When the target local structure loses stability, the magnitude of the external force acting on the global structure could be calculated.

- The computational efficiency was improved by use of the model reduction technique for large FE models.

Future studies

- The method can be applied to demonstrate strength proofs of irregularly reinforced structures and weakened plate openings which are frequently required to be evaluated, not predefined in design formulas.

- In addition, the residual strength can be identified to determine the reinforcement of local structures after structural damages from grounding or collision accidents.

감사합니다.