Continuum mechanics based beam elements for linear and nonlinear analyses of multi-layer beam structures

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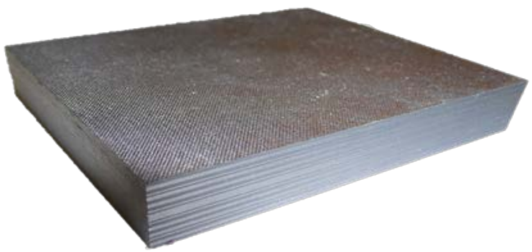
1. Introduction

Applications of layered structures

- **Layered beam structures**
  - ✓ Capability to combine several components depending on specific design requirements
  - ✓ High strength-to-weight ratio
  - ✓ High durability
  - ✓ High resistance
  - ✓ Required design property (minimum bending radius, flexibility…)
  - ✓ Cost-effective

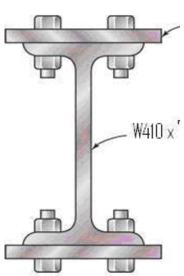
<Laminated wood>

<Allied steel-wood>

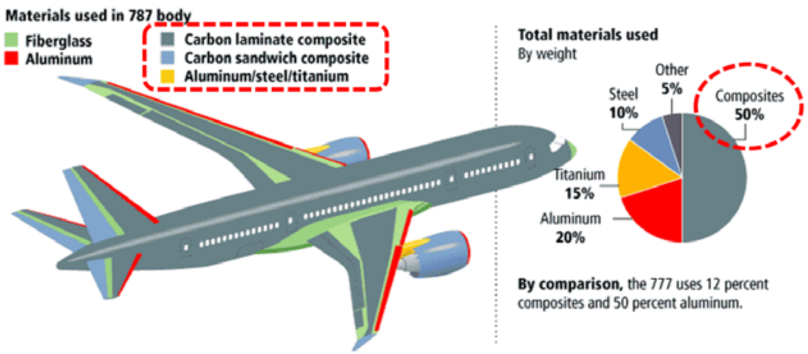


<Carbon-epoxy laminate>

<Steel-concrete beam>



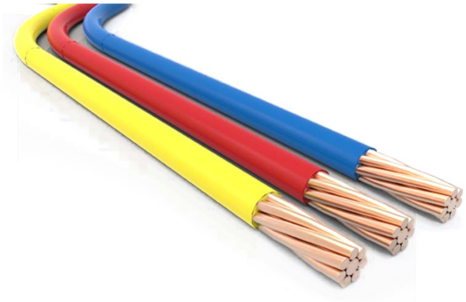
<Bolted steel beam>



<Airbus 787>

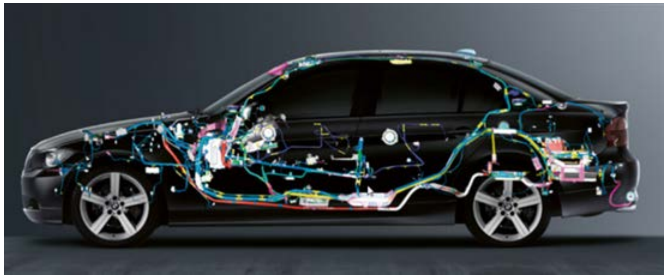
Applications of layered structures

- **Layered beam structures – helically stranded geometry**
  - ✓ Capability to combine several components depending on specific design requirements
  - ✓ High strength-to-weight ratio
  - ✓ High durability
  - ✓ High resistance
  - ✓ Required design property (minimum bending radius, flexibility…)
  - ✓ Cost-effective



<copper electrical conductor>

<optical fiber cable>



<Automobile cable>

<Aluminum alloy conductors>

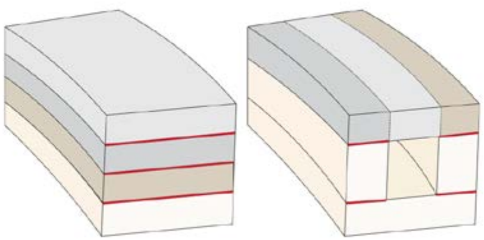
<Submarine power cable>

<Medical cable>

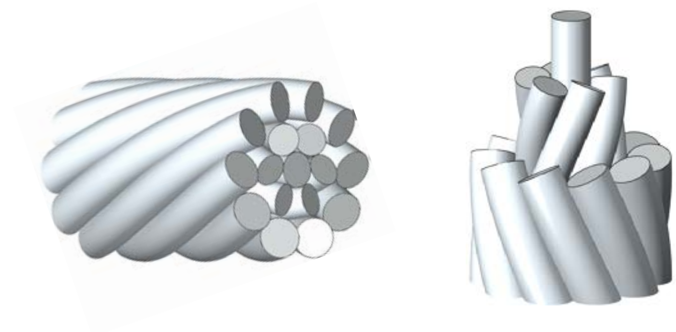
Research purpose

- **Research purpose**

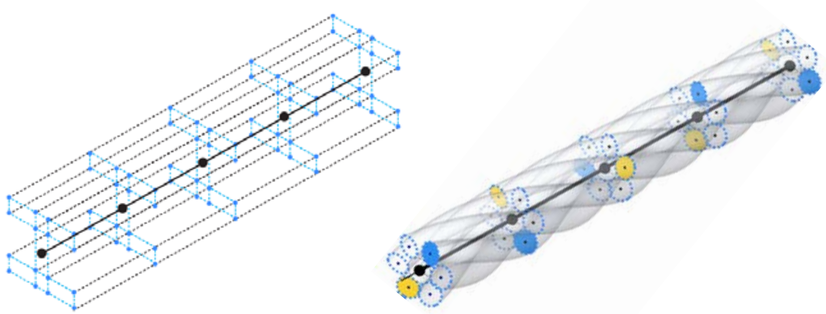
<Multi-layered beam structures>



<Multi-layered helically stranded cable structure>



Develop efficient beam finite element for multi-layered beam structures based on continuum mechanics formulation



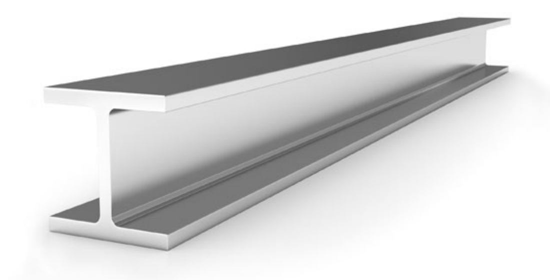
A single beam model

✓ Model the complex multi-layered beam geometries as a single beam unit

- Arbitrary number of layers and interlayers
- Complex geometries and various loading/boundary conditions
- Geometrical and material nonlinearities
- Nonlinear load-slip relation at interlayers

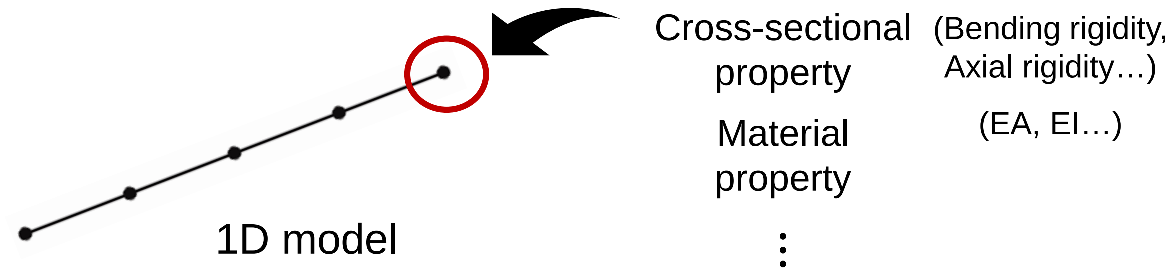
2. Concepts of continuum beam finite elements

Concept of continuum beam finite elements

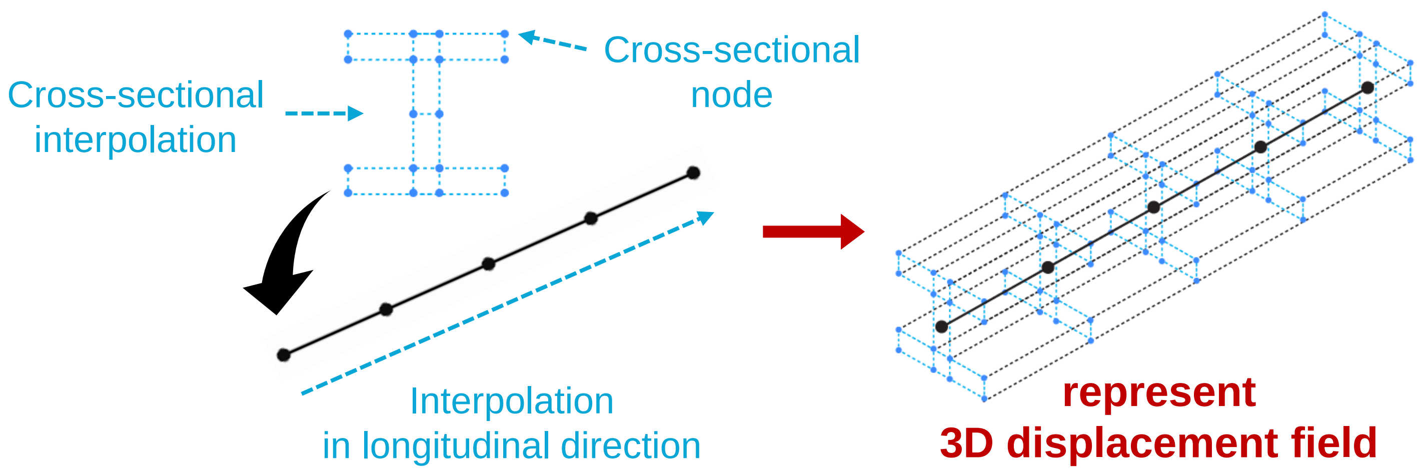


<Beam structure>

- **Classical beam**

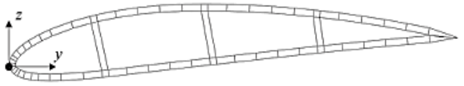


- **Continuum beam**

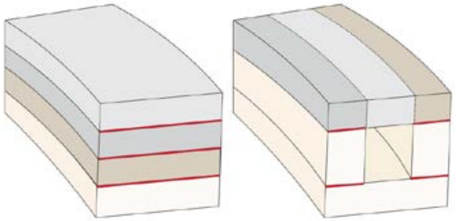


Features of continuum beam finite elements

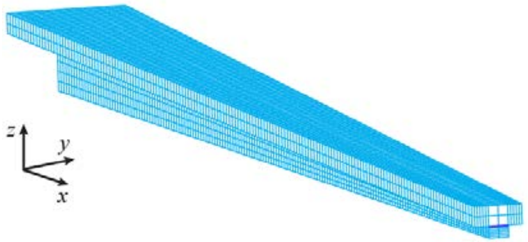
- **Great modeling capability**



<Complicated cross-section>

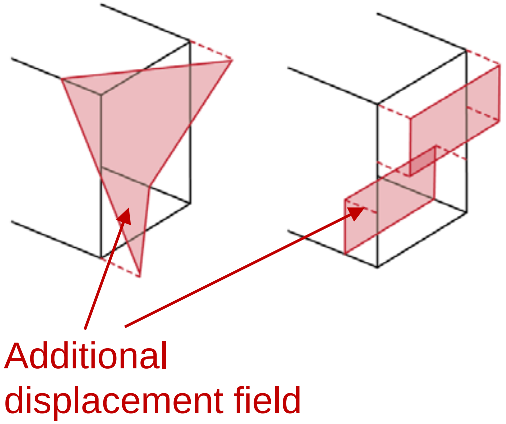


<Composite cross-section>



<Tapered geometry>

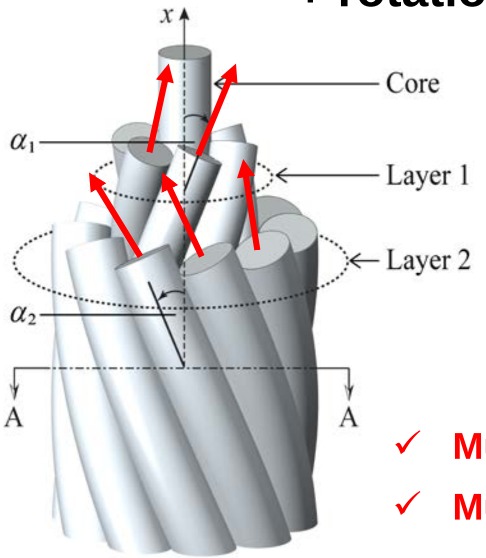
- **Additional displacement field**



Out-of-plane displacement

- ✓ Warping mode
- ✓ Interlayer slip
- ✓ Distorted mode

- **Individual cross-section direction + rotation direction**



- ✓ **Multi-layer**
- ✓ **Multi-director**

Features of continuum beam finite elements

- **Composition of continuum displacement fields**

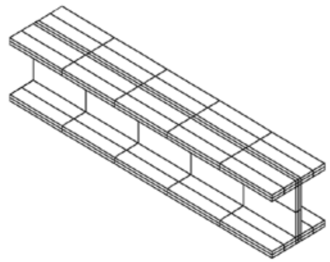
$$ {}_0\mathbf{U}_k = \left[{}_0u_k \quad {}_0v_k \quad {}_0w_k \quad {}_0\theta_x^k \quad {}_0\theta_y^k \quad {}_0\theta_z^k \right]^{\mathrm{T}} + $$ Additional displacements fields defined by additional DOFs → Modes are independent each other

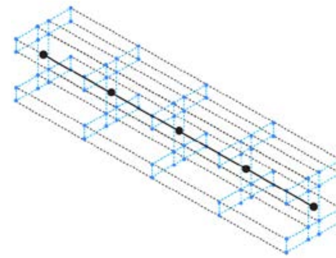
Basic 6DOFs

- **3D Beam**

  (6DOF + warping)

  Warping DOF: ${}_0\alpha_k$

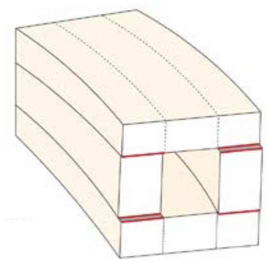


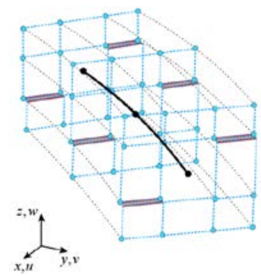


- **Layered beam**

  (6DOF + Interlayer slip)

  Layer DOF: ${}_0\boldsymbol{\varphi}_k = \left[{}_0\varphi_k^{(1)} \quad \cdots \quad {}_0\varphi_k^{(l-1)} \right]^{\mathrm{T}}$





- **Helically stranded cable**

  (6DOF + Local wire rotation + Inter-wire slip)

  Wire rotation DOF: ${}_0\boldsymbol{\theta}_w^{k(n)} = \left[{}_0\boldsymbol{\theta}_w^{k(1)} \quad \cdots \quad {}_0\boldsymbol{\theta}_w^{k(n_L)} \right]^{\mathrm{T}}$

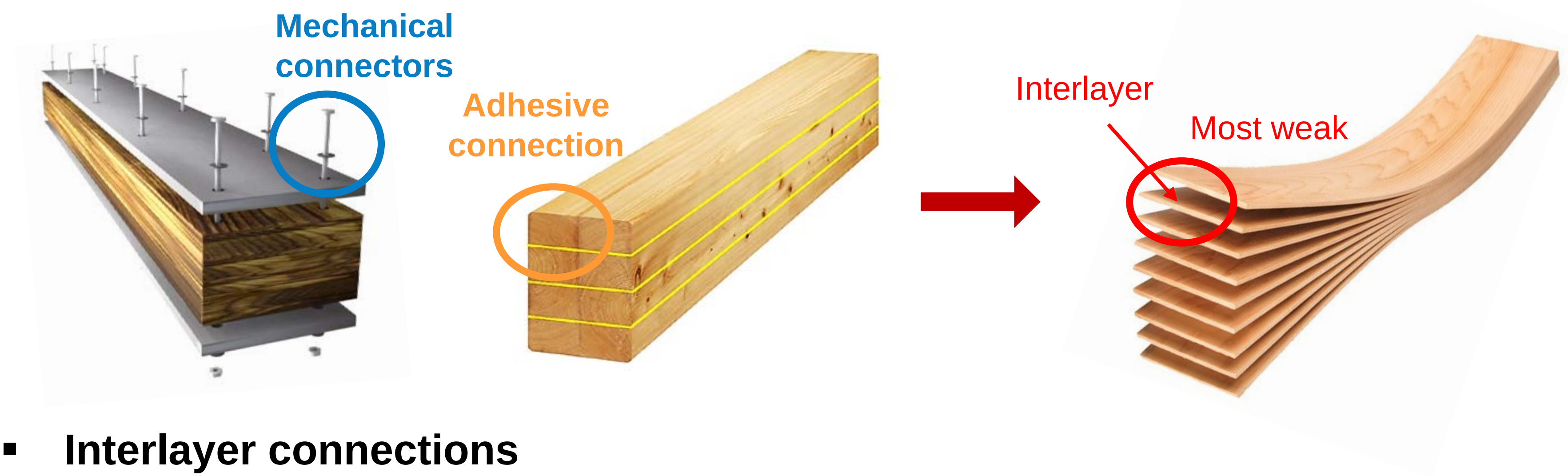
  Wire DOF: ${}_0\boldsymbol{\varphi}_k = \left[{}_0\varphi_k^{(1)} \quad \cdots \quad {}_0\varphi_k^{(n_w)} \right]^{\mathrm{T}}$

3. Development of continuum mechanics based beam finite elements

3.1 Continuum mechanics based beam formulation for multi-layered beams

Interlayer connections of layered structures

- **Composition of layered structures**
  - ✓ Loss of stiffness
  - ✓ Increase in deflections
  - ✓ More flexible behaviors
  - ✓ Increase nonlinearities



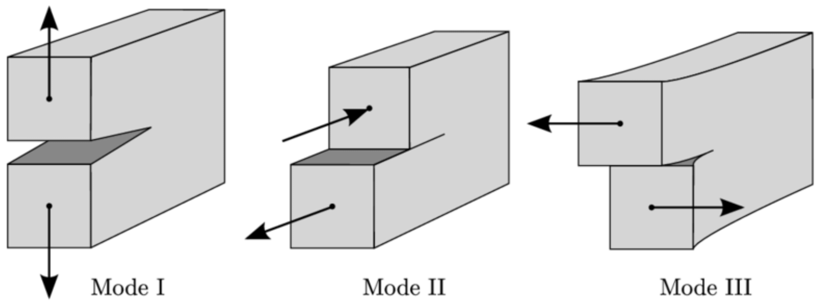
- **Interlayer connections**
  - Mechanical connectors: bolts, nails, etc.

  - Adhesive connections: epoxy, wood glue, etc.

→ **Behaviors of layered structures significantly depend on interlayer connections**

Research history of layered structures

- **Three fundamental failure modes at interlayers**



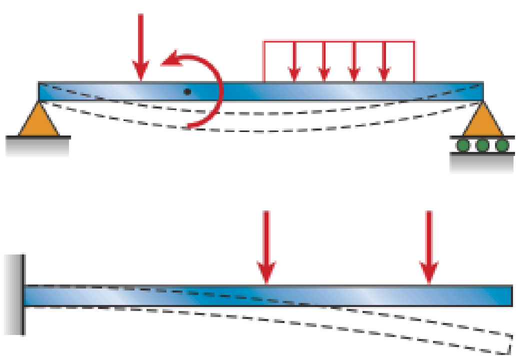
Normal mode (Opening mode)	Shear mode (In plane mode)	Shear mode (out-of-plane mode)

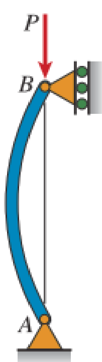
Important modes depend on

- ✓ Dominant loading conditions
- ✓ Structural member property
- ✓ Interlayer connection property

- **Beam structures:**

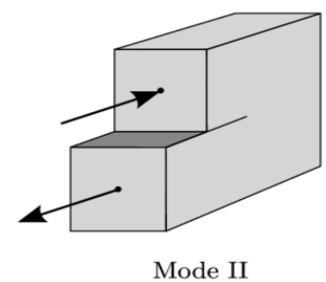
Structural element that primarily resists loads applied laterally to the beam's axis





<Beam-column>

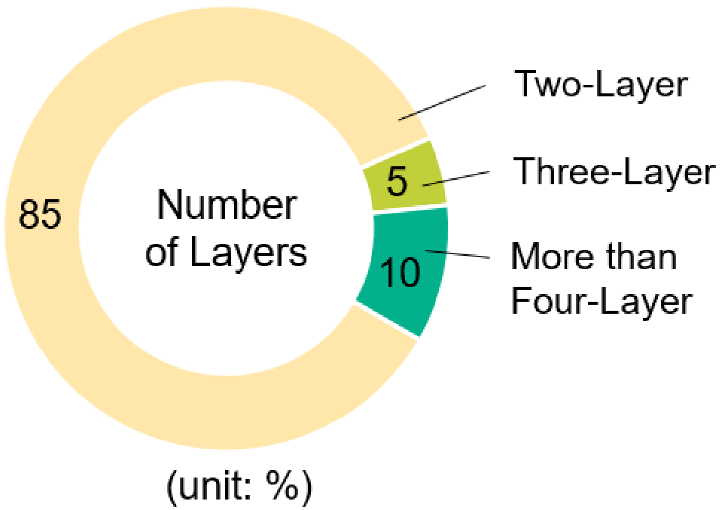
Dominant

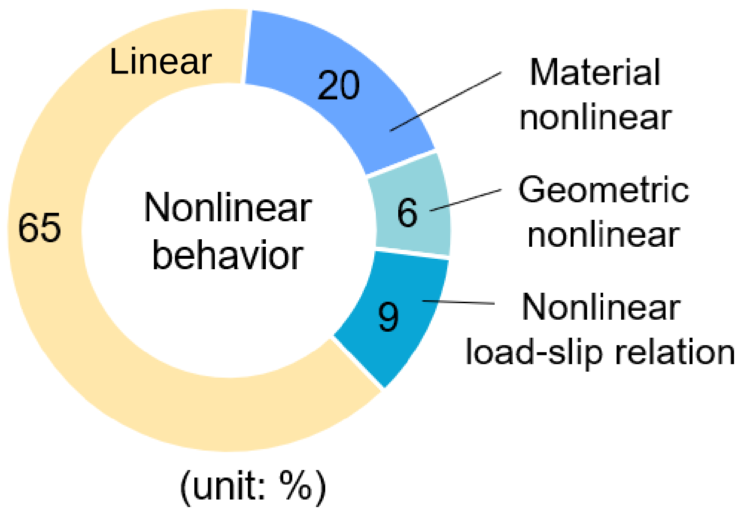


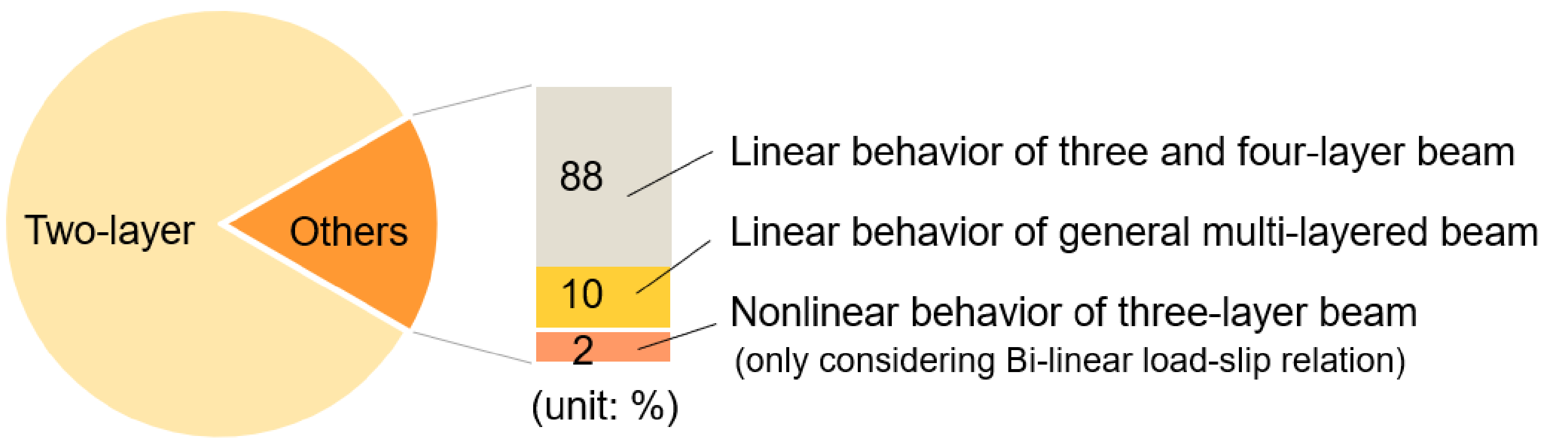
Interlayer slip

Research distribution – Layered beam behavior

- **Analysis model for layered beam including interlayer slips**





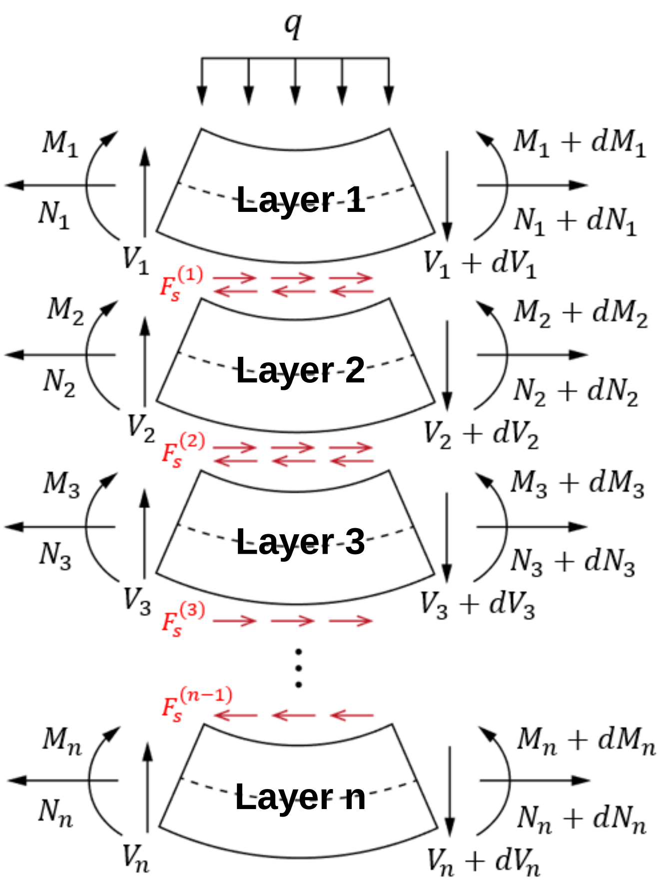


**Most researches only can considers two- and three- layer beam.
Nonlinear behavior of multi-layered beam is very rarely discussed.**

Multi-layered structures

- **Challenging issue: Nonlinear behavior of multi-layered beam structures**

As the number of Layers increases → the complexity of problem increases extremely



✓ Composite interaction between the layers

✓ Interlayer behavior

+ Nonlinearities

✓ Nonlinear load-slip relation at interlayers

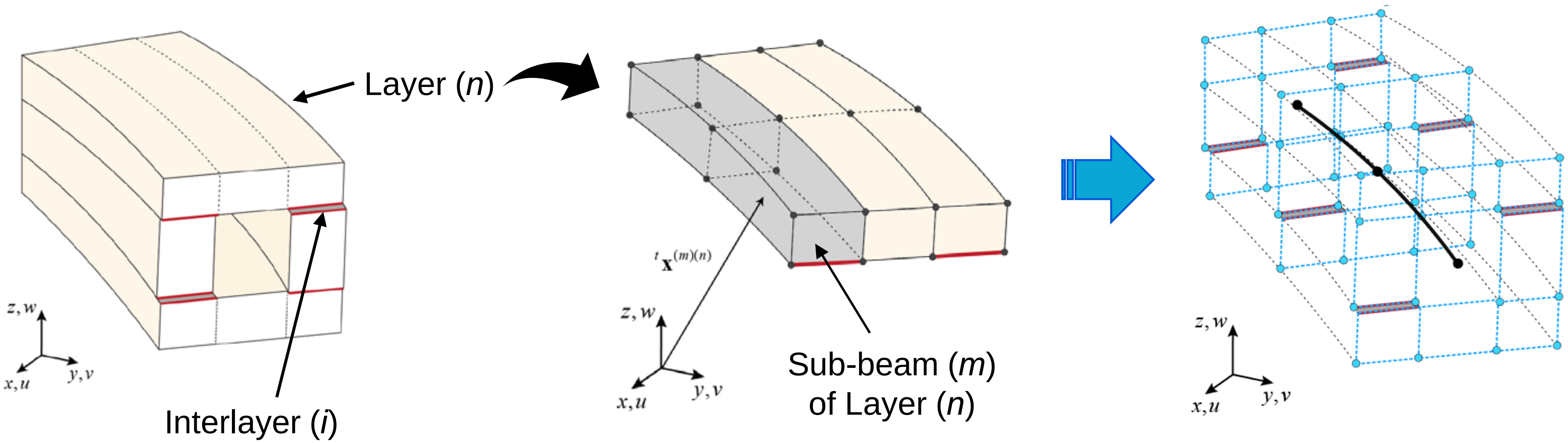
✓ Geometrical nonlinearity

✓ Material nonlinearity

✓ Nonlinear convergence problem

Continuum mechanics based beam formulation

▪ Concept of continuum mechanics based beam formulation



${}^{t}F_{s}^{(i)}$: slip force

${}^{t}s^{(i)}$: interlayer slip displ.

✓ Arbitrary number of layers and interlayers

✓ Complicated cross-sectional geometries

▪ Incremental nonlinear analysis

Incremental load-slip relation

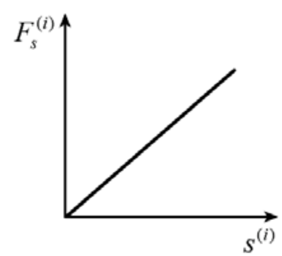
$${}_{0}F_{s}^{(i)} = {}_{0}K_{s}^{(i)}\,{}_{0}s^{(i)}$$

where

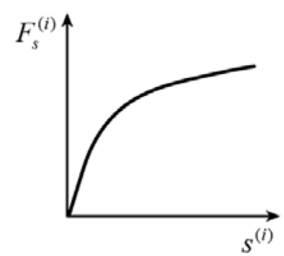
$${}^{t+\Delta t}F_{s}^{(i)} = {}^{t}F_{s}^{(i)} + {}_{0}F_{s}^{(i)}$$

$${}^{t+\Delta t}s^{(i)} = {}^{t}s^{(i)} + {}_{0}s^{(i)}$$

- **Linear load-slip**

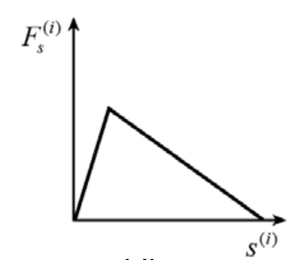


- **Mechanical connectors**



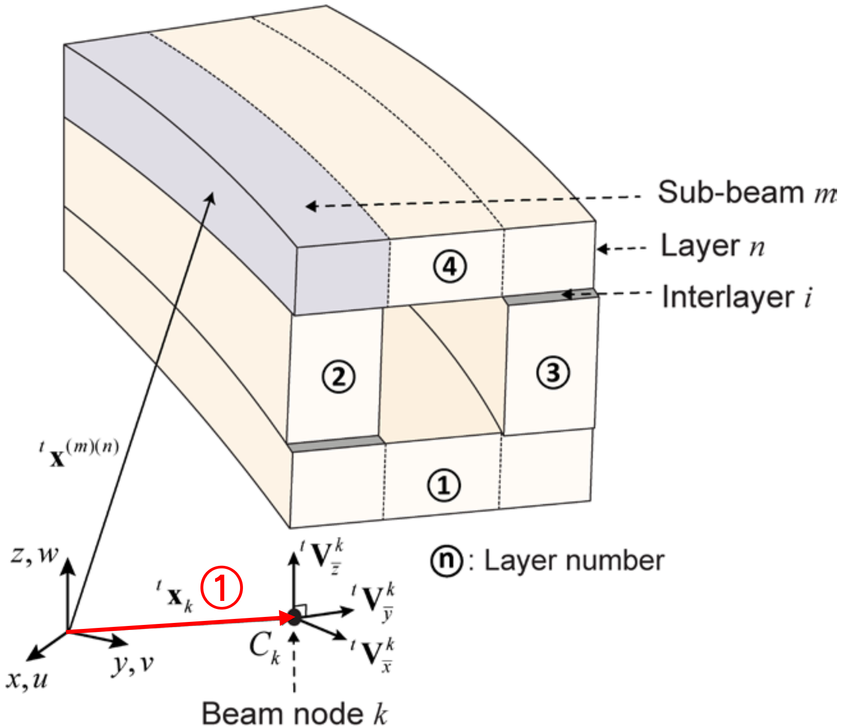
Nonlinear load-slip relation based on experimental study

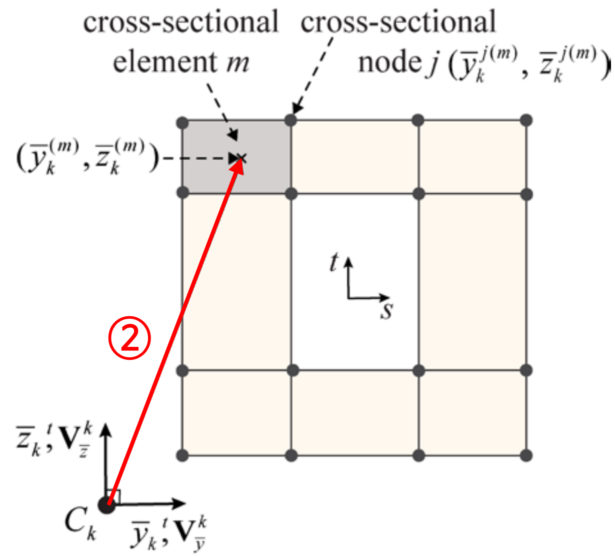
- **Adhesive connection**



Bi-linear cohesive model

Continuum mechanics based beam formulation



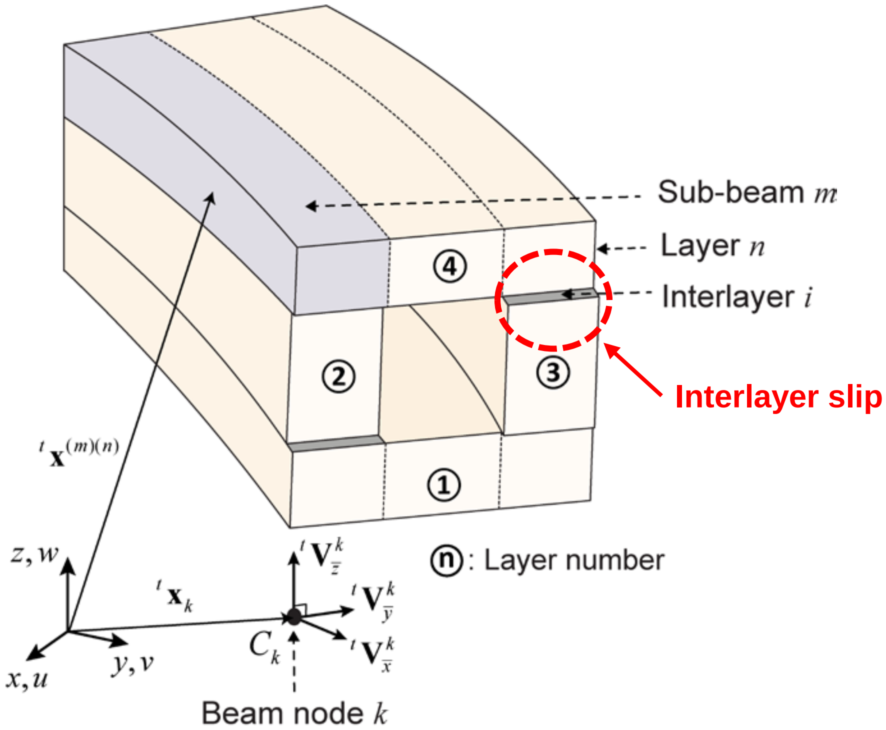


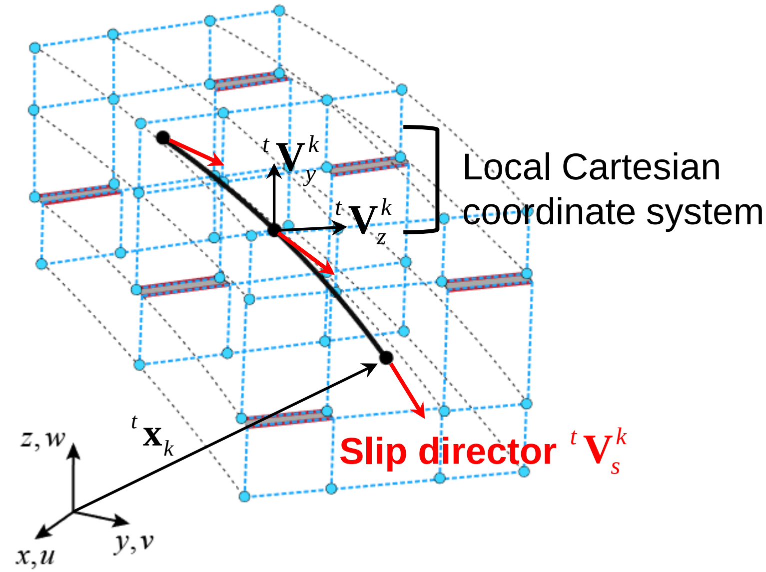
$$\left[\begin{aligned} \bar{y}_k^{(m)} &= \sum_{j=1}^{p} h_j(s,t)\bar{y}_k^{j(m)} \\ \bar{z}_k^{(m)} &= \sum_{j=1}^{p} h_j(s,t)\bar{z}_k^{j(m)} \end{aligned}\right.$$

- **Geometry interpolation**

$$ {}^t\mathbf{x}^{(m)(n)} = \underbrace{\sum_{k=1}^{q} h_k(r)\,{}^t\mathbf{x}_k}_{1} + \underbrace{\sum_{k=1}^{q} h_k(r)\bar{y}_k^{(m)(n)}\,{}^t\mathbf{V}_{\bar{y}}^k + \sum_{k=1}^{q} h_k(r)\bar{z}_k^{(m)(n)}\,{}^t\mathbf{V}_{\bar{z}}^k}_{2} + \sum_{k=1}^{q} h_k(r)\,{}^t\varphi_k^{(n)}\,{}^t\mathbf{V}_s^k $$

Continuum mechanics based beam formulation





$${}^{t}\mathbf{V}_{s}^{k} = \frac{{}^{t}\mathbf{g}_{x}^{k}}{\left\| {}^{t}\mathbf{g}_{x}^{k} \right\|} \quad \text{where} \quad {}^{t}\mathbf{g}_{x}^{k} = \sum_{\xi=1}^{q} \left. \frac{\partial h_{\xi}(r)}{\partial r} \right|_{r=r_k} {}^{t}\mathbf{x}_{\xi}$$

- **Geometry interpolation**

$${}^{t}\mathbf{x}^{(m)(n)} = \sum_{k=1}^{q} h_k(r)\, {}^{t}\mathbf{x}_k + \sum_{k=1}^{q} h_k(r)\, \bar{y}_k^{(m)(n)}\, {}^{t}\mathbf{V}_{\bar{y}}^{k} + \sum_{k=1}^{q} h_k(r)\, \bar{z}_k^{(m)(n)}\, {}^{t}\mathbf{V}_{\bar{z}}^{k} + \underbrace{\sum_{k=1}^{q} h_k(r)\, {}^{t}\varphi_k^{(n)}\, {}^{t}\mathbf{V}_{s}^{k}}_{③}$$

Continuum mechanics based beam formulation

▪ Incremental displacement

- Geometry interpolation

$$ {}^{t}\mathbf{x}^{(m)(n)} = \sum_{k=1}^{q} h_k(r)\,{}^{t}\mathbf{x}_k + \sum_{k=1}^{q} h_k(r)\bar{y}_k^{(m)(n)}\,{}^{t}\mathbf{V}_{\bar{y}}^{k} + \sum_{k=1}^{q} h_k(r)\bar{z}_k^{(m)(n)}\,{}^{t}\mathbf{V}_{\bar{z}}^{k} + \sum_{k=1}^{q} h_k(r)\,{}^{t}\varphi_k^{(n)}\,{}^{t}\mathbf{V}_{s}^{k} $$

$$\downarrow$$

- Linearized incremental displacement

$$ {}_{0}\mathbf{u}^{(m)(n)} \approx \sum_{k=1}^{q} h_k(r)\,{}_{0}\mathbf{u}_k + \sum_{k=1}^{q} h_k(r)\bar{y}_k^{(m)(n)}\hat{\mathbf{R}}({}_{0}\boldsymbol{\theta}^{k})\,{}^{t}\mathbf{V}_{\bar{y}}^{k} + \sum_{k=1}^{q} h_k(r)\bar{z}_k^{(m)(n)}\hat{\mathbf{R}}({}_{0}\boldsymbol{\theta}^{k})\,{}^{t}\mathbf{V}_{\bar{z}}^{k} $$

$$ + \sum_{k=1}^{q} h_k(r)\left({}_{0}\varphi_k^{(n)}\,{}^{t}\mathbf{V}_{s}^{k} + ({}^{t}\varphi_k^{(n)} + {}_{0}\varphi_k^{(n)})\left(\frac{1}{\left\|{}^{t}\mathbf{g}_{x}^{k}\right\|}\sum_{\xi=1}^{q} h'_{\xi}(r)\,{}_{0}\mathbf{u}_{\xi}\right)\right) $$

$$ = \begin{bmatrix} \mathbf{L}_1^{(m)(n)} & \mathbf{L}_2^{(m)(n)} & \dots & \mathbf{L}_q^{(m)(n)} \end{bmatrix} {}_{0}\mathbf{U} = \mathbf{L}^{(m)(n)}\,{}_{0}\mathbf{U} $$

where

Incremental nodal DOFs vector

$$ \underbrace{{}_{0}\mathbf{U}} = \begin{bmatrix} {}_{0}\mathbf{U}_1^{\mathrm{T}} & {}_{0}\mathbf{U}_2^{\mathrm{T}} & \dots & {}_{0}\mathbf{U}_q^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} $$

$$ {}_{0}\mathbf{U}_k = \begin{bmatrix} {}_{0}u_k & {}_{0}v_k & {}_{0}w_k & {}_{0}\theta_x^k & {}_{0}\theta_y^k & {}_{0}\theta_z^k & {}_{0}\varphi_k^{(2)} & \dots & {}_{0}\varphi_k^{(l)} \end{bmatrix}^{\mathrm{T}} $$

6+(l-1) DOFs per beam node

- Total Lagrangian formulation

$$ \sum_{n}\sum_{m}\int_{{}^{0}V^{(m)(n)}} \bar{C}_{ijkl}^{(m)(n)}\,{}_{0}\bar{e}_{ij}^{(m)(n)}\delta\,{}_{0}\bar{e}_{kl}^{(m)(n)}d\,{}^{0}V^{(m)(n)} + \sum_{n}\sum_{m}\int_{{}^{0}V^{(m)(n)}} {}_{0}^{t}\bar{S}_{ij}^{(m)(n)}\delta\,{}_{0}\bar{\eta}_{ij}^{(m)(n)}d\,{}^{0}V^{(m)(n)} $$

$$ + \sum_{i}\int_{{}^{0}L} K_s^{(i)}\,{}_{0}u_s^{(i)}\delta\,{}_{0}u_s^{(i)}d\,{}^{0}L = {}^{t+\Delta t}\Re - \sum_{n}\sum_{m}\int_{{}^{0}V^{(m)(n)}} {}_{0}^{t}\bar{S}_{ij}^{(m)(n)}\delta\,{}_{0}\bar{e}_{ij}^{(m)(n)}d\,{}^{0}V^{(m)(n)} - \sum_{i}\int_{{}^{0}L} {}^{t}F_s^{(i)}\delta\,{}_{0}u_s^{(i)}d\,{}^{0}L $$

Constitutive relations

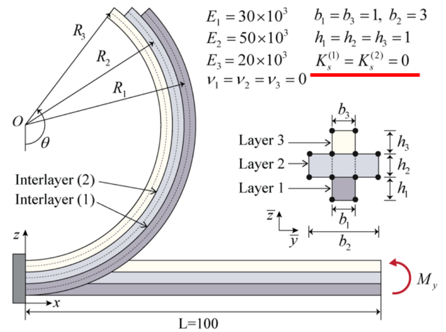
For the layers: ${}_{0}S_{ij}^{(m)(n)} = \bar{C}_{ijkl}^{(m)(n)}\,{}_{0}\bar{e}_{kl}^{(m)(n)}$

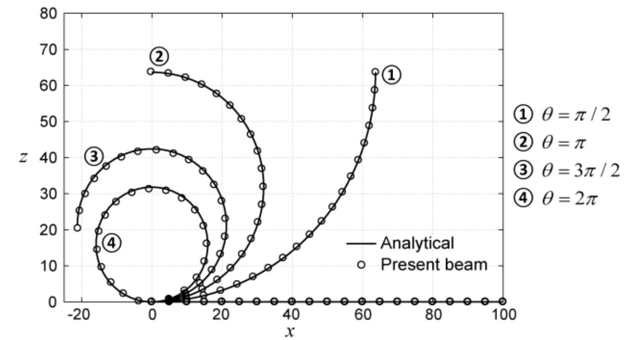
For *i* th interlayers: ${}_{0}F_s^{(i)} = {}_{0}K_s^{(i)}\,{}_{0}u_s^{(i)}$

3.2 Numerical examples 1

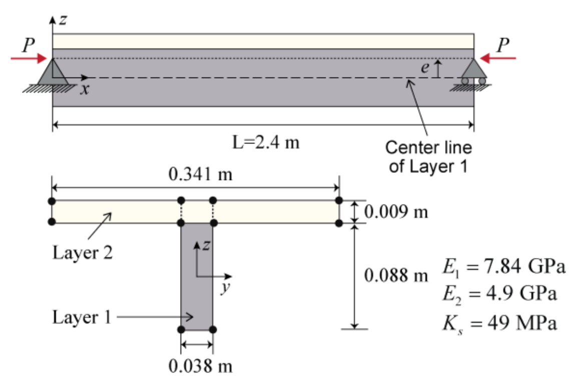
Numerical examples

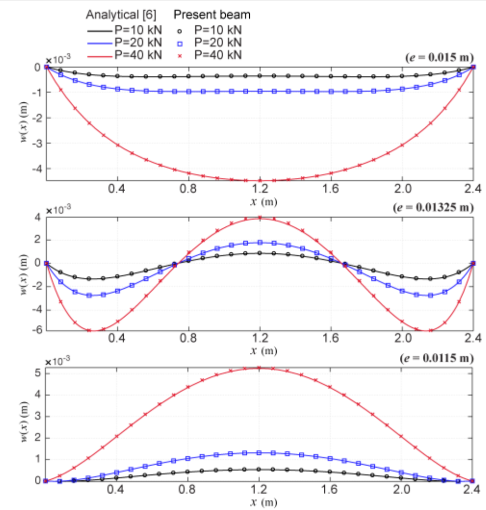
✓ **Large rotation of layered beam**



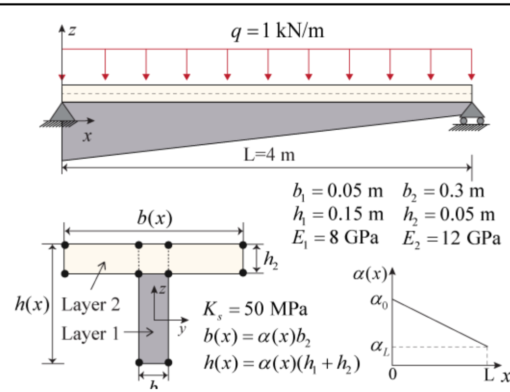


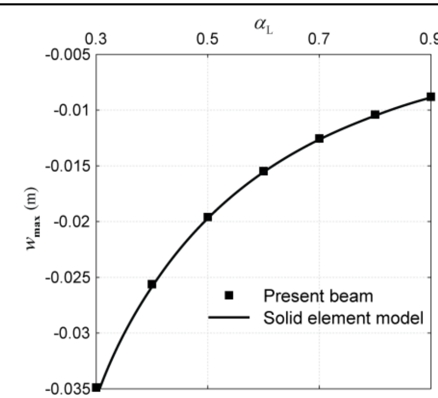
✓ **Buckling of layered beam**





✓ **Tapered layered beam**

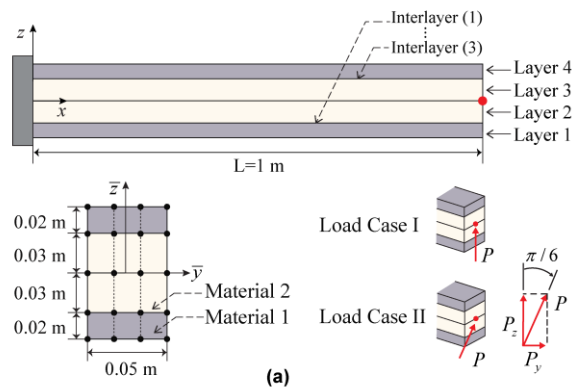


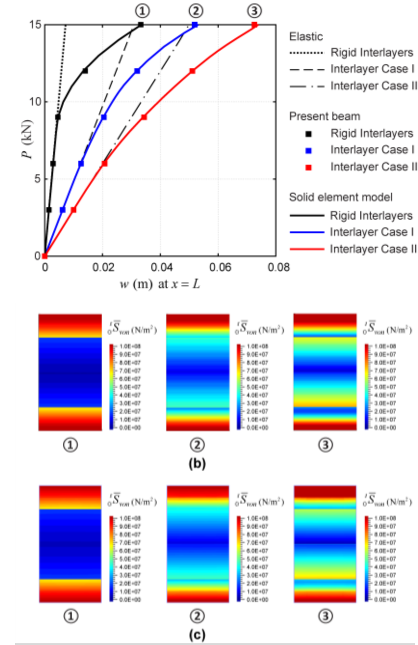


Numerical examples

✓ **Four-layer cantilever beam**

Elasto-plastic material





Von-mises stress plot

Solid model

(6,500 elements)
(178,200 DOFs)

Beam model

(99 DOFs)

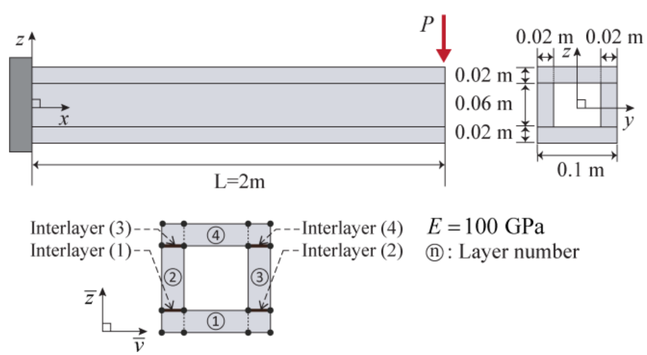
Elasto-plastic materials

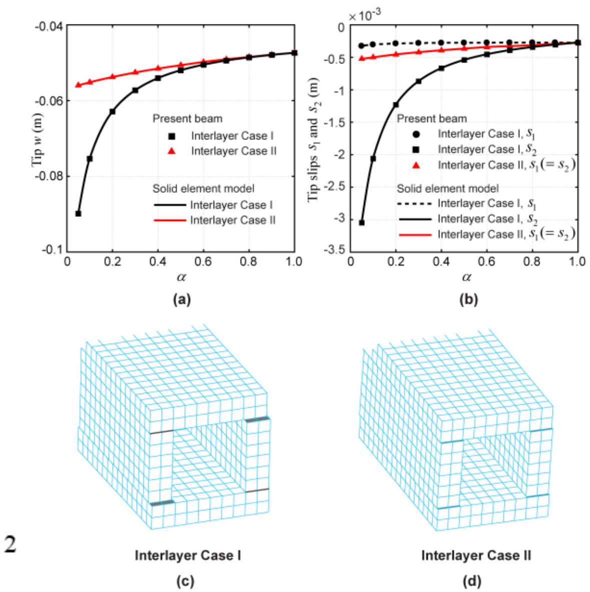
$E_1 = 200\ \text{GPa} \quad Y_1 = 100\ \text{MPa} \quad H_1 = 5\ \text{GPa}$

$E_2 = 50\ \text{GPa} \quad Y_2 = 80\ \text{MPa} \quad H_2 = 20\ \text{GPa}$

✓ **Box-shape cantilever beam**

Four-interlayers located through two columns





Four interlayer connections

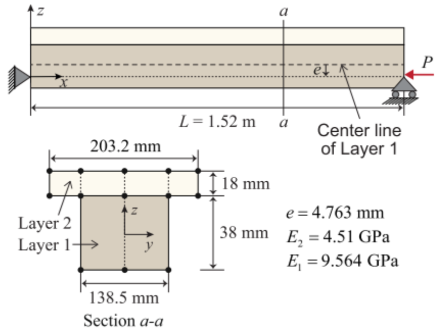
Interlayer case I : $K_s^{(2)} = K_s^{(3)} = K, \quad K_s^{(1)} = K_s^{(4)} = \alpha K$

Interlayer case II : $K_s^{(2)} = K_s^{(2)} = K_s^{(3)} = K_s^{(4)} = (1+\alpha)K/2$

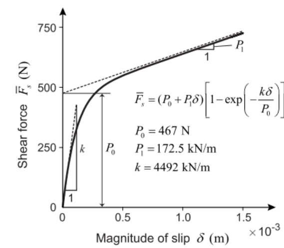
Numerical examples

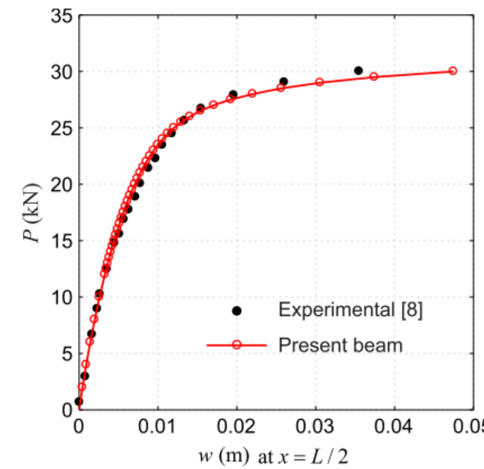
✓ **Comparison with experimental study1**

Two-layer wood beam
connected by mechanical connectors



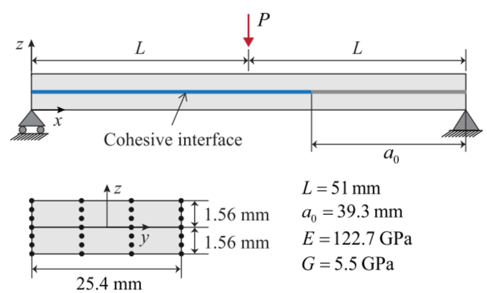
Nonlinear load-slip relation for mechanical connectors



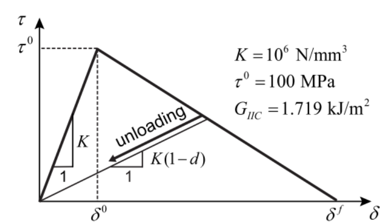


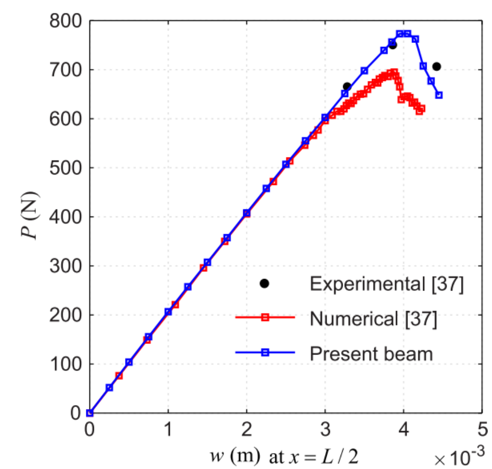
✓ **Comparison with experimental study2**

Two-layer AS4/PEEK carbon-fiber reinforced composite
connected by cohesive interface
with an initial delamination



Bi-linear load-slip relation for cohesive interlayer

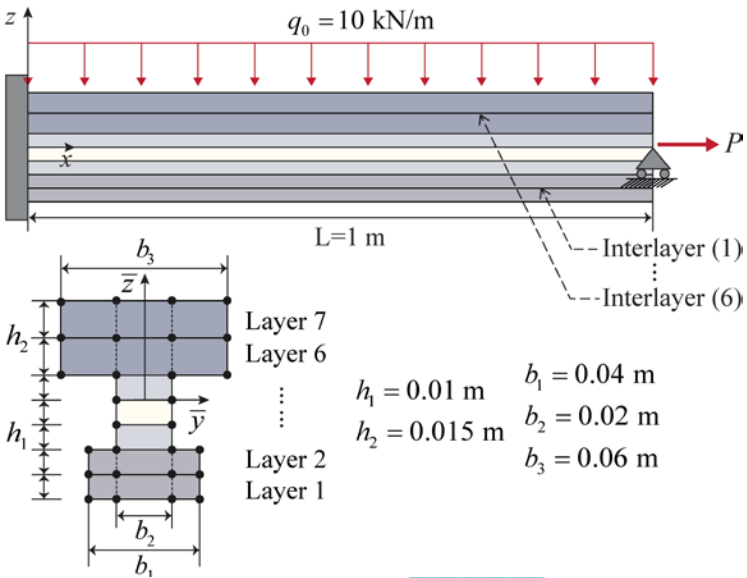


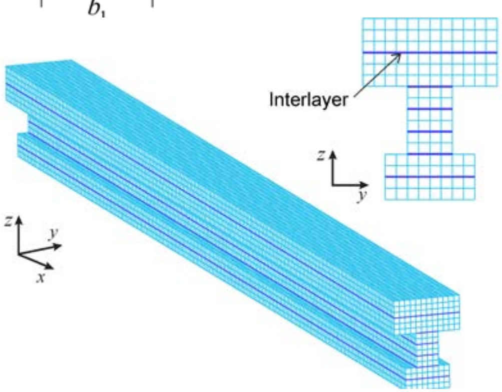


Numerical examples

▪ Seven-layer beam under transverse and axial load

- Problem description



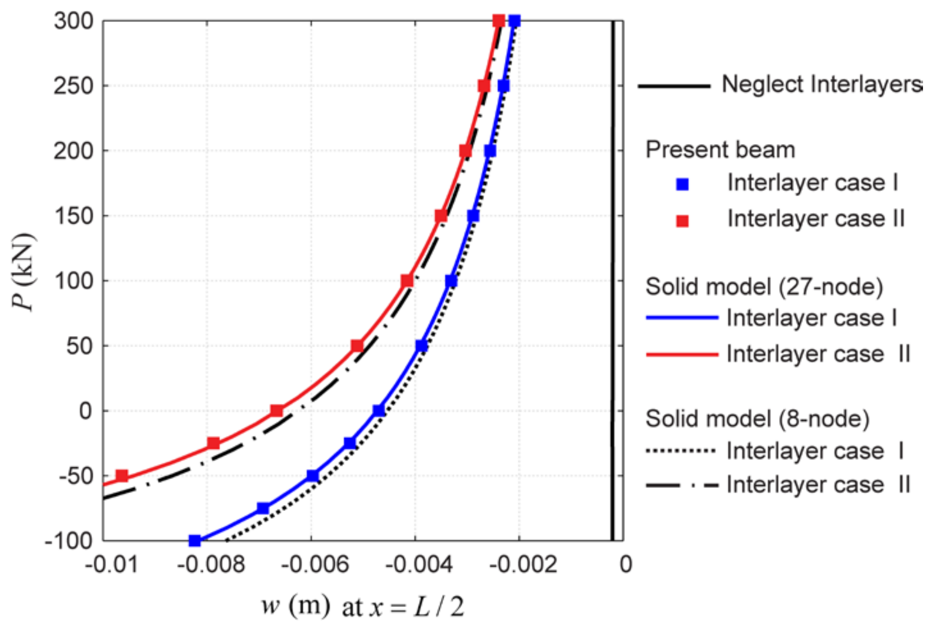


- Numerical results

Interlayer case I: Rigid connections

Interlayer case II: Deformable connections

Interlayer case III: No connections (separate)



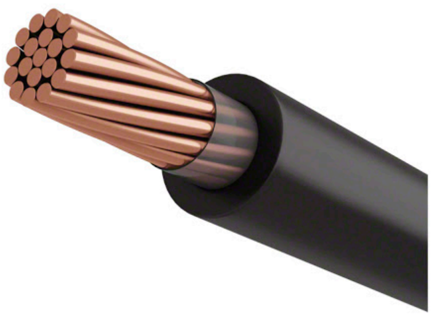
– Beam model: 20 continuum mechanics based beam elements (147 DOFs)
– Solid model : 32,800 8-node and 27-node solid elements (318,138 DOFs and 878,134 DOFs)

✓ **Geometrical nonlinear behavior of multi-layer beam**

3.3 Helically stranded cable structures

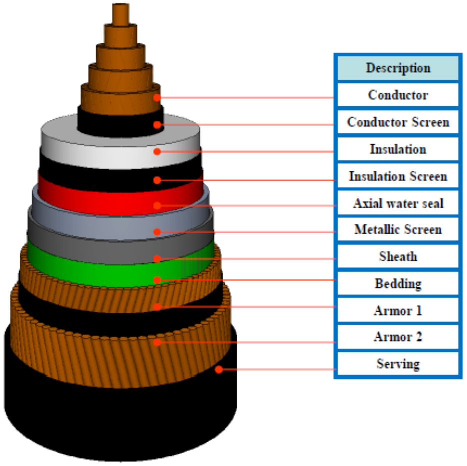
Helical cable structures

- **Helical cable structures**



<Electrical power conductor>

<Stainless steel rope wire>

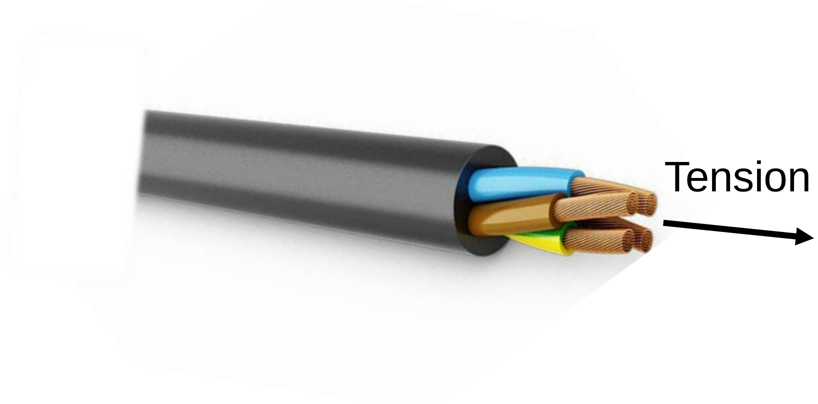


<Sub-marine power cable>

- **Resist large tension but flexible**

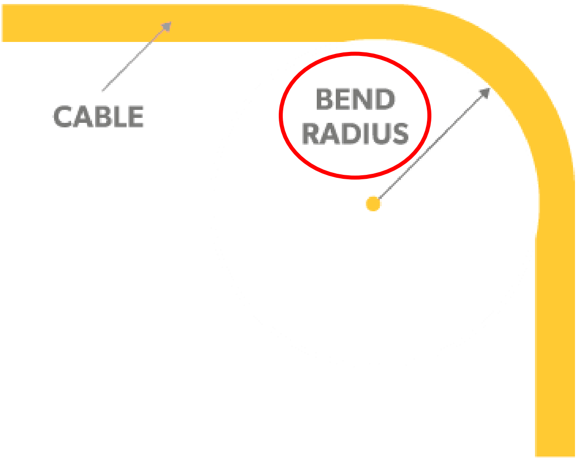
✓ Resist large axial load

✓ Flexible (small bending rigidity)



Helical cable structures

- **Main design criterion**
  - ✓ Resist large axial load
  - ✓ Low bending rigidity
    (MBR: minimum bending radius)
  - ✓ Cost-effective
  - ✓ High durability, High resistance



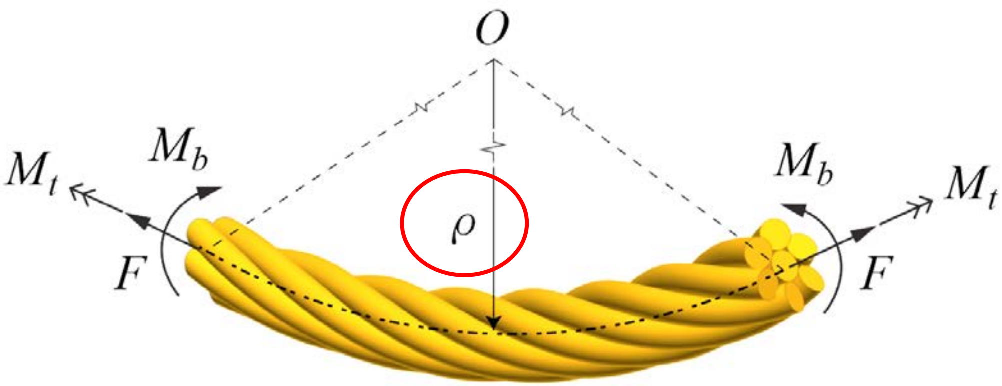
<winded electric cable>

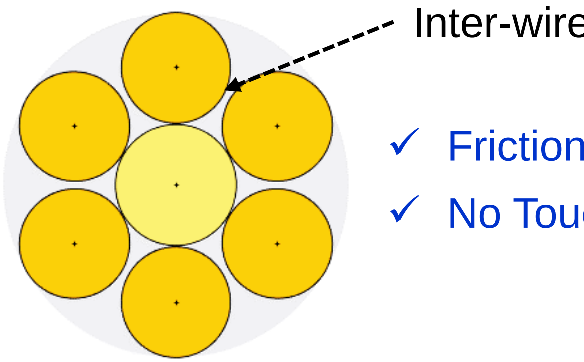
<winded subsea cable>

Helical cable structures

- **Inter-wire connections**

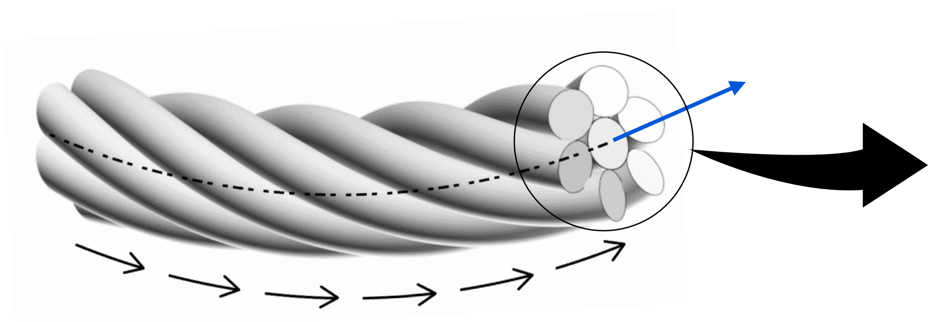
Minimum inter-wire connection → Minimum bending radius (MBR, ρ)



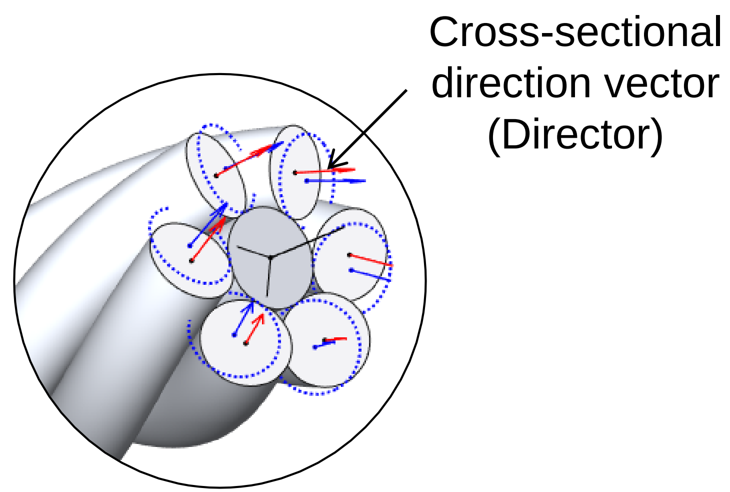


✓ Frictionless
✓ No Touching

- **Local wire behaviors**



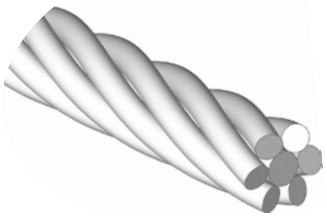
Inter-wire slips
in longitudinal direction

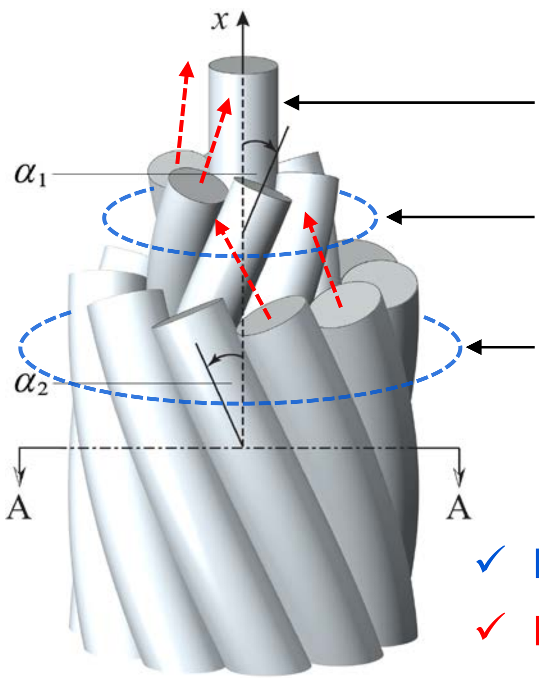


Cable rotation + Local wire rotations

Helical cable structures

- **Basic geometry of helically stranded cables**

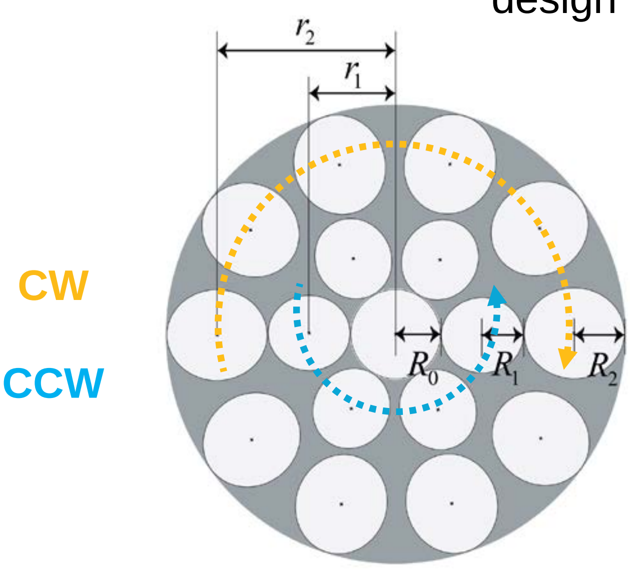




✓ Multi-layer

✓ Multi-director

Reaction torque → Torque balance design



Section A-A

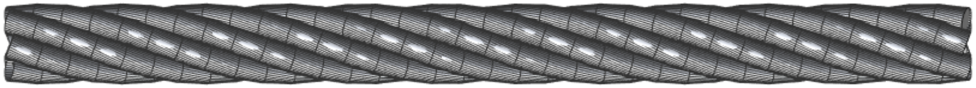
Research History

Analytical models

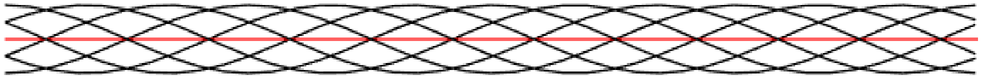
- Hruska (1951) : The simplest model ignoring the wire bending and torsion effect.
- Thin rod theory (Love), McConnell and Zemek, Machida and Durelli, Knapp models…
- Costello (1997): The linearized theories including the curvature and twist variations for each wire in cables.

Numerical models

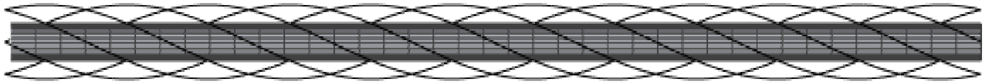
- Full solid model



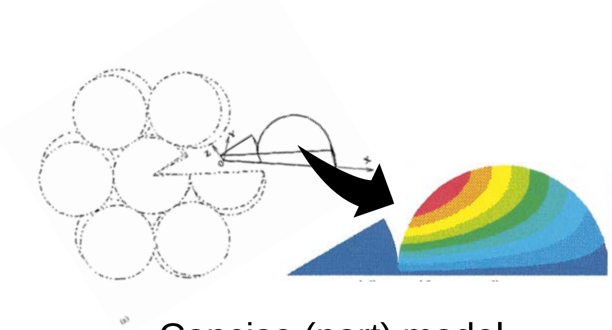
- Beam to beam model
  (Wire-wise beam model)



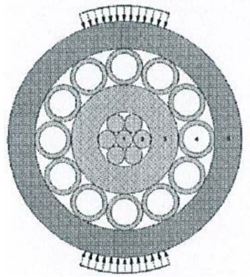
- Mixed model (Beam + Solid)



- Simplified model
  (for only cross-sectional analysis)



Concise (part) model



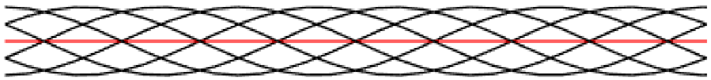
Ring model

Research purpose

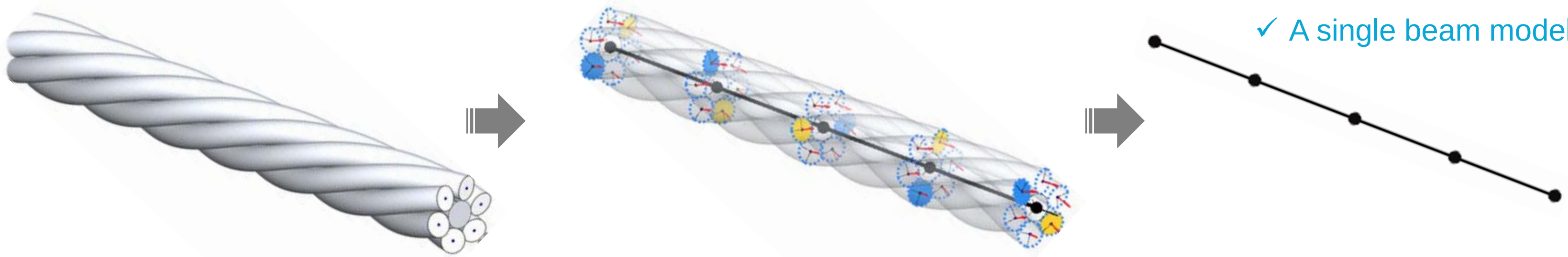
▪ Research purpose

- Solid model

- Beam to beam model



- <u>Continuum beam model</u>



For a 1-Layer (7-wires), Lay angle=18 dgr, 2-Pitch length model

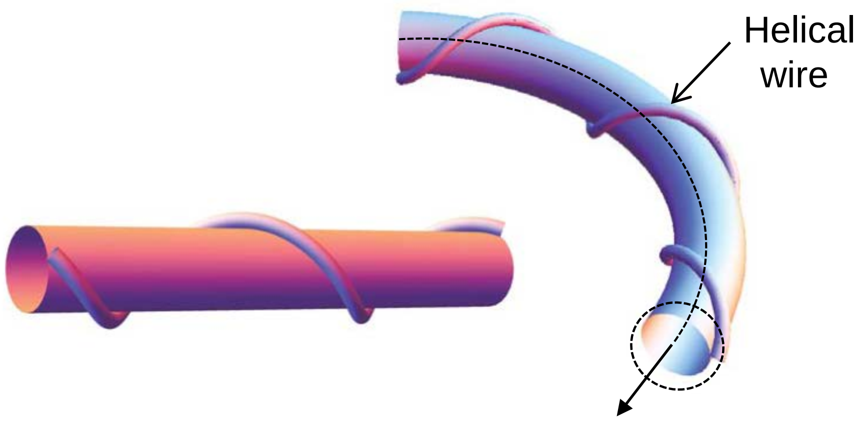
Full solid model	Beam to beam model	Continuum beam model
(96,556 elements, 343,104 DOFs)	(1,099 elements, 6,636 DOFs)	(40 element, 615 DOFs)

→ Develop continuum beam finite elements for accurate and efficient analysis of helically stranded cable structures

3.4 Continuum mechanics based beam formulation for helically stranded beam structures

Continuum mechanics based beam formulation

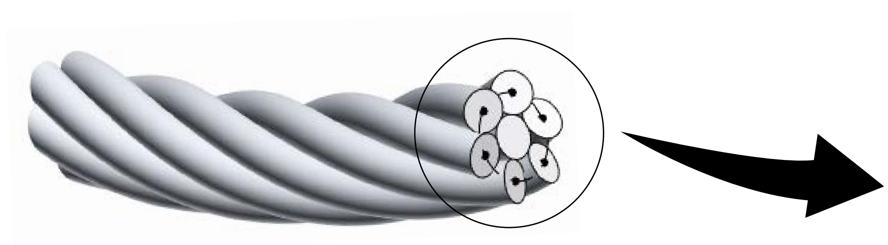
- **Basic kinematics of helical beam structures**



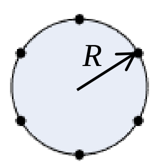
Without local failure

<Local fracture >

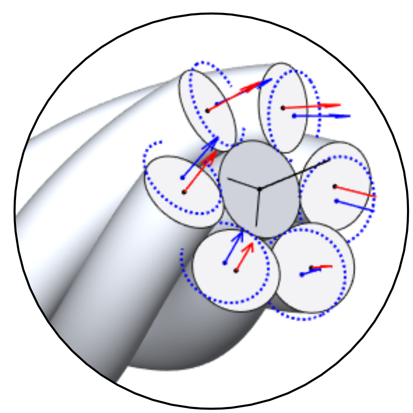
<Bird-caging phenomenon>

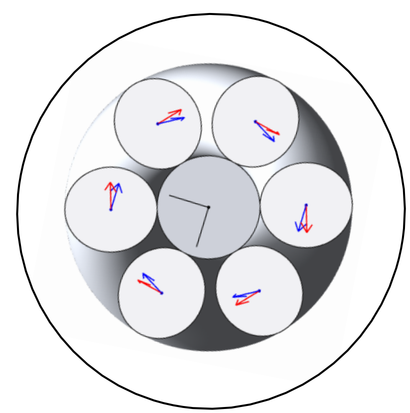


Flexible cylinder



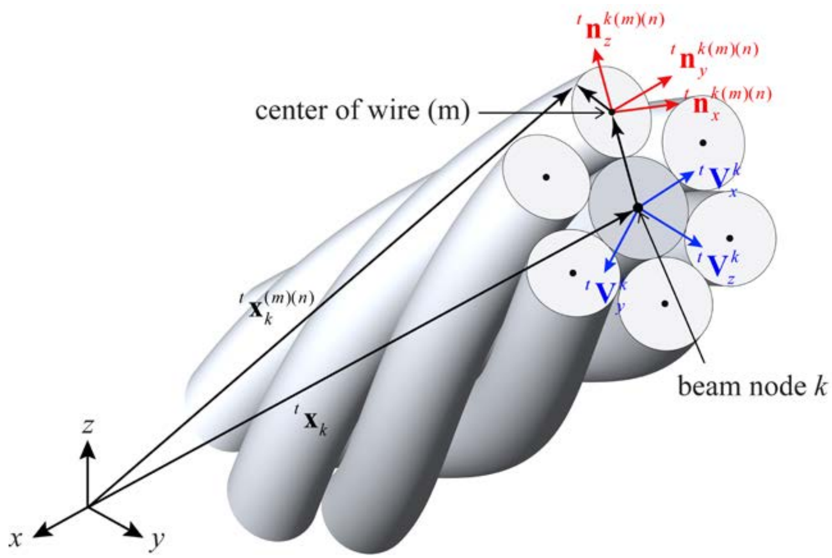
R: winding radius

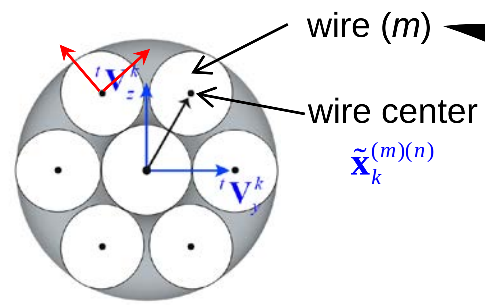


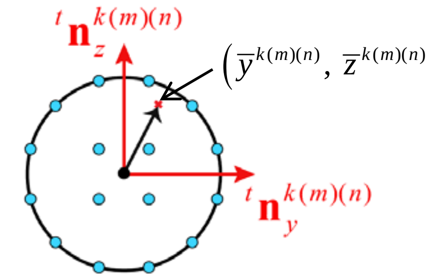


without cable rotation

Continuum mechanics based beam formulation







cross-sectional interpolation

$$
\begin{cases}
\bar{y}^{k(m)(n)} = \sum_{j=1}^{p} h_j(s,t)\,\bar{y}_k^{j(m)(n)} \\
\bar{z}^{k(m)(n)} = \sum_{j=1}^{p} h_j(s,t)\,\bar{z}_k^{j(m)(n)}
\end{cases}
$$

<0> : Global coordinate system

- Beam node k position on <0>

$$ {}^{t}\mathbf{x}_k = \begin{bmatrix} {}^{t}x_k & {}^{t}y_k & {}^{t}z_k \end{bmatrix} $$

<1> Cable local coordinate system (at beam node k) → "~"

- wire center position on <1>

$$ \tilde{\mathbf{x}}_k^{(m)(n)} = \begin{bmatrix} \tilde{x}^{k(m)(n)} & \tilde{y}^{k(m)(n)} & \tilde{z}^{k(m)(n)} \end{bmatrix}^{\mathrm{T}} $$

- Cable rotation w.r.t. <1>

$$ \tilde{\boldsymbol{\theta}}^{k} = \begin{bmatrix} \tilde{\theta}_x^{k} & \tilde{\theta}_y^{k} & \tilde{\theta}_z^{k} \end{bmatrix}^{\mathrm{T}} $$

<2> Wire local coordinate system) (on wire m of layer n) → "–"

- wire cross-sectional position on <2>

$$ \bar{\mathbf{x}}_k^{(m)(n)} = \begin{bmatrix} \bar{x}^{k(m)(n)} & \bar{y}^{k(m)(n)} & \bar{z}^{k(m)(n)} \end{bmatrix}^{\mathrm{T}} $$

- Layer-wise wire rotation w.r.t. <2>

$$ \bar{\boldsymbol{\theta}}^{k(n)} = \begin{bmatrix} \bar{\theta}_x^{k(n)} & \bar{\theta}_y^{k(n)} & \bar{\theta}_z^{k(n)} \end{bmatrix}^{\mathrm{T}} $$

- Slip director on <2>

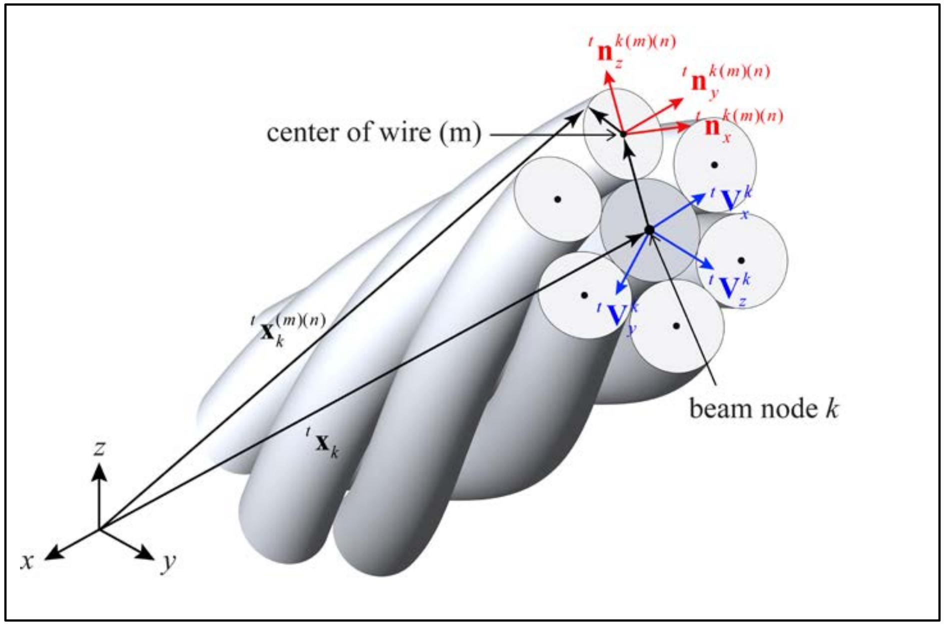
$$ {}^{t}\bar{\mathbf{V}}_s^{k} = \begin{bmatrix} {}^{t}\bar{V}_{sx}^{k} & {}^{t}\bar{V}_{sy}^{k} & {}^{t}\bar{V}_{sz}^{k} \end{bmatrix}^{\mathrm{T}} $$

Coordinate transformation matrix :

${}^{0}\mathbf{T}_{1(k)}$ = Transformation <0> → <1>

${}^{1(k)}\mathbf{T}_{2(k)(m)}$ = Transformation <1> → <2>

Continuum mechanics based beam formulation



<0>: Global Coordinate system

${}^t\mathbf{x}_k$: Beam node k position on <0>

<1>: cable local coordinate system (at beam node k)

$\tilde{\mathbf{x}}_k^{(m)(n)}$: Wire center position on <1>

$\tilde{\boldsymbol{\theta}}_c^k$: Cable rotation w.r.t. <1>

<2>: wire local coordinate system) (on wire m of layer n)

$\bar{\mathbf{x}}_k^{(m)(n)}$: Wire cross-sectional position on <2>

$\bar{\boldsymbol{\theta}}_w^{k(n)}$: Layer-wise wire rotation w.r.t. <2>

${}^t\bar{\mathbf{V}}_s^k$: Slip director on <2>

< at time = t >

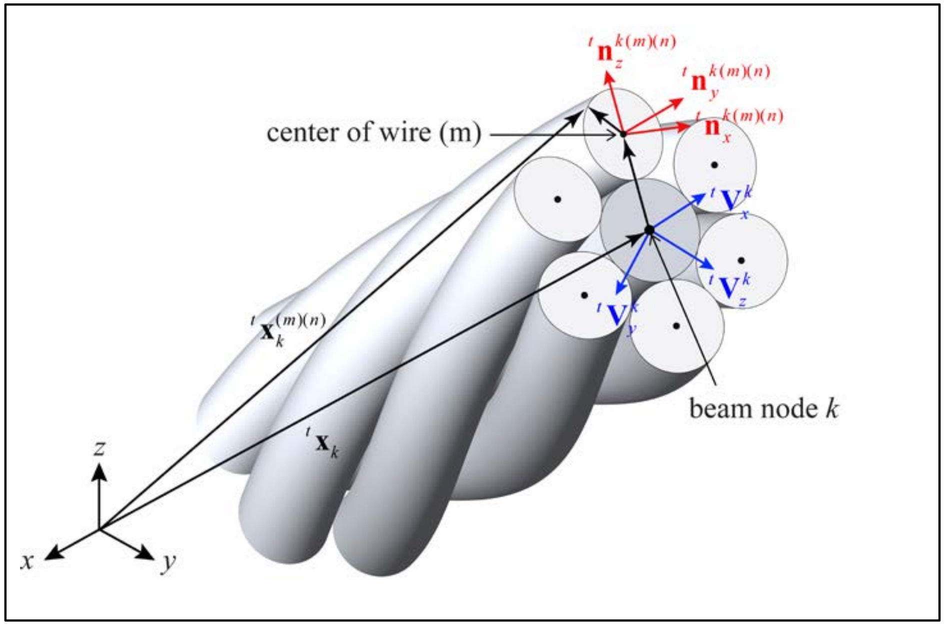
$$\begin{bmatrix} {}^t\mathbf{x}_k^{(m)(n)} \\ 1 \end{bmatrix} = \begin{bmatrix} [\mathbf{I}] & {}^t\mathbf{x}_k \\ 0 & 1\end{bmatrix} \begin{bmatrix} {}^t\left[{}^0\mathbf{T}_{1(k)}\right] & \mathbf{0} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} [\mathbf{I}] & \tilde{\mathbf{x}}_k^{(m)(n)} \\ 0 & 1\end{bmatrix} \begin{bmatrix} {}^t\left[{}^{1(k)}\mathbf{T}_{2(k)(m)}\right] & \mathbf{0} \\ 0 & 1\end{bmatrix}\begin{bmatrix} [\mathbf{I}] & \bar{\mathbf{x}}_k^{(m)(n)} \\ 0 & 1\end{bmatrix}\begin{bmatrix} {}^t\varphi_k^{(m)(n)}\,{}^t\bar{\mathbf{V}}_s^k \\ 1\end{bmatrix}$$

$${}^t\mathbf{x}^{(m)(n)} = \sum_{k=1}^{q} h_k(r)\left\langle {}^t\mathbf{x}_k^{(m)(n)}\right\rangle = \sum_{k=1}^{q} h_k(r)\left\langle {}^t\mathbf{x}_k + {}^t\left[{}^0\mathbf{T}_{1(k)}\right]\tilde{\mathbf{x}}_k^{(m)(n)} + {}^t\left[{}^0\mathbf{T}_{1(k)}\right]{}^t\left[{}^1\mathbf{T}_{2(k)(m)}\right]\bar{\mathbf{x}}_k^{(m)(n)} + {}^t\varphi_k^{(m)(n)}\,{}^t\left[{}^0\mathbf{T}_{1(k)}\right]{}^t\left[{}^1\mathbf{T}_{2(k)(m)}\right]{}^t\bar{\mathbf{V}}_s^k\right\rangle$$

where $\quad {}^t\left[{}^0\mathbf{T}_{1(k)}\right] = \left[{}^t\mathbf{V}^k\right] = \left[{}^t\mathbf{V}_x^k \quad {}^t\mathbf{V}_y^k \quad {}^t\mathbf{V}_z^k\right]$

$${}^t\left[{}^0\mathbf{T}_{1(k)}\right]{}^t\left[{}^1\mathbf{T}_{2(k)(m)}\right] = \left[{}^t\mathbf{V}^k\right]\left[{}^t\tilde{\mathbf{n}}^{k(m)(n)}\right] = \left[{}^t\mathbf{n}^{k(m)(n)}\right] = \left[{}^t\mathbf{n}_x^{k(m)(n)} \quad {}^t\mathbf{n}_y^{k(m)(n)} \quad {}^t\mathbf{n}_z^{k(m)(n)}\right]$$

Continuum mechanics based beam formulation



<0>: Global Coordinate system

${}^{t}\mathbf{x}_k$: Beam node k position on <0>

<1>: cable local coordinate system (at beam node k)

$\tilde{\mathbf{x}}_k^{(m)(n)}$: Wire center position on <1>

$\tilde{\boldsymbol{\theta}}_c^k$: Cable rotation w.r.t. <1>

<2>: wire local coordinate system) (on wire m of layer n)

$\bar{\mathbf{x}}_k^{(m)(n)}$: Wire cross-sectional position on <2>

$\bar{\boldsymbol{\theta}}_w^{k(n)}$: Layer-wise wire rotation w.r.t. <2>

${}^{t}\bar{\mathbf{V}}_s^k$: Slip director on <2>

< **at time =** $t+\Delta t$ >

$$\begin{bmatrix} {}^{t+\Delta t}\mathbf{x}_k^{(m)(n)} \\ 1\end{bmatrix} = \begin{bmatrix}[\mathbf{I}] & {}^{t+\Delta t}\mathbf{x}_k \\ \mathbf{0} & 1\end{bmatrix} \begin{bmatrix} {}^{t}[{}^{0}\mathbf{T}_{1(k)}] & \mathbf{0} \\ \mathbf{0} & 1\end{bmatrix} \begin{bmatrix}\mathbf{R}({}_0\tilde{\boldsymbol{\theta}}_c^k) & \mathbf{0}\\ \mathbf{0} & 1\end{bmatrix}\begin{bmatrix}[\mathbf{I}] & \tilde{\mathbf{x}}_k^{(m)(n)}\\ \mathbf{0} & 1\end{bmatrix}\begin{bmatrix}{}^{t}[{}^{1(k)}\mathbf{T}_{2(k)(m)}] & \mathbf{0}\\ \mathbf{0} & 1\end{bmatrix}\begin{bmatrix}\mathbf{R}({}_0\bar{\boldsymbol{\theta}}_w^{k(n)}) & \mathbf{0}\\ \mathbf{0} & 1\end{bmatrix}\begin{bmatrix}[\mathbf{I}] & \bar{\mathbf{x}}_k^{(m)(n)}\\ \mathbf{0} & 1\end{bmatrix}\begin{bmatrix}{}^{t+\Delta t}\varphi_k^{(m)(n)}\,{}^{t+\Delta t}\bar{\mathbf{V}}_s^k \\ 1\end{bmatrix}$$

$${}^{t+\Delta t}\mathbf{x}^{(m)(n)} = \sum_{k=1}^{q} h_k(r)\left\langle {}^{t+\Delta t}\mathbf{x}_k^{(m)(n)}\right\rangle = \sum_{k=1}^{q} h_k(r)\left\langle {}^{t+\Delta t}\mathbf{x}_k + {}^{t}[{}^{0}\mathbf{T}_{1(k)}]\mathbf{R}({}_0\tilde{\boldsymbol{\theta}}_c^k)\tilde{\mathbf{x}}_k^{(m)(n)} + {}^{t}[{}^{0}\mathbf{T}_{1(k)}]\mathbf{R}({}_0\tilde{\boldsymbol{\theta}}_c^k)\,{}^{t}[{}^{1(k)}\mathbf{T}_{2(k)(m)}]\mathbf{R}({}_0\bar{\boldsymbol{\theta}}_w^{k(n)})\bar{\mathbf{x}}_k^{(m)(n)} + {}^{t+\Delta t}\varphi_k^{(m)(n)}\,{}^{t+\Delta t}\mathbf{V}_s^k\right\rangle$$

$$\mathbf{R}({}_0\boldsymbol{\theta}_c^k) = {}^{t}[{}^{0}\mathbf{T}_{1(k)}]\mathbf{R}({}_0\tilde{\boldsymbol{\theta}}_c^k)\,{}^{t}[{}^{0}\mathbf{T}_{1(k)}]^{\mathrm{T}}$$: Cable rotation w.r.t. <0>

$$= \sum_{k=1}^{q} h_k(r)\left\langle {}^{t+\Delta t}\mathbf{x}_k + \mathbf{R}({}_0\boldsymbol{\theta}_c^k)\,{}^{t}[{}^{0}\mathbf{T}_{1(k)}]\tilde{\mathbf{x}}_k^{(m)(n)} + \mathbf{R}({}_0\boldsymbol{\theta}_c^k)\,{}^{t}[{}^{0}\mathbf{T}_{1(k)}]\,{}^{t}[{}^{1(k)}\mathbf{T}_{2(k)(m)}]\mathbf{R}({}_0\bar{\boldsymbol{\theta}}_w^{k(n)})\bar{\mathbf{x}}_k^{(m)(n)} + {}^{t+\Delta t}\varphi_k^{(m)(n)}\,{}^{t+\Delta t}\mathbf{V}_s^k\right\rangle$$

Continuum mechanics based beam formulation

- **Incremental displacement**
  - Geometry interpolation

$$
{}^{t+\Delta t}\mathbf{x}^{(m)(n)} = \sum_{k=1}^{q} h_k(r)\Big\langle {}^{t+\Delta t}\mathbf{x}_k + \mathbf{R}({}_0\boldsymbol{\theta}_c^k)\big[{}^{t}\mathbf{V}^k\big]\tilde{\mathbf{x}}_k^{(m)(n)} + \mathbf{R}({}_0\boldsymbol{\theta}_c^k)\big[{}^{t}\mathbf{V}^k\big]\big[{}^{t}\tilde{\mathbf{n}}^{k(m)(n)}\big]\mathbf{R}({}_0\overline{\boldsymbol{\theta}}_w^{k(n)})\overline{\mathbf{x}}_k^{(m)(n)} + {}^{t+\Delta t}\varphi_k^{(m)(n)}\,{}^{t+\Delta t}\mathbf{V}_s^k \Big\rangle
$$

  - Linearized incremental displacement

$$
{}_0\mathbf{u}^{(m)(n)} = {}^{t+\Delta t}\mathbf{x}^{(m)(n)} - {}^{t}\mathbf{x}^{(m)(n)}
$$

$$
= \sum_{k=1}^{q} h_k(r)\Big\langle ({}_0\mathbf{u}_k) + \hat{\mathbf{R}}({}_0\boldsymbol{\theta}_c^k)\big[{}^{t}\mathbf{V}^k\big]\tilde{\mathbf{x}}_k^{(m)(n)} + \hat{\mathbf{R}}({}_0\boldsymbol{\theta}_c^k)\big[{}^{t}\mathbf{n}^{k(m)(n)}\big]\overline{\mathbf{x}}_k^{(m)(n)} + \big[{}^{t}\mathbf{n}^{k(m)(n)}\big]\hat{\mathbf{R}}({}_0\boldsymbol{\theta}_w^{k(n)})\overline{\mathbf{x}}_k^{(m)(n)} + \hat{\mathbf{R}}({}_0\boldsymbol{\theta}_c^k)\big[{}^{t}\mathbf{n}^{k(m)(n)}\big]\hat{\mathbf{R}}({}_0\boldsymbol{\theta}_w^{k(n)})\overline{\mathbf{x}}_k^{(m)(n)} \Big\rangle
$$

$$
= \sum_{k=1}^{q} h_k(r)\Big\langle ({}_0\mathbf{u}_k) + \hat{\mathbf{R}}({}_0\boldsymbol{\theta}_c^k)\,{}^{t}\mathbf{V}_y^k\,\tilde{y}_k^{(m)(n)} + \hat{\mathbf{R}}({}_0\boldsymbol{\theta}_c^k)\,{}^{t}\mathbf{V}_z^k\,\tilde{z}_k^{(m)(n)} + \hat{\mathbf{R}}({}_0\boldsymbol{\theta}_c^k)\,{}^{t}\mathbf{n}_y^{k(m)(n)}\,\overline{y}_k^{(m)(n)} + \hat{\mathbf{R}}({}_0\boldsymbol{\theta}_c^k)\,{}^{t}\mathbf{n}_z^{k(m)(n)}\,\overline{z}_k^{(m)(n)} \Big\rangle
$$

$$
+ \sum_{k=1}^{q} h_k(r)\Big\langle {}^{t}\mathbf{n}_y^{k(m)(n)}\hat{\mathbf{R}}({}_0\boldsymbol{\theta}_w^{k(n)})\overline{y}_k^{(m)(n)} + {}^{t}\mathbf{n}_z^{k(m)(n)}\hat{\mathbf{R}}({}_0\boldsymbol{\theta}_w^{k(n)})\overline{z}_k^{(m)(n)} + \hat{\mathbf{R}}({}_0\boldsymbol{\theta}_c^k)\,{}^{t}\mathbf{n}_y^{k(m)(n)}\hat{\mathbf{R}}({}_0\boldsymbol{\theta}_w^{k(n)})\overline{y}_k^{(m)(n)} + \hat{\mathbf{R}}({}_0\boldsymbol{\theta}_c^k)\,{}^{t}\mathbf{n}_z^{k(m)(n)}\hat{\mathbf{R}}({}_0\boldsymbol{\theta}_w^{k(n)})\overline{z}_k^{(m)(n)} \Big\rangle
$$

  - Nodal DOFs $(= 6 + 3n_L + n_w)$ n_L: number of layers, n_w: number of wires

$$
{}_0\mathbf{U}_k = \begin{bmatrix} {}_0\mathbf{u}_k & {}_0\boldsymbol{\theta}_c^k & {}_0\boldsymbol{\theta}_w^{k(1)} & \cdots & {}_0\boldsymbol{\theta}_w^{k(n_L)} & {}_0\boldsymbol{\varphi}_k \end{bmatrix}^{\mathrm{T}} \quad \text{where} \quad \left[\begin{array}{ll} {}_0\mathbf{u}_k = \begin{bmatrix} {}_0u_k & {}_0v_k & {}_0w_k \end{bmatrix}^{\mathrm{T}} & {}_0\boldsymbol{\theta}_w^{k(n)} = \begin{bmatrix} {}_0\theta_{wx}^k & {}_0\theta_{wy}^k & {}_0\theta_{wz}^k \end{bmatrix}^{\mathrm{T}} \\ {}_0\boldsymbol{\theta}_c^k = \begin{bmatrix} {}_0\theta_{cx}^k & {}_0\theta_{cy}^k & {}_0\theta_{cz}^k \end{bmatrix}^{\mathrm{T}} & {}_0\boldsymbol{\varphi}_k = \begin{bmatrix} {}_0\varphi_k^{(1)} & \cdots & {}_0\varphi_k^{(n_w)} \end{bmatrix}^{\mathrm{T}} \end{array}\right.
$$

- **Finite element formulation**
  - Total Lagrangian formulation (for l-layer cable, $n = 0 \ldots l$)

$$
\sum_n \sum_m \int_{{}^0V^{(m)(n)}} \overline{C}_{ijkl}^{(m)(n)}\,{}_0\overline{e}_{ij}^{(m)(n)}\,\delta\,{}_0\overline{e}_{kl}^{(m)(n)}\,d\,{}^0V^{(m)(n)} + \sum_n \sum_m \int_{{}^0V^{(m)(n)}} {}_0^{t}\overline{S}_{ij}^{(m)(n)}\,\delta\,{}_0\overline{\eta}_{ij}^{(m)(n)}\,d\,{}^0V^{(m)(n)}
$$

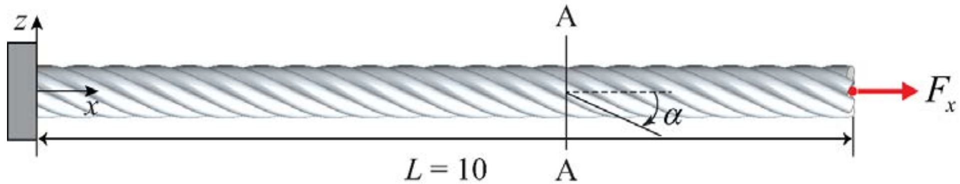
$$
= {}^{t+\Delta t}\Re - \sum_n \sum_m \int_{{}^0V^{(m)(n)}} {}_0^{t}\overline{S}_{ij}^{(m)(n)}\,\delta\,{}_0\overline{e}_{ij}^{(m)(n)}\,d\,{}^0V^{(m)(n)}
$$

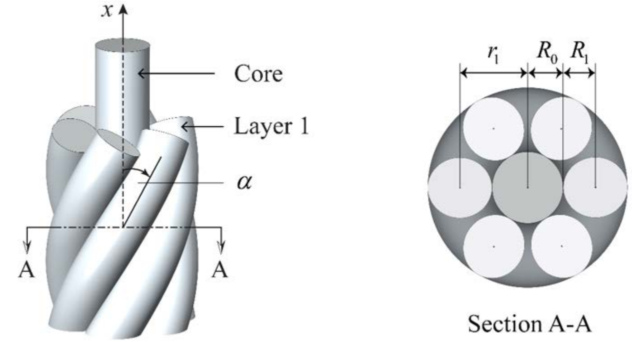
* Frictionless condition between inter-wires

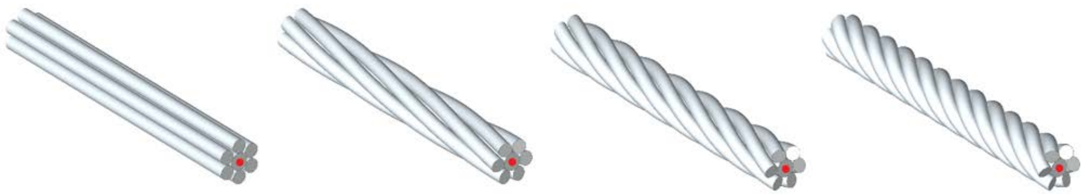
7. Numerical examples 2

Numerical examples

- **7-wire cantilever cable subjected to tip Tension**

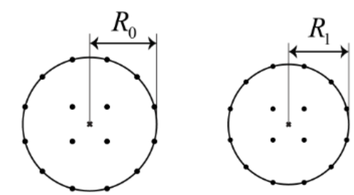






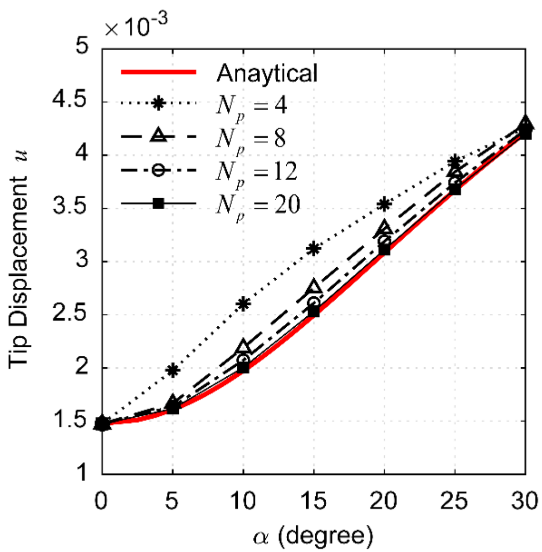
$\alpha = 0°$ $\alpha = 10°$ $\alpha = 20°$ $\alpha = 30°$

16 cross-sectional node

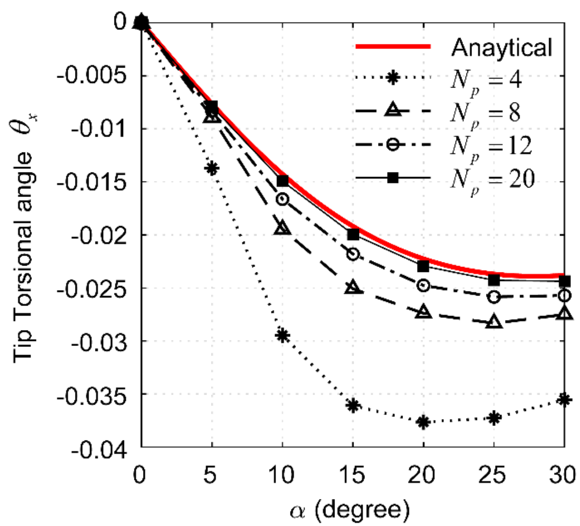


Core Wires in Layer 1

$R_0 = 0.125$
$R_1 = 0.1$



(a)



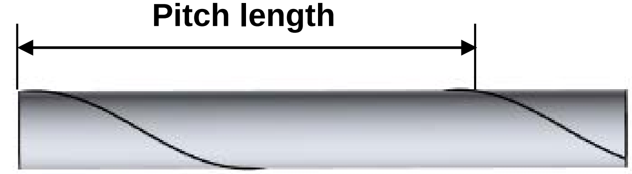
(b)

N_p : **Number of element per pitch length**

$N_p = 4,\ 8,\ 12,\ 20$ **Max difference<2%**

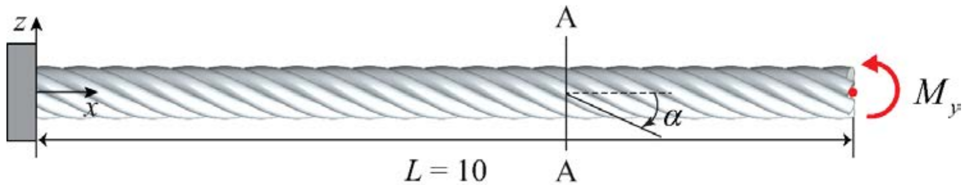
(1 element for 18° rotation)

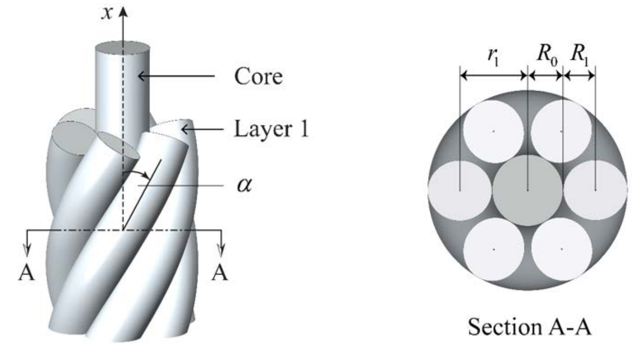
Pitch length

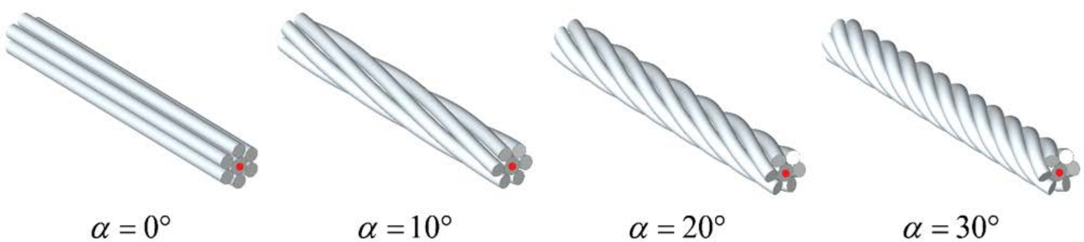


Numerical examples

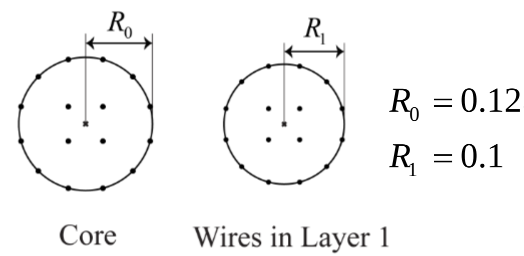
- **7-wire cantilever cable subjected to tip Bending**





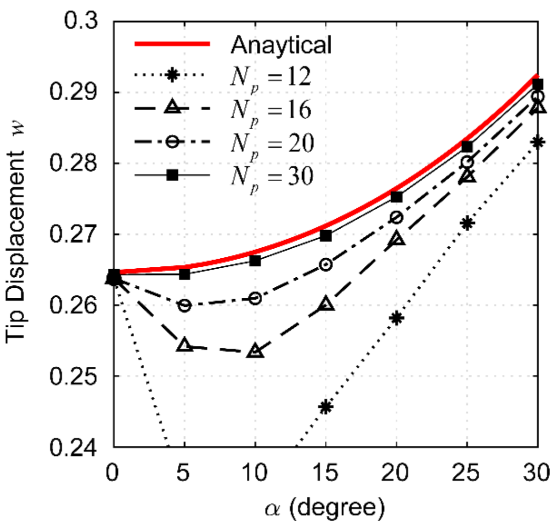


16 cross-sectional node

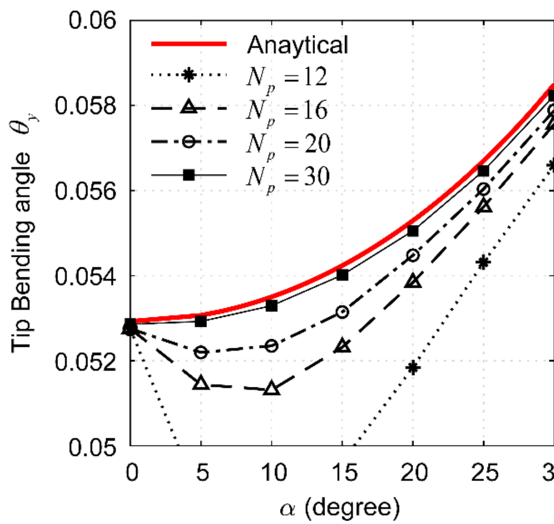


$R_0 = 0.125$

$R_1 = 0.1$



(a)



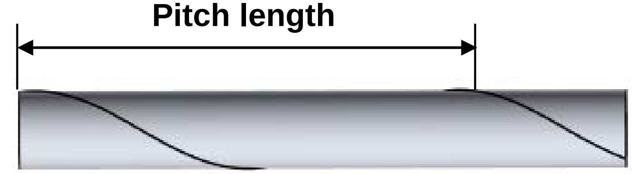
(b)

N_p : **Number of element per pitch length**

$N_p = 4,\ 8,\ 12,\ 20,\ 30$ **Max difference<2%**

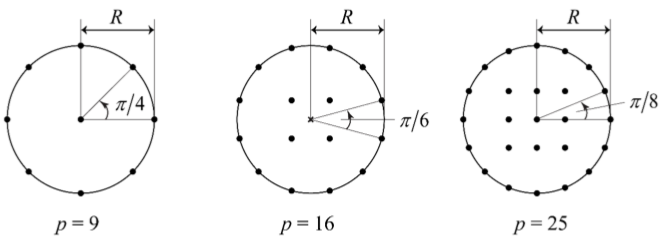
(1 element for 12° rotation)

Pitch length

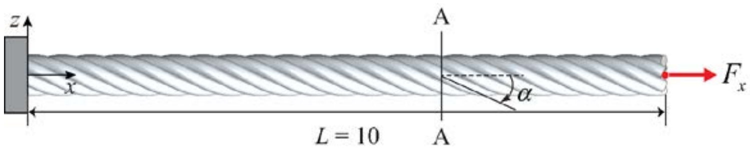


Numerical examples

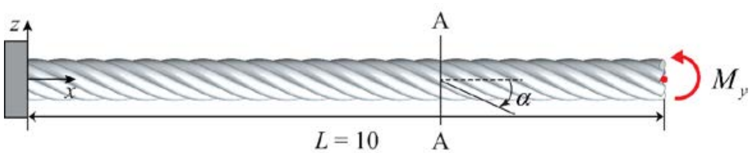
- **7-wire cantilever cable – cross-sectional discretization test**



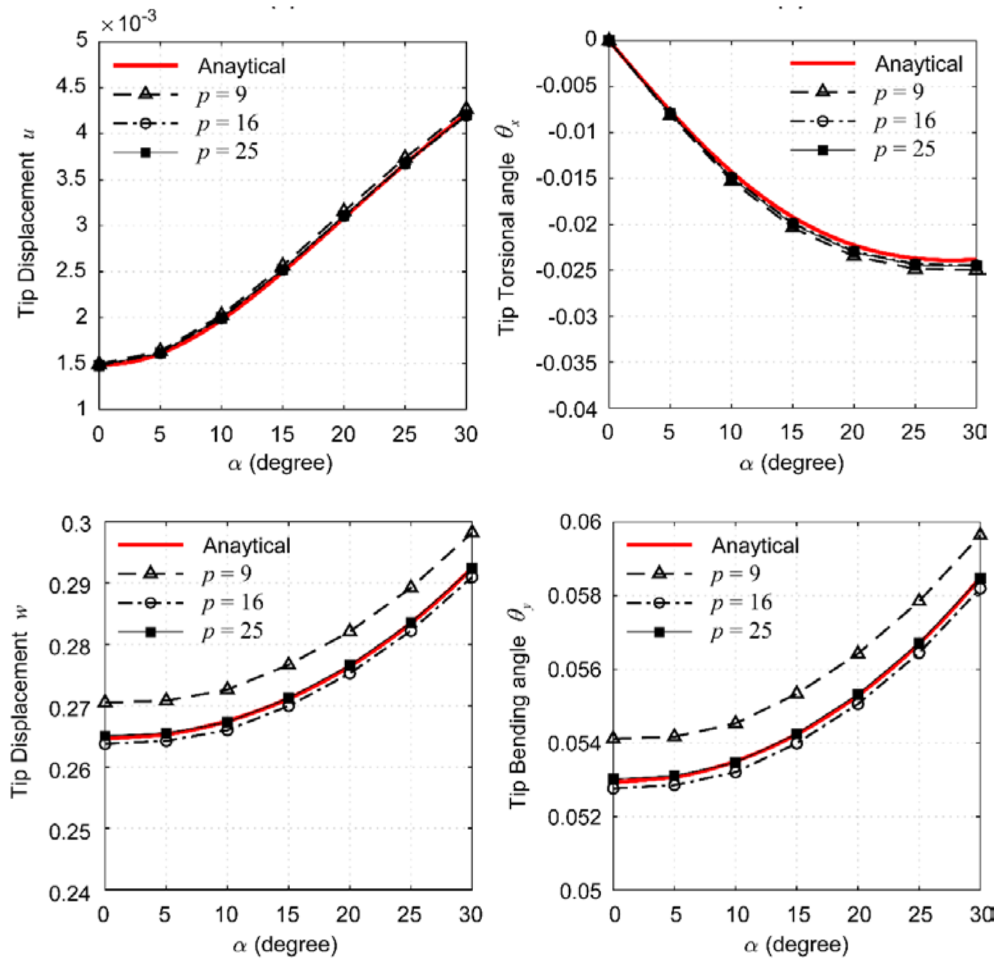
and $N_p = 30$ **(** N_p **: Number of element per pitch length)**



✓ Axial rigidity= EA
✓ Torsional rigidity=GJ



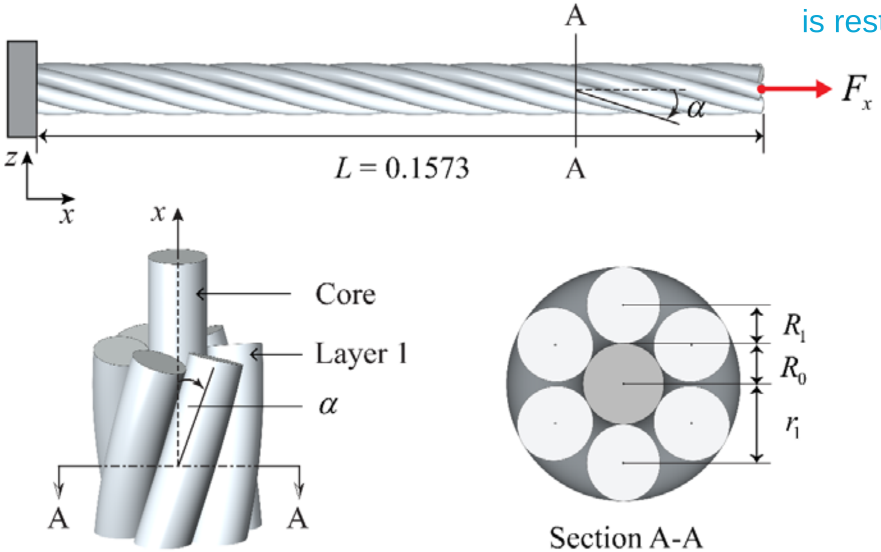
✓ Bending rigidity= EI



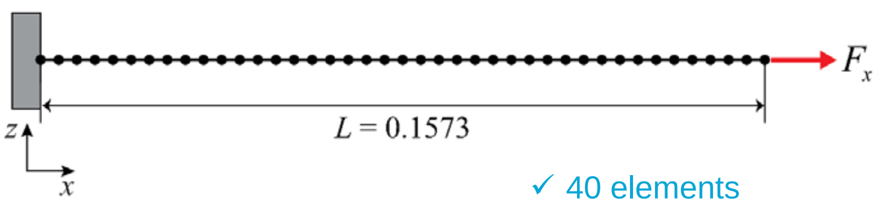
Numerical examples

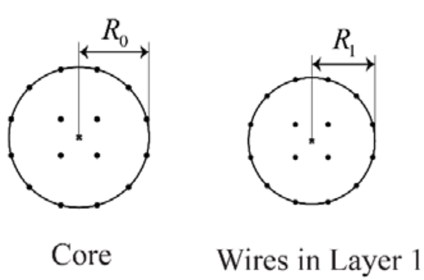
- **Benchmark problem1: Comparison with experimental test**

✓ Torsional angle is restrained



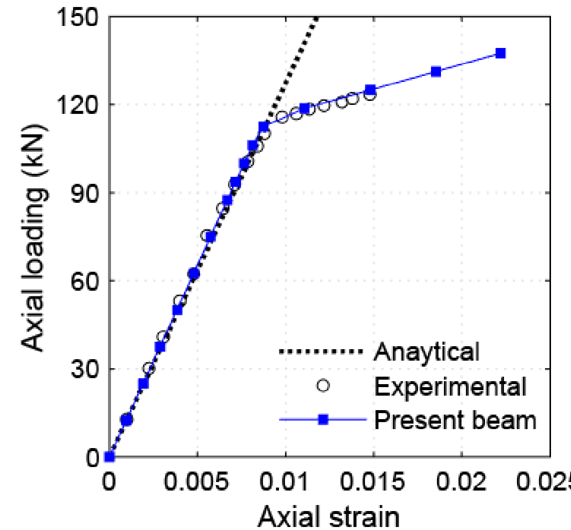
(a)



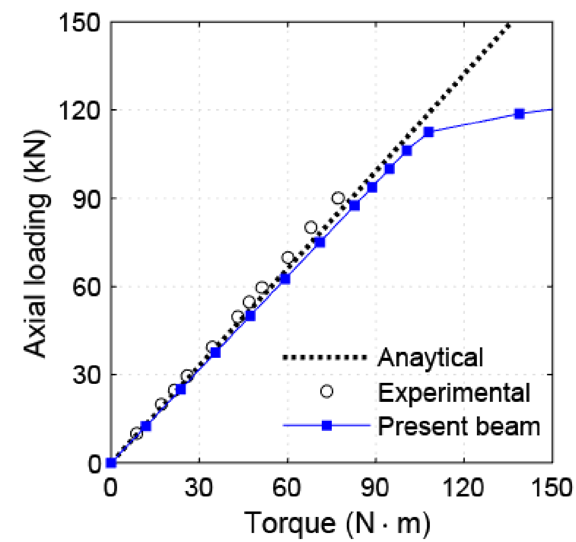


✓ 40 elements (20 elements per pitch)

$$R_0 = 1.970\times10^{-3} \quad E = 188\ \text{GPa}$$
$$R_1 = 1.860\times10^{-3} \quad Y_0 = 1.54\ \text{GPa}$$
$$H = 24.6\ \text{GPa}$$



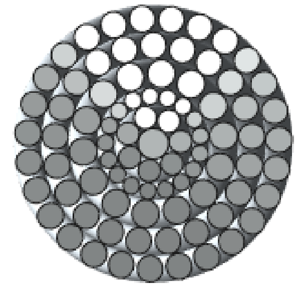
(a)



(b)

Numerical examples

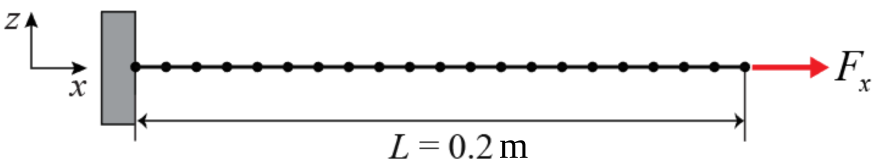
- **Benchmark problem2: 6-Layer, 120-Wire cable**



#	Wire radius (mm)	Number of wires
Core	2.9	1
Layer 1	2.15	7
Layer 2	1.6	17
Layer 3	2.65	14
Layer 4	2.5	21
Layer 5	2.5	27
Layer 6	2.5	33

✓ Uniform tension problem
✓ Torsional angle is restrained

- **Beam model:** 20 elements, 483 DOFs (23 DOFs per node) (<1 min)



- **8-node solids model:** 2,520,000 elements
8,393,760 DOFs

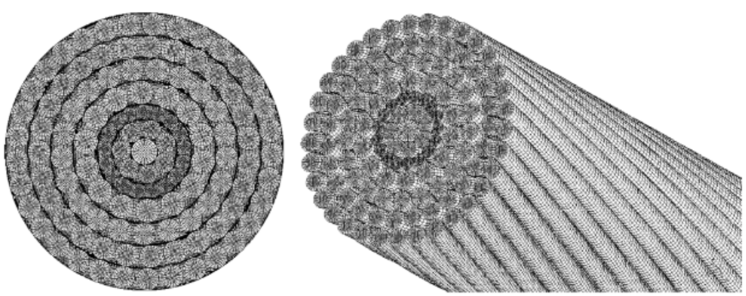
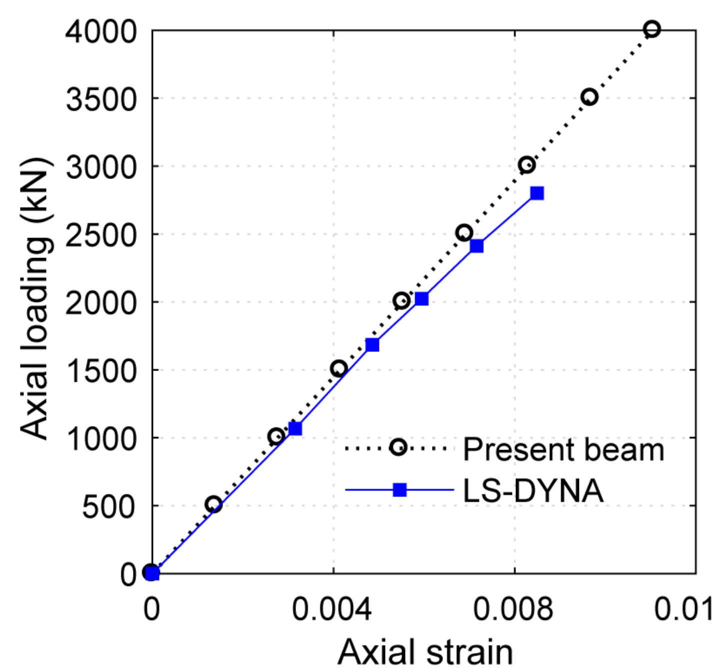


Fig. 3. The FE model of 120-wire spiral strand cable.

(Bridon International Ltd)



7. Conclusions & Future works

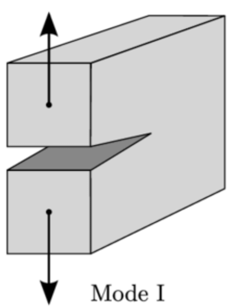
Conclusions & Future works

- **Continuum mechanics based beam elements for <u>multi-layered beams</u>**

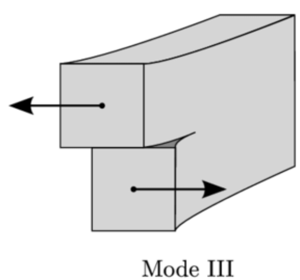
  ✓ Develop a new beam finite element for **multi-layered beams**

  - Arbitrary number of layers and interlayers can be modeled as a single beam
  - Complex geometries and boundary conditions
  - Geometrical and material nonlinear behavior
  - Nonlinear load-slip relation at interlayers

- More general interlayer behaviors

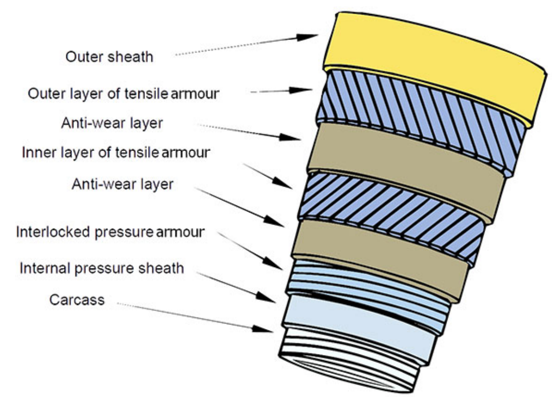


Normal mode
(Opening mode)



Shear mode
(out-of-plane mode)

- Other multi-layered structures



<Multi-layer pipe structure>

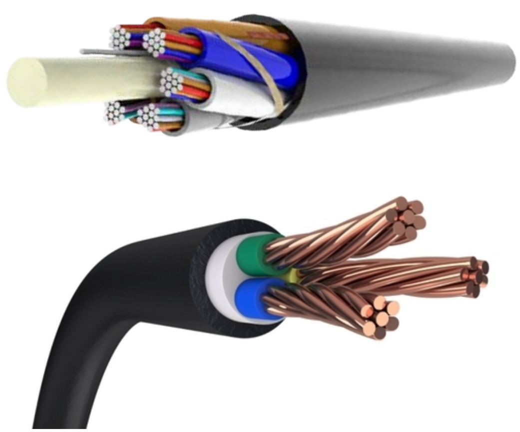
Conclusions & Future works

- **Continuum mechanics based beam elements for <u>helically stranded cables</u>**

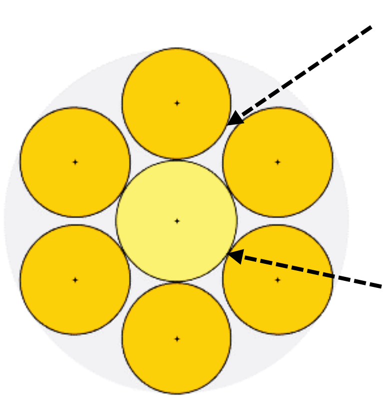
  ✓ Develop a new beam finite element for **helically stranded cables**

  - Arbitrary number of layers and wires can be modeled as a single beam
  - Complex geometries and boundary conditions
  - Geometrical and material nonlinear behavior
  - Simple modeling procedure & Low computational cost

- More general helical geometries



- Friction, contact effect



- Bird-caging (un-winding)

Appendix