Ph.D. dissertation presentation

2D-MITC4 솔리드 및 MITC4+ 쉘 유한요소의 개선

Improvement of 2D-MITC4 solid and MITC4+ shell finite elements

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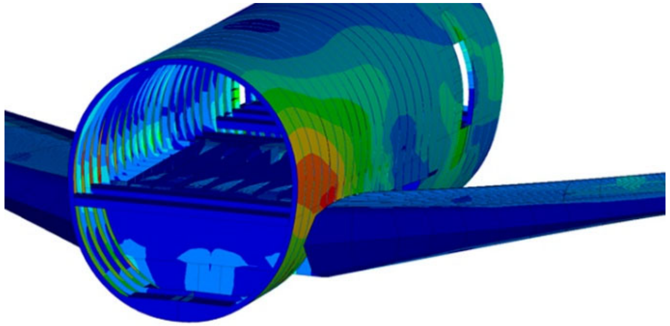
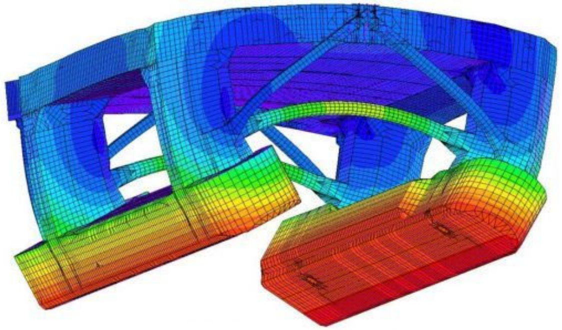
Contents

1. Introduction

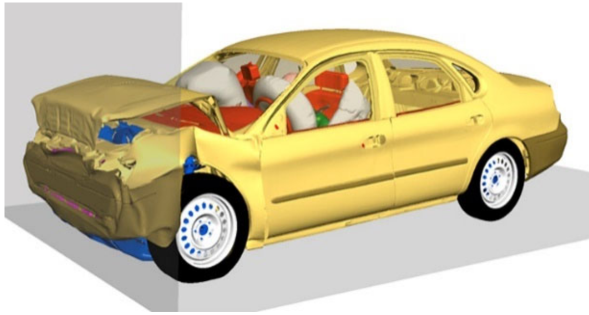
❖ FEM in various engineering fields



Aerospace engineering



Offshore engineering



Automotive engineering

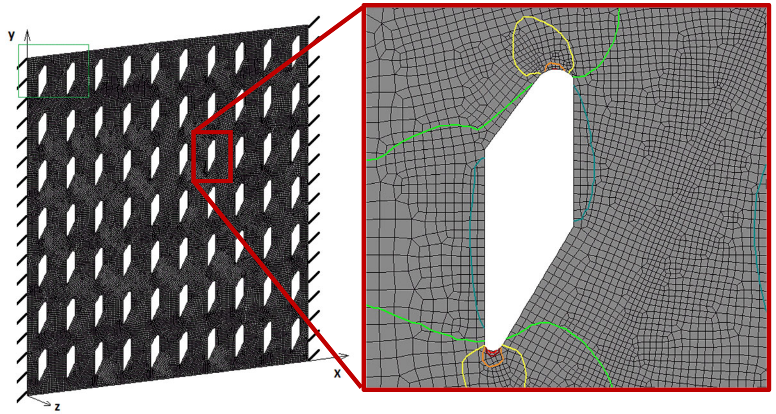
Digital twin

Structural health monitoring

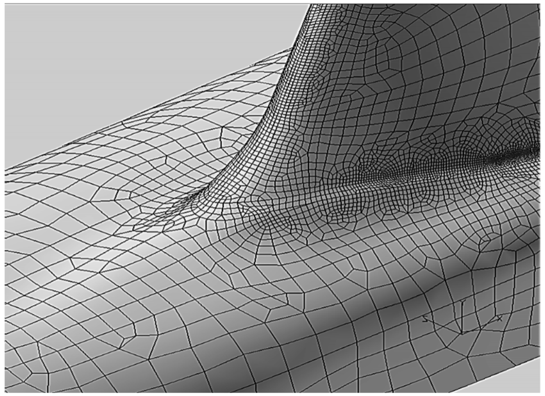
https://altairhyperworks.com/industry/ https://zhuanlan.zhihu.com/p/20465355
http://www.lminnomaritime.com/application-of-fem-on-ships-structural-design/
https://www.hartenergy.com/exclusives/ep-solutions-reduce-costly-regrets-through-ai-powered-digital-twins-188772

http://members.ozemail.com.au/~comecau
http://www.centaursoft.com/structured-surface-mesh

❖ Distorted mesh

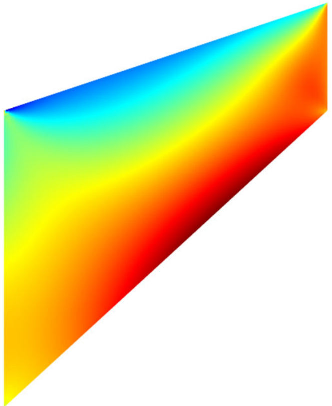


Perforated panel (hole)

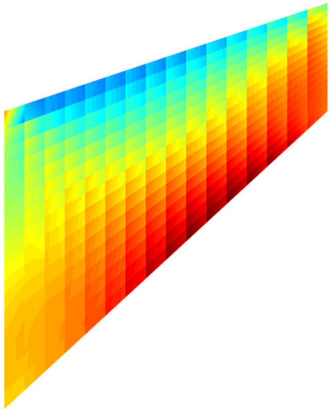


Aircraft wing (curved geometry)

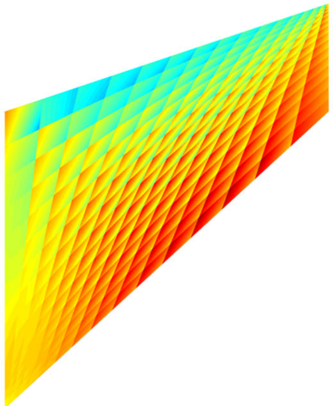
❖ Performance deterioration



Reference solution



Regular mesh



Distorted mesh

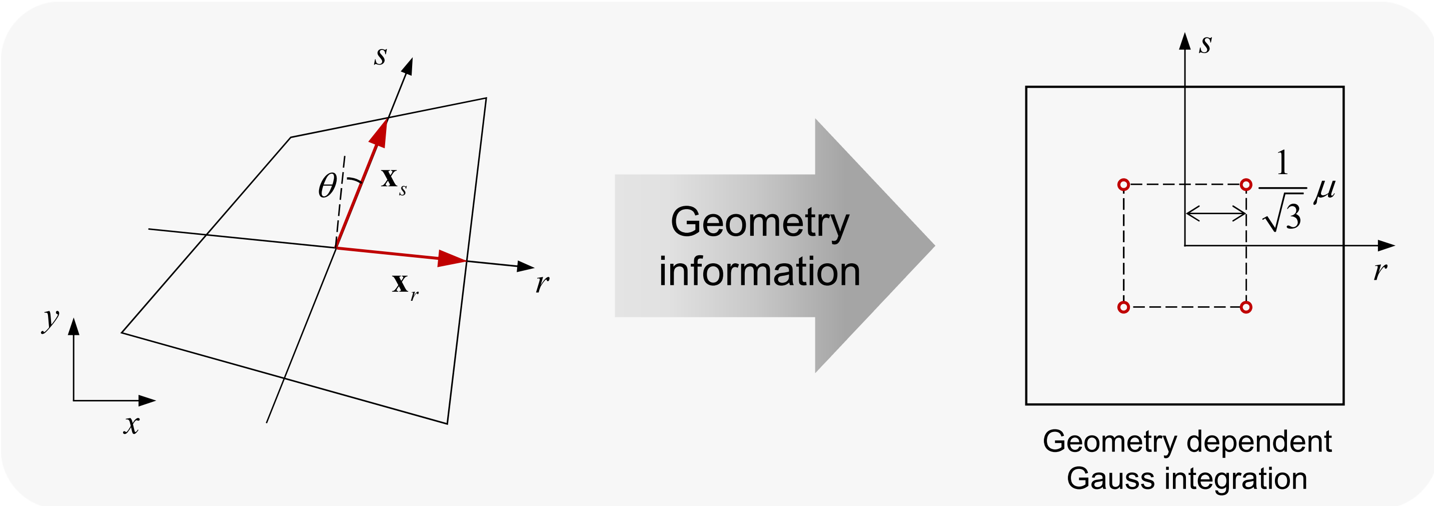
Research purpose

Goal

- ✓ **Improvement of the performance on distorted mesh**
- ✓ **Simplification of the formulation**

❖ Topic 1. new 2D-MITC4 solid element

- Simplification of the previous formulation
- Adjustment of Gauss points considering element distortion

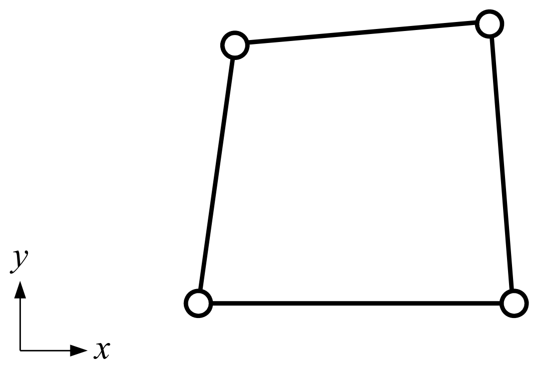


❖ Topic 2. new MITC4+ shell element

- New assumed membrane strain field
- Extension of the concept of “geometry dependent Gauss integration”

Topic 1

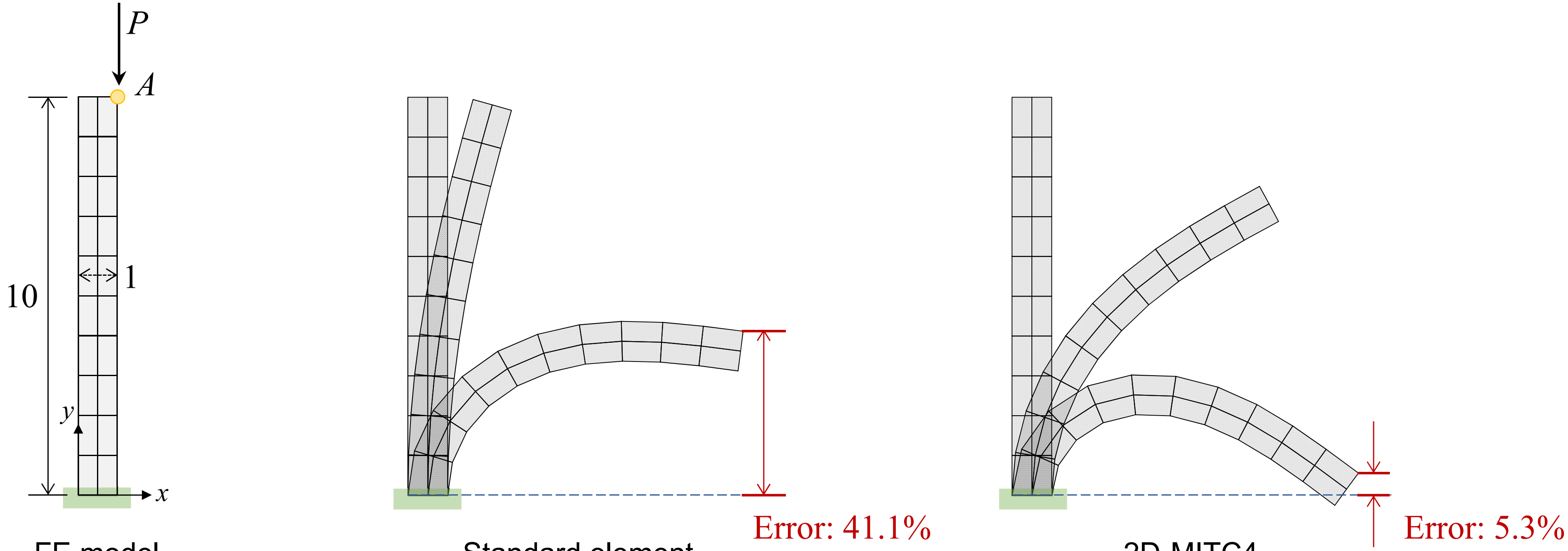
New 2D-MITC4 solid element



4-node quadrilateral 2D solid element

Research motivations

❖ Development of a new finite element



FE model
(Regular mesh)

Standard element

2D-MITC4

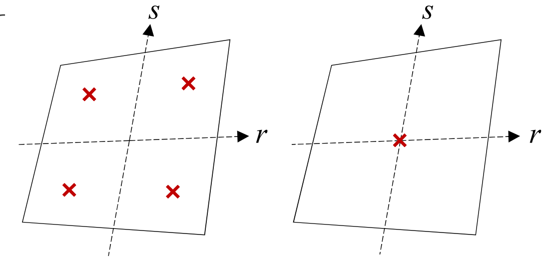
❖ Major considerations

- Basic tests (patch, zero energy mode, and isotropy tests)
- Treatment of locking phenomena
- Reliable and accurate solution

Related studies

❖ Reduced integration

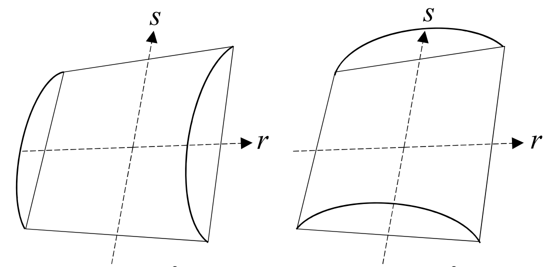
- **Zienkiewicz et al. (1971)**
  - Reduced integration technique
  - Spurious zero energy modes problem
- **Malkus and Huges (1978)**
  - Selective integration technique
  - Spatially isotropy problem



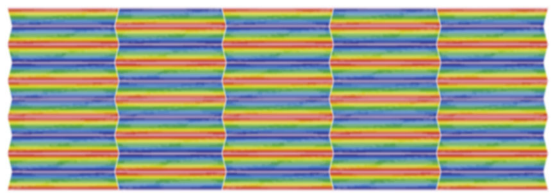
Standard   Reduced

❖ Incompatible modes (EAS)

- **Wilson et al. (1973) / Taylor et al. (1976)**
  - Modified incompatible modes element which passes the patch test
  - One of the most-widely used elements in commercial software
- **Sussman and Bathe (2014)**
  - Numerical instabilities of incompatible modes element



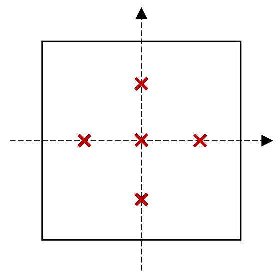
$h_5 = 1 - s^2$ $h_6 = 1 - r^2$



Spurious mode in the incompatible modes element

❖ Assumed strain method

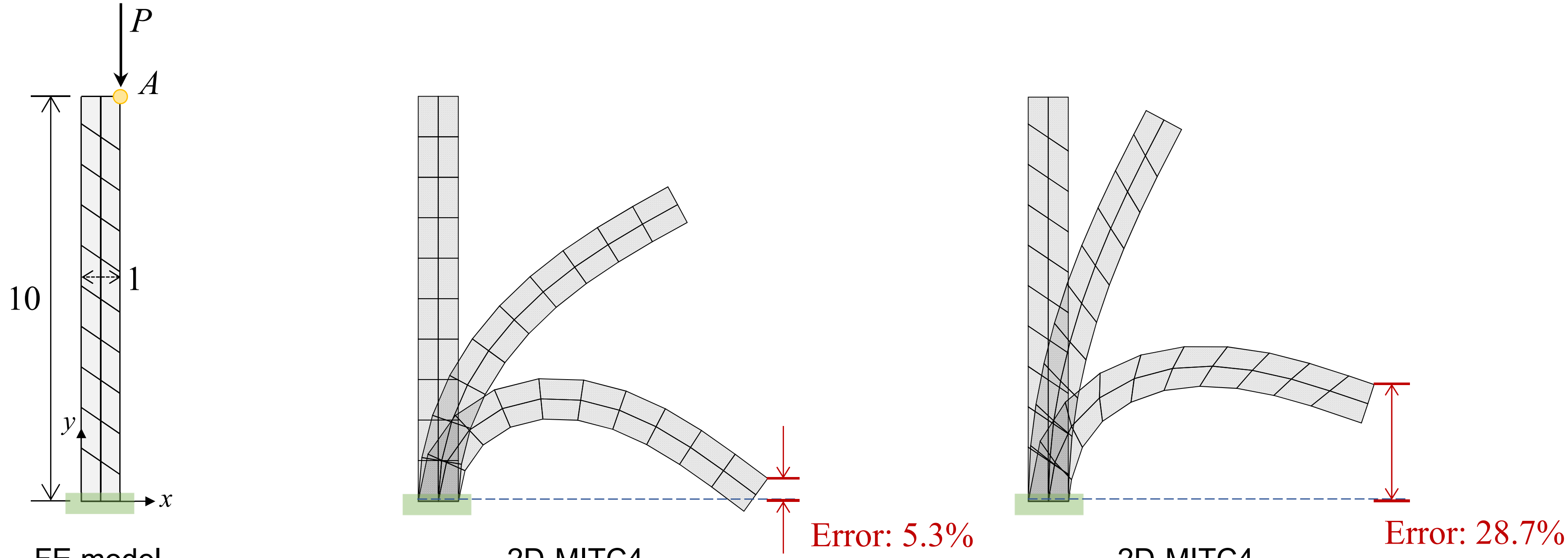
- **Dvorkin and Bathe (1984)**
  - Mixed interpolation of tensorial components(MITC) method
  - Improved performance by alleviating locking phenomena
- **Ko et al. (2017) → 2D-MITC4**
  - Alleviation of *in-plane shear locking* using MITC method
  - Superior convergence behavior in regular meshes
  - No spurious zero energy modes in nonlinear analysis



Tying point

Research motivations

❖ Performance deterioration in distorted mesh



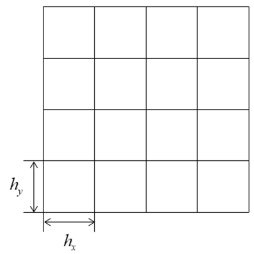
FE model (Distorted mesh) | 2D-MITC4 (Regular) | 2D-MITC4 (Distorted)

✓ **To use in engineering practice, we need to improve the performance of the 2D-MITC4 element in 'distorted mesh'.**

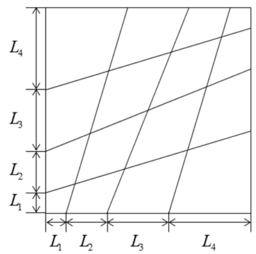
Related studies

❖ Investigation of accuracy loss in distorted mesh

- **Harder and MacNeal (1985) / MacNeal (1989)**
  - Accuracy of finite elements with nonstandard shape
- **Lee and Bathe (1993)**
  - Effects of element distortion on the performance



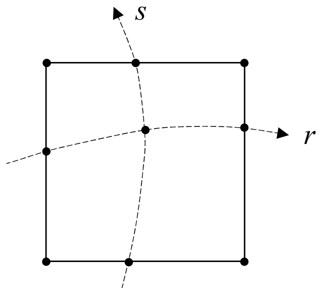
Regular mesh



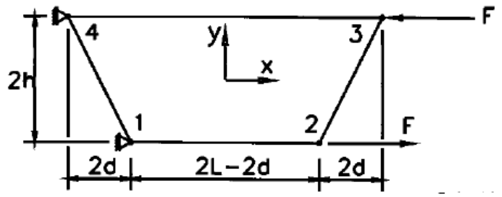
Distorted mesh

❖ Improvement of performance in distorted mesh

- **Celia and Gray (1984) / Wisniewski and Turska (2018)**
  - Corrected shape functions (high-order element)
- **Sze (2000)**
  - Alleviation of trapezoidal locking

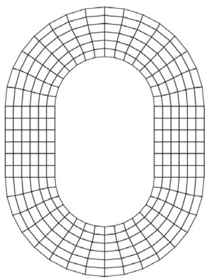


Unevenly-spaced-nodes distortion
(high-order element)

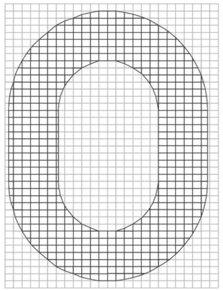


❖ Meshing techniques

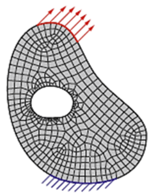
- **Kendhe et al. (2005) / Peto et al. (2020)**
  - Structured mesh generation
  - Employing background imaginary grid



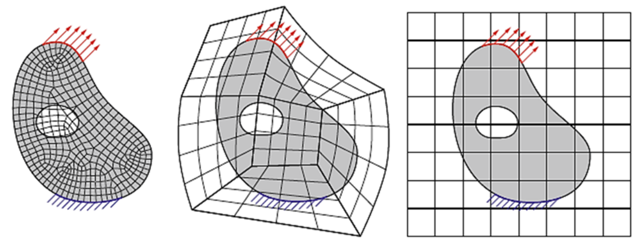
Structured mesh



Conformal decomposition
with Cartesian grid



Conventional



Unfitted mesh with fictitious domain

Previous 2D-MITC4 solid element

❖ Geometry and displacement interpolations

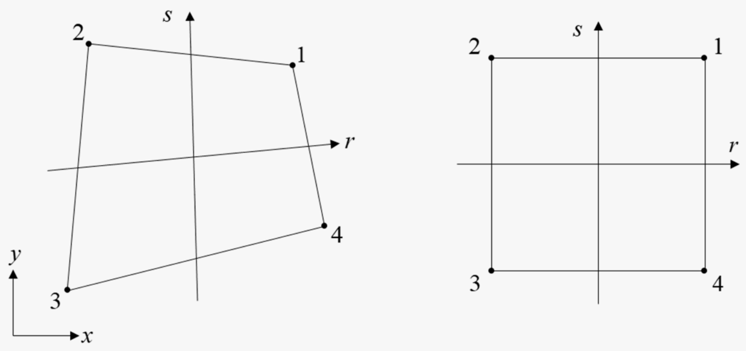
- $\mathbf{x} = \sum_{i=1}^{4} h_i(r,s)\mathbf{x}_i$ with $\mathbf{x}_i = \begin{bmatrix} x_i & y_i \end{bmatrix}^T$.

- $\mathbf{u} = \sum_{i=1}^{4} h_i(r,s)\mathbf{u}_i$ with $\mathbf{u}_i = \begin{bmatrix} u_i & v_i \end{bmatrix}^T$.

- Shape functions: $h_i = \frac{1}{4}(1+\xi_i r)(1+\eta_i s)$

  with $\begin{bmatrix} \xi_1 & \xi_2 & \xi_3 & \xi_4 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -1 & 1 \end{bmatrix}$
  $\begin{bmatrix} \eta_1 & \eta_2 & \eta_3 & \eta_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 & -1 \end{bmatrix}$

A 4-node quadrilateral 2D solid element



Global Cartesian coordinate Natural coordinate

❖ Strain components

- $e_{ij} = \frac{1}{2}(\mathbf{g}_i \cdot \mathbf{u}_{,j} + \mathbf{g}_j \cdot \mathbf{u}_{,i})$ with $i,\ j = 1,2$

  where $\mathbf{g}_i = \frac{\partial \mathbf{x}}{\partial r_i}$, $\mathbf{u}_{,i} = \frac{\partial \mathbf{u}}{\partial r_i}$ with $r_1 = r,\ r_2 = s$

2D-MITC4 element has constant base vectors.

$\widehat{e}_{ij} = e_{kl} g_i^k g_j^l$ with $g_i^j = \widehat{\mathbf{g}}_i \cdot \mathbf{g}^j$ and $\widehat{\mathbf{g}}_i = \mathbf{g}_i(0,0)$

Previous 2D-MITC4 solid element

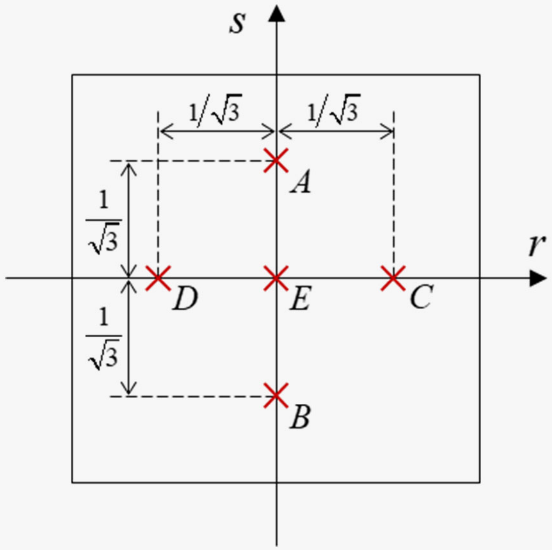
❖ Assumed strain field for the 2D-MITC4 element

- $\hat{e}_{rr} = \hat{e}_{rr}^{(E)} + \frac{\sqrt{3}}{2}(\hat{e}_{rr}^{(A)} - \hat{e}_{rr}^{(B)})\lambda(r,s)s$

- $\hat{e}_{ss} = \hat{e}_{ss}^{(E)} + \frac{\sqrt{3}}{2}(\hat{e}_{ss}^{(C)} - \hat{e}_{ss}^{(D)})\lambda(r,s)r$

- $\hat{e}_{rs} = \hat{e}_{rs}^{(E)}$ with $\lambda(r,s) = \frac{\det(\mathbf{J}(0,0))}{\det(\mathbf{J}(r,s))}$

Tying points



To reduce the computational cost

- $\hat{e}_{rr} = e_{rr}\big|_{con} + \hat{e}_{rr}^{lin}\lambda(r,s)s$

- $\hat{e}_{ss} = e_{ss}\big|_{con} + \hat{e}_{ss}^{lin}\lambda(r,s)r$

- $\hat{e}_{rs} = e_{rs}\big|_{con}$

Procedure

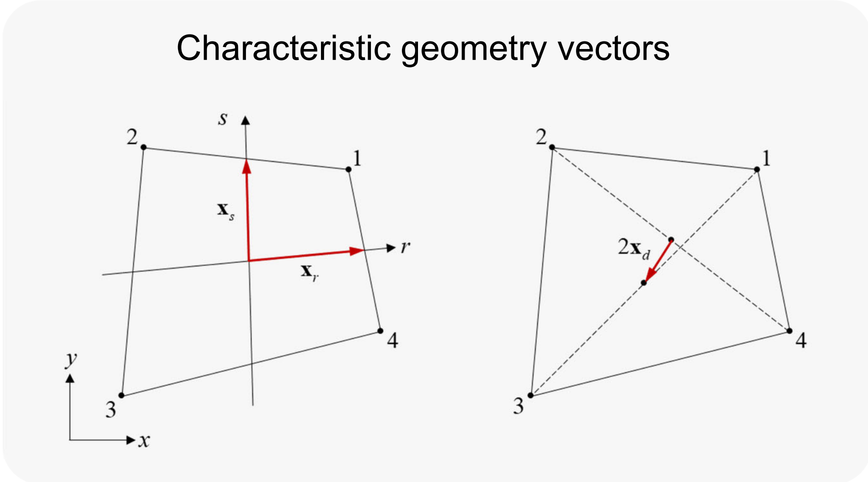
1. Sampling at tying point: $e_{ij}^{(\cdot)}$
2. Coordinate transformation: $e_{ij}^{(\cdot)} \to \hat{e}_{ij}^{(\cdot)}$
3. Representation using strain coefficients: $\hat{e}_{ij}^{(\cdot)} \to e_{ij}\big|_{coefficients}$

Previous 2D-MITC4 solid element

❖ Characteristic geometry and displacement vectors

- $\mathbf{x}_r = \frac{1}{4}\sum_{i=1}^{4}\xi_i \mathbf{x}_i$, $\mathbf{x}_s = \frac{1}{4}\sum_{i=1}^{4}\eta_i \mathbf{x}_i$, $\mathbf{x}_d = \frac{1}{4}\sum_{i=1}^{4}\xi_i \eta_i \mathbf{x}_i$

- $\mathbf{u}_r = \frac{1}{4}\sum_{i=1}^{4}\xi_i \mathbf{u}_i$, $\mathbf{u}_s = \frac{1}{4}\sum_{i=1}^{4}\eta_i \mathbf{u}_i$, $\mathbf{u}_d = \frac{1}{4}\sum_{i=1}^{4}\xi_i \eta_i \mathbf{u}_i$

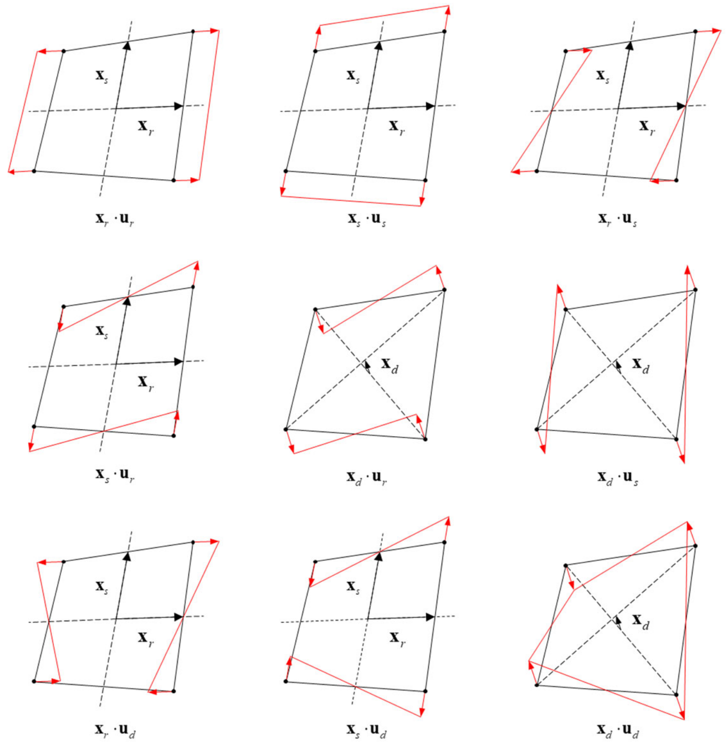


❖ Strain coefficients

- $e_{rr}\big|_{con} = \mathbf{x}_r \cdot \mathbf{u}_r$, $e_{rr}\big|_{lin} = \mathbf{x}_r \cdot \mathbf{u}_d + \mathbf{x}_d \cdot \mathbf{u}_r$

- $e_{ss}\big|_{con} = \mathbf{x}_s \cdot \mathbf{u}_s$, $e_{ss}\big|_{lin} = \mathbf{x}_s \cdot \mathbf{u}_d + \mathbf{x}_d \cdot \mathbf{u}_s$

- $e_{rs}\big|_{con} = \frac{1}{2}\left(\mathbf{x}_r \cdot \mathbf{u}_s + \mathbf{x}_s \cdot \mathbf{u}_r\right)$, $e_{rs}\big|_{bil} = \mathbf{x}_d \cdot \mathbf{u}_d$



Proposed 2D-MITC4 solid element

❖ Simplified formulation

Previous formulation

$$\hat{e}_{rr}^{lin} = \frac{n_1}{\sqrt{3}} e_{rs}\big|_{bil} + \sqrt{3}n_1\, e_{rr}\big|_{con} + \sqrt{3}n_2\, e_{ss}\big|_{con} + n_3\, e_{rr}\big|_{lin} + n_4\, e_{ss}\big|_{lin} + 2\sqrt{3}n_5\, e_{rs}\big|_{con}$$

$$\hat{e}_{ss}^{lin} = \frac{m_1}{\sqrt{3}} e_{rs}\big|_{bil} + \sqrt{3}m_1\, e_{ss}\big|_{con} + \sqrt{3}m_2\, e_{rr}\big|_{con} + m_3\, e_{ss}\big|_{lin} + m_4\, e_{rr}\big|_{lin} + 2\sqrt{3}m_5\, e_{rs}\big|_{con}$$

with

$$n_1 = \frac{1}{2}\left[\left(g_r^r\big|_{(A)}\right)^2 - \left(g_r^r\big|_{(B)}\right)^2\right] \qquad n_2 = \frac{1}{2}\left[\left(g_r^s\big|_{(A)}\right)^2 - \left(g_r^s\big|_{(B)}\right)^2\right] \qquad n_3 = \frac{1}{2}\left[\left(g_r^r\big|_{(A)}\right)^2 + \left(g_r^r\big|_{(B)}\right)^2\right]$$

$$n_4 = \frac{1}{2}\left[g_r^r\big|_{(A)} \cdot g_r^s\big|_{(A)} + g_r^r\big|_{(B)} \cdot g_r^s\big|_{(B)}\right] \qquad n_5 = \frac{1}{2}\left[g_r^r\big|_{(A)} \cdot g_r^s\big|_{(A)} - g_r^r\big|_{(B)} \cdot g_r^s\big|_{(B)}\right]$$

$$m_1 = \frac{1}{2}\left[\left(g_s^s\big|_{(C)}\right)^2 - \left(g_s^s\big|_{(D)}\right)^2\right] \qquad m_2 = \frac{1}{2}\left[\left(g_s^r\big|_{(C)}\right)^2 - \left(g_s^r\big|_{(D)}\right)^2\right] \qquad m_3 = \frac{1}{2}\left[\left(g_s^s\big|_{(C)}\right)^2 + \left(g_s^s\big|_{(D)}\right)^2\right]$$

$$m_4 = \frac{1}{2}\left[g_s^r\big|_{(C)} \cdot g_s^s\big|_{(C)} + g_s^r\big|_{(D)} \cdot g_s^s\big|_{(D)}\right] \qquad m_5 = \frac{1}{2}\left[g_s^r\big|_{(C)} \cdot g_s^s\big|_{(C)} - g_s^r\big|_{(D)} \cdot g_s^s\big|_{(D)}\right]$$

Proposed formulation

$$\hat{e}_{rr}^{lin} = \frac{3}{3-\alpha^2}\left(-\alpha \mathbf{x}_r \cdot \mathbf{u}_r - \beta \mathbf{x}_r \cdot \mathbf{u}_s + \mathbf{x}_r \cdot \mathbf{u}_d\right)$$

$$\hat{e}_{ss}^{lin} = \frac{3}{3-\beta^2}\left(-\beta \mathbf{x}_s \cdot \mathbf{u}_s - \alpha \mathbf{x}_s \cdot \mathbf{u}_r + \mathbf{x}_s \cdot \mathbf{u}_d\right)$$

with $\mathbf{x}_d = \alpha \mathbf{x}_r + \beta \mathbf{x}_s$

Calculated directly
from the characteristic vectors

- $\hat{e}_{rr} = e_{rr}\big|_{con} + \hat{e}_{rr}^{lin}\lambda(r,s)s$
- $\hat{e}_{ss} = e_{ss}\big|_{con} + \hat{e}_{ss}^{lin}\lambda(r,s)r$
- $\hat{e}_{rs} = e_{rs}\big|_{con}$

- Simple implementation
- Almost same performance
- Promising feature in the shell

Proposed 2D-MITC4 solid element

$\mathbf{u} = [u_1 \ \cdots \ u_8 \ \ v_1 \ \cdots \ v_8]^{\mathrm{T}}$

$\mathbf{e} = [\varepsilon_{xx} \ \ \varepsilon_{yy} \ \ \gamma_{xy}]^{\mathrm{T}}$

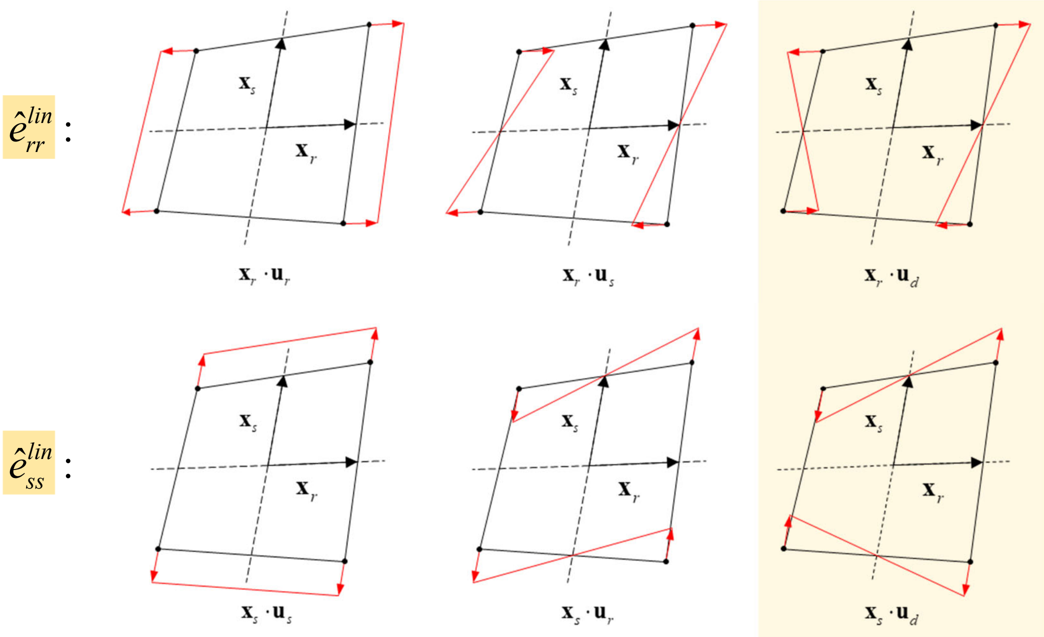
❖ Investigation of the assumed strain field

- $\mathbf{K} = \int_{V_e} \widehat{\mathbf{B}}^{\mathrm{T}} \mathbf{C} \widehat{\mathbf{B}} \, dV_e$, $\widehat{\mathbf{B}}(r,s)$: assumed strain - displacement matrix ($\mathbf{e} = \widehat{\mathbf{B}}\mathbf{u}$)

$$\hat{e}_{rr} = e_{rr}\big|_{con} + \boxed{\hat{e}_{rr}^{lin} \lambda(r,s) s}$$
$$\hat{e}_{ss} = e_{ss}\big|_{con} + \boxed{\hat{e}_{ss}^{lin} \lambda(r,s) r}$$
$$\hat{e}_{rs} = e_{rs}\big|_{con}$$

Bending related term

Bending behavior



- ✓ Bending performance of the element depends on the **bending related term**.
- ✓ The $\hat{e}_{rr}^{lin}$ and $\hat{e}_{ss}^{lin}$ are determined by the element geometry only.

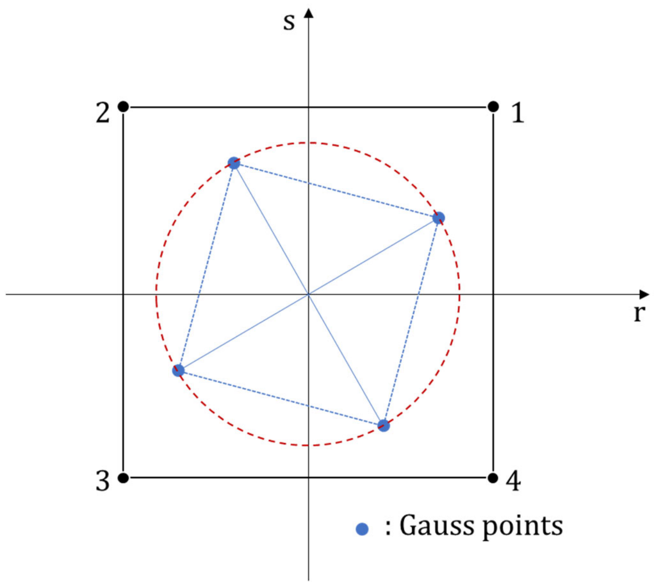
When evaluating the stiffness matrix

- ✓ The element geometry should be considered when choose the r and s.
- ✓ Depending on which r and s are employed, we can selectively adjust the bending stiffness.

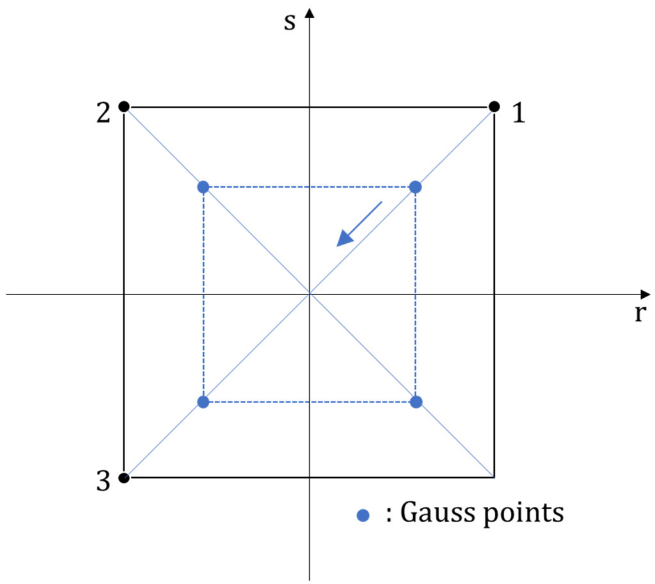
Proposed 2D-MITC4 solid element

❖ How to adjust Gauss point

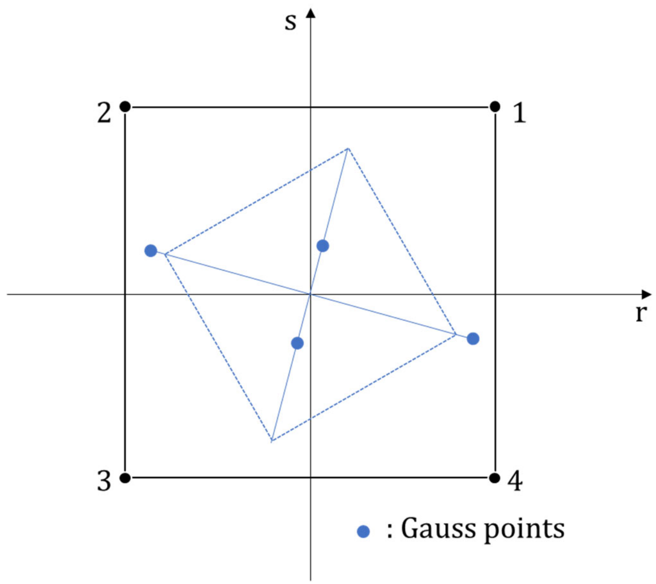
- Although there are infinitely many ways to modify Gauss point, only a few cases can pass the patch tests



Rotation

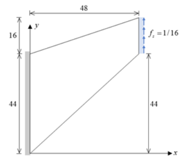


Scaling



Rotation and scaling

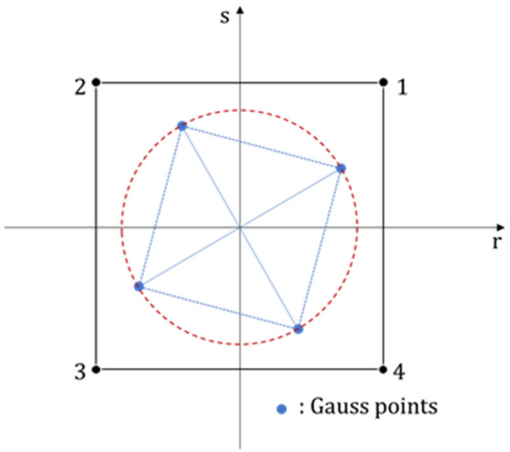
Proposed 2D-MITC4 solid element

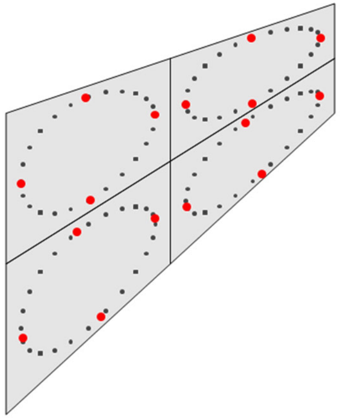


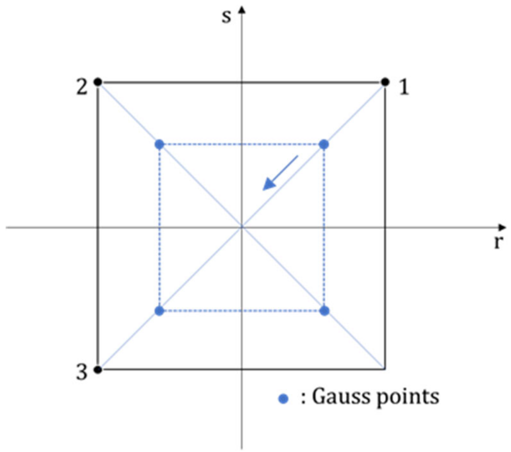
Q4	: standard quadrilateral element
ICM-Q4	: incompatible modes element
2D-MITC4	: previous 2D-MITC4 element
Reference	: 9-node quadrilateral element

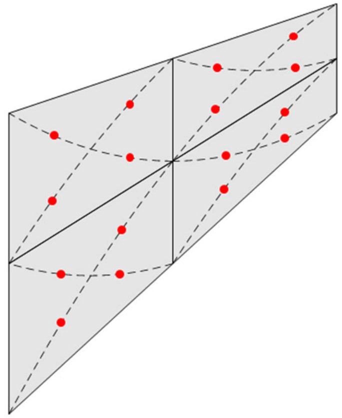
❖ How to adjust Gauss point

- Rotation vs scaling?









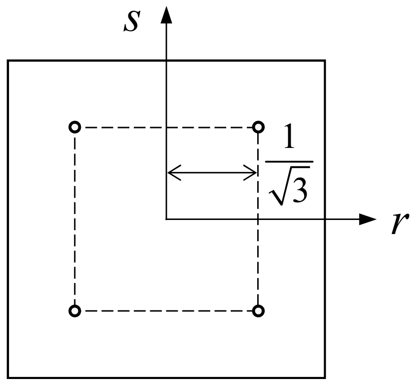
Strain energy for Cook's skew beam with 4×4 mesh

Element	Gauss integration	Strain energy	Normalization
Q4	Standard	5.899	0.491
2D-MITC4	Standard	8.676	0.722
2D-MITC4	Rotation(optimal)	8.719	0.726
2D-MITC4	Scaling(optimal)	11.177	0.930
Reference	-	12.017	1

Proposed 2D-MITC4 solid element

❖ How to adjust Gauss points

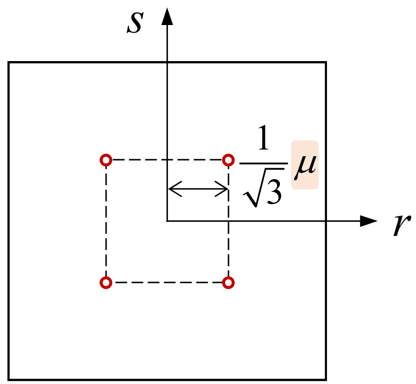
- $\mathbf{K} = t\sum_{i=1}^{2}\sum_{j=1}^{2} w_i w_j \mathbf{F}(\xi_i, \xi_j)$ with $\mathbf{F}(r,s) = t\,\widehat{\mathbf{B}}^{\mathrm{T}}(r,s)\mathbf{C}\widehat{\mathbf{B}}(r,s)\det(\mathbf{J}(r,s))$



Standard Gauss points

$\xi_1 = \frac{1}{\sqrt{3}}, \quad \xi_2 = -\frac{1}{\sqrt{3}}, \quad w_1 = w_2 = 1$ with 2 x 2 Gauss integration (standard)

Adjusting parameter μ

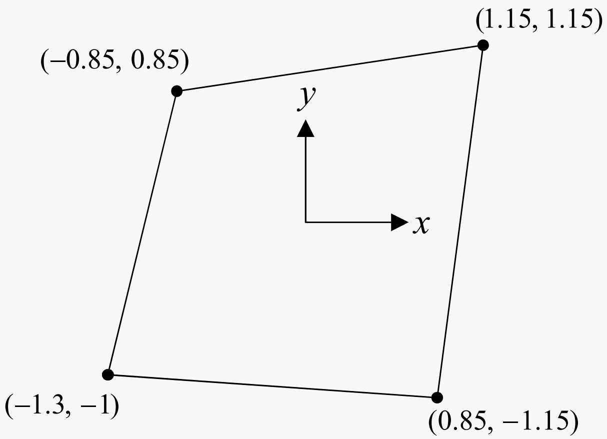


Adjusted Gauss points

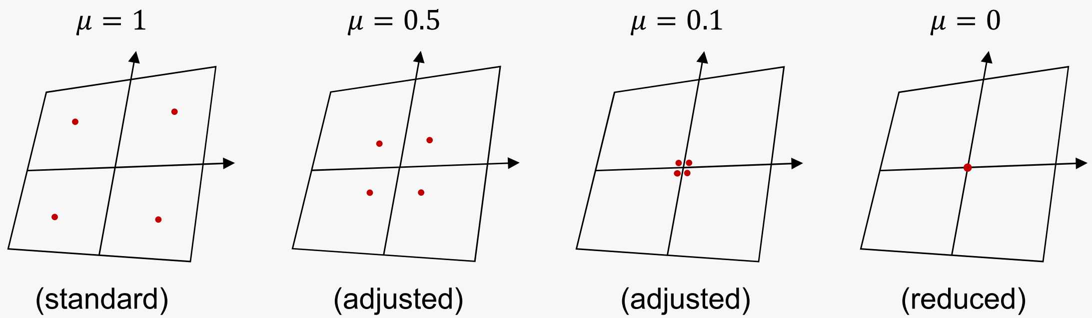
$\hat{\xi}_i = \mu\xi_i , \qquad w_i = 1$ with 2 x 2 Gauss integration (adjusted)

Proposed 2D-MITC4 solid element

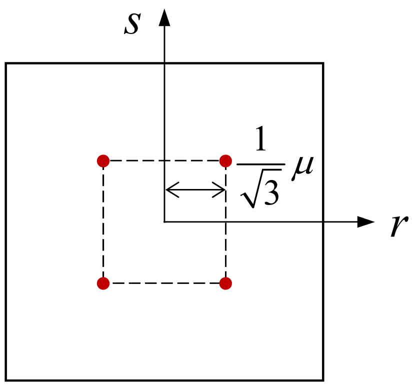
❖ Effect of adjusting parameter, μ



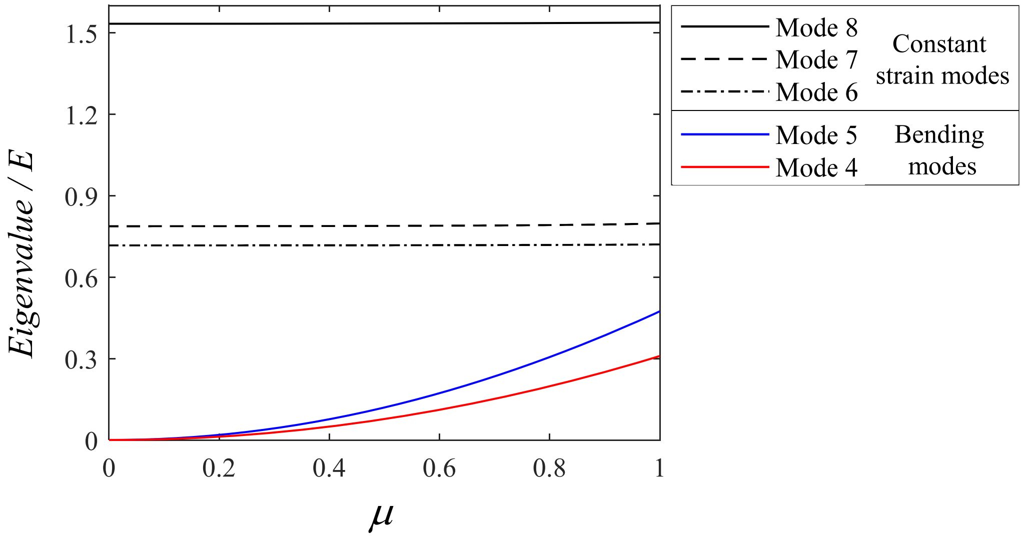
Geometry of a single element



Gauss points according to adjusting parameter, μ



Adjusted Gauss points



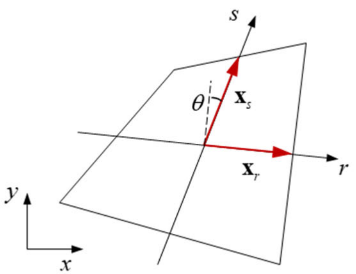
Proposed 2D-MITC4 solid element

❖ Practical requirements for the adjusting parameter, μ

- For undistorted element geometries ➡ $\mu = 1$ (equivalent to standard Gauss points)
- For distorted element geometries ➡ $\mu \leq 1$ ($\mu > 1$ causes stiff behavior)
  $\mu > 0$ (to avoid spurious zero energy mode)
- For the geometries in similarity ➡ $\mu_{\square ABCD} = \mu_{\square EFGH}$ when $\square ABCD \sim \square EFGH$
- $\mu = f(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4)$ should be uniquely and adaptively determined according to geometry distortion

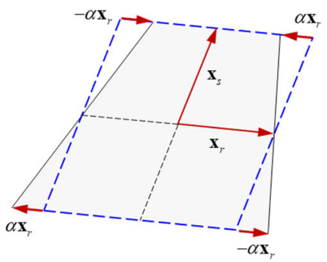
❖ Distortion measures

Skewness

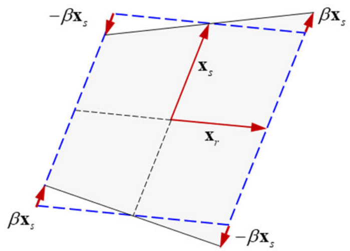


θ

Taper

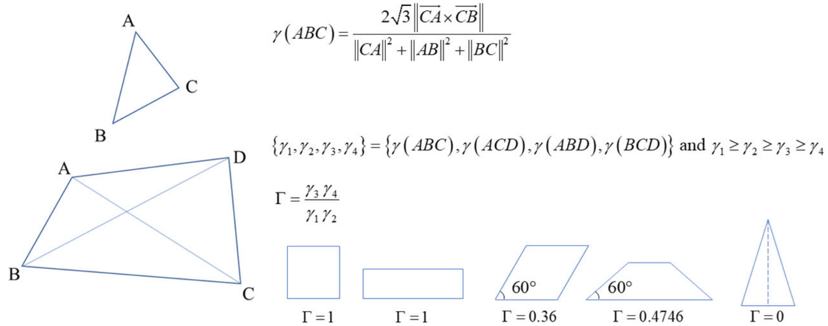


α



β

Overall



$$\gamma(ABC) = \frac{2\sqrt{3}\left\|\overrightarrow{CA} \times \overrightarrow{CB}\right\|}{\|CA\|^2 + \|AB\|^2 + \|BC\|^2}$$

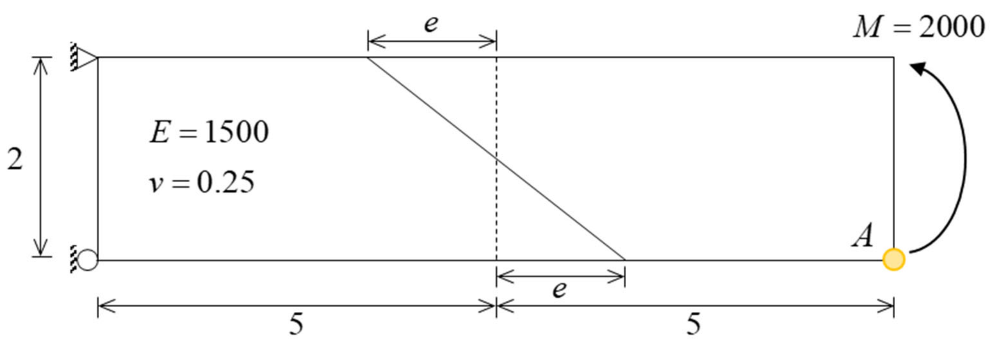
$$\{\gamma_1, \gamma_2, \gamma_3, \gamma_4\} = \{\gamma(ABC), \gamma(ACD), \gamma(ABD), \gamma(BCD)\} \text{ and } \gamma_1 \geq \gamma_2 \geq \gamma_3 \geq \gamma_4$$

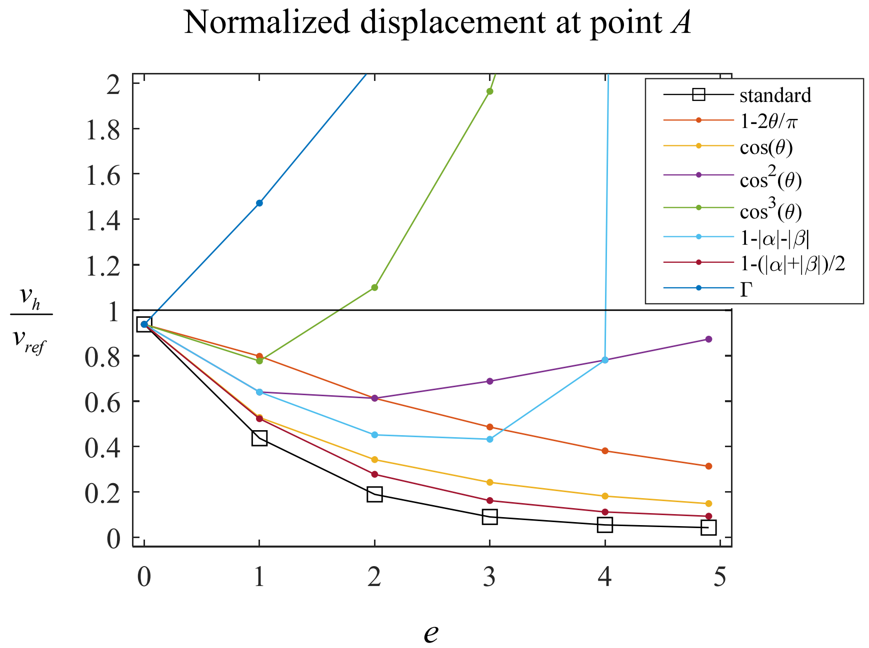
$$\Gamma = \frac{\gamma_3 \gamma_4}{\gamma_1 \gamma_2}$$

Γ

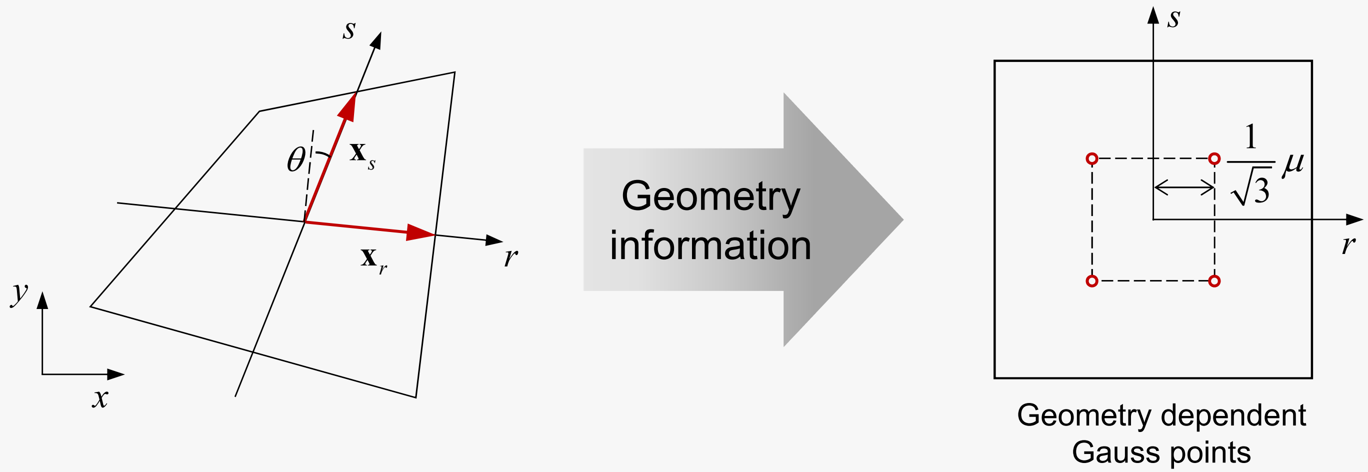
Proposed 2D-MITC4 solid element

❖ Numerical investigation





❖ Proposed function for adjusting parameter



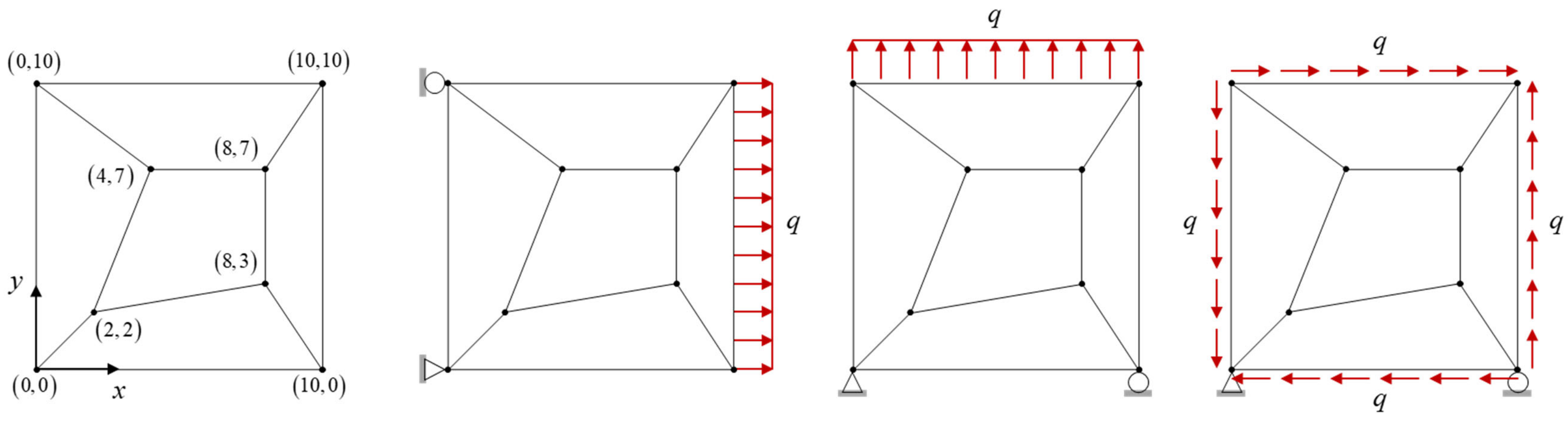
$$\mu = \cos^2(\theta)$$

- ✓ meets the requirements
- ✓ generally provides the best solution among the candidates

Basic numerical tests

❖ Patch tests

- The minimum number of constraints are given to prevent rigid body motions.
- Proper loadings are applied to produce a constant stress field.
- In this tests, a constant stress field should be represented exactly.



Geometry Stretching test Stretching test Shearing test

❖ Zero energy mode test

- A single unsupported 2D 4-node quadrilateral element should have only three zero energy modes.

❖ Isotropy test

- The behavior of finite elements should not be affected by the node numbering sequence.

Numerical examples

❖ Finite elements considered

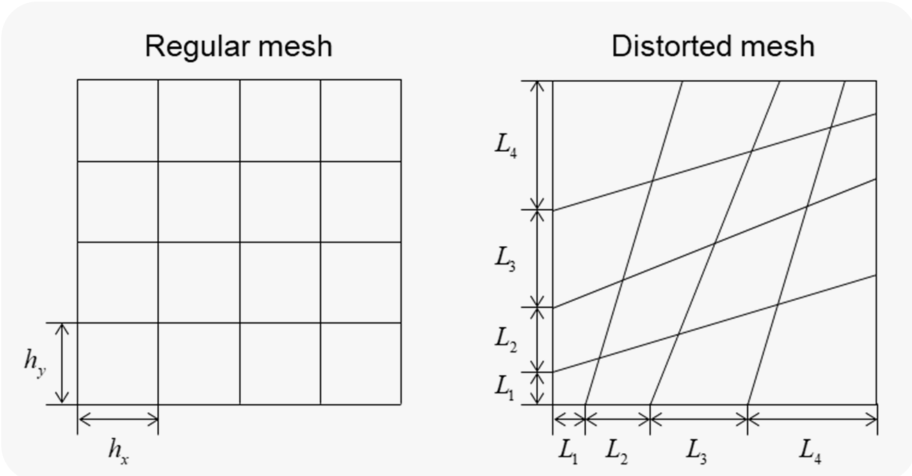
- **Q4** : standard quadrilateral element
- **ICM-Q4** : incompatible modes element (EAS element, QM6)
- **2D-MITC4** : previous 2D-MITC4 element
- **New 2D-MITC4** : proposed 2D-MITC4 element

❖ Evaluation method

- Convergence of the relative error in strain energy $E_r = \frac{\left|E_{ref} - E_h\right|}{E_{ref}}$
- Displacements and stresses
- Reference solutions are obtained by using 9-node solid element with 64×64 mesh

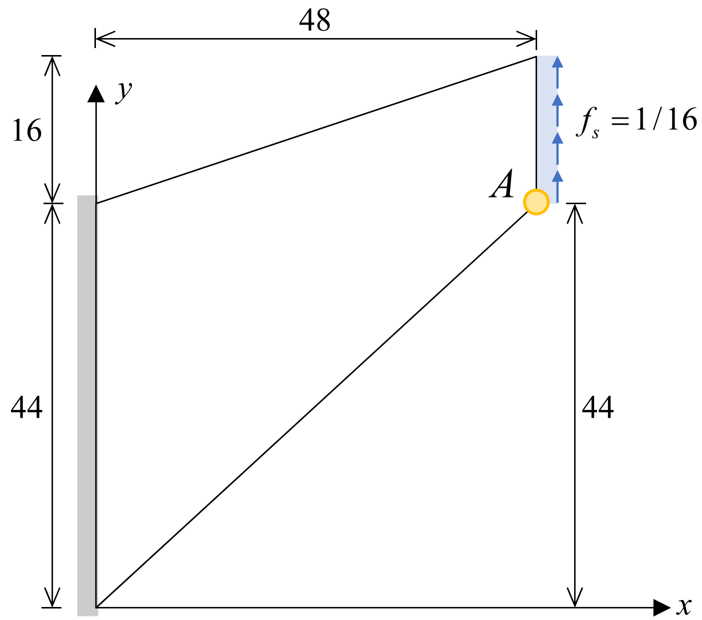
❖ Mesh patterns

- Regular and distorted meshes
- $h = 1/N$ for the element size
- $L_1 : L_2 : \ldots L_N = 1 : 2 : \ldots N$
- Arbitrary distorted meshes



Numerical examples (Plane stress)

❖ Cook's skew beam



- **Force**

  Distributed shear force $f_s = 1/16$ (force/length)

- **Boundary condition**

  Left edge is clamped.

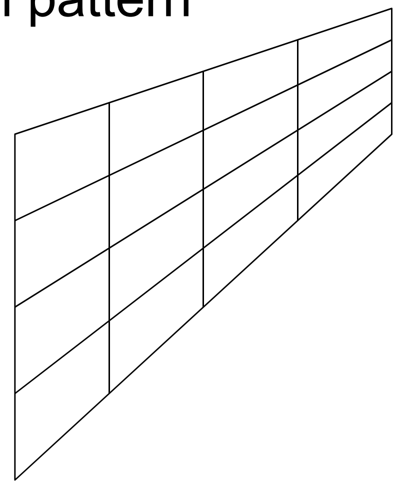
- **Material property**

  Plane stress condition with $E = 1, \nu = 0.3$

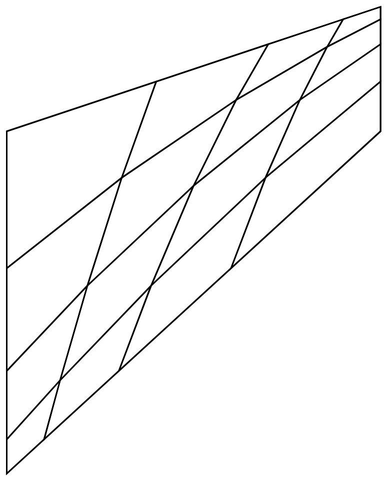
- **Regular and two distorted meshes** ($N \times N$)

  $N = 2, 4, 8, 16$

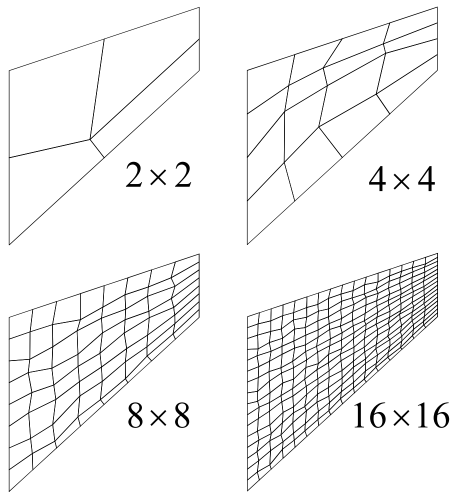
\- Mesh pattern



Regular



Distorted I



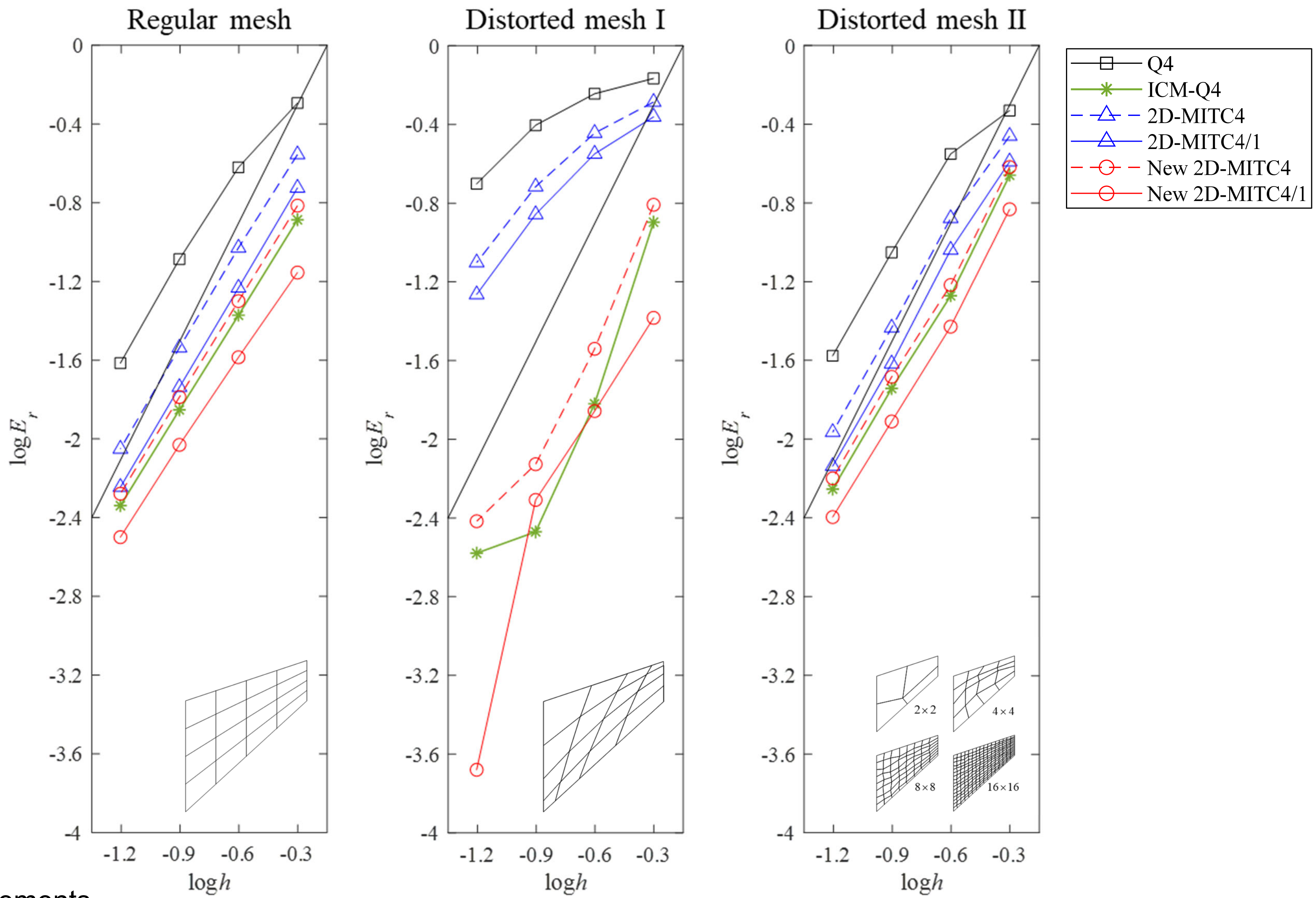
Distorted II

Numerical examples (Plane stress)

2D-MITC4/1 : 2D-MITC4 with treatment of volumetric locking

❖ Cook's skew beam

- Convergence curves

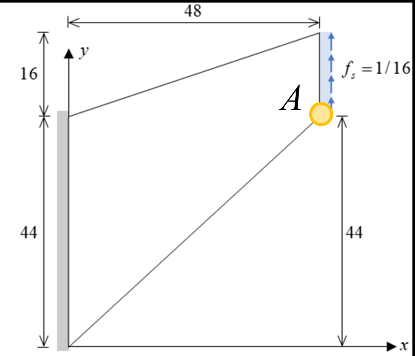


More accurate

More elements

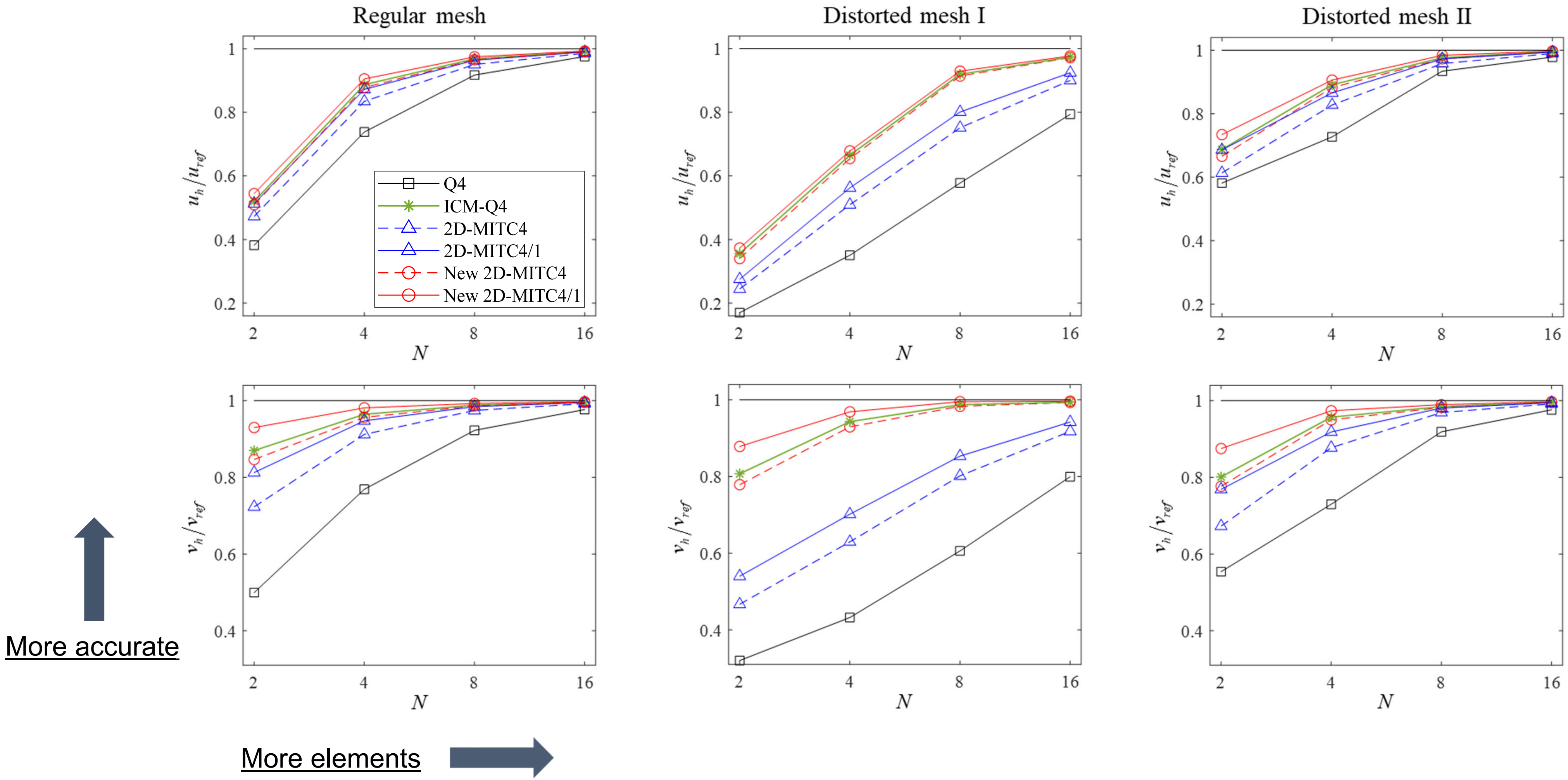
h=1/*N*

Numerical examples (Plane stress)



❖ Cook's skew beam

- Normalized horizontal and vertical displacements at point A.

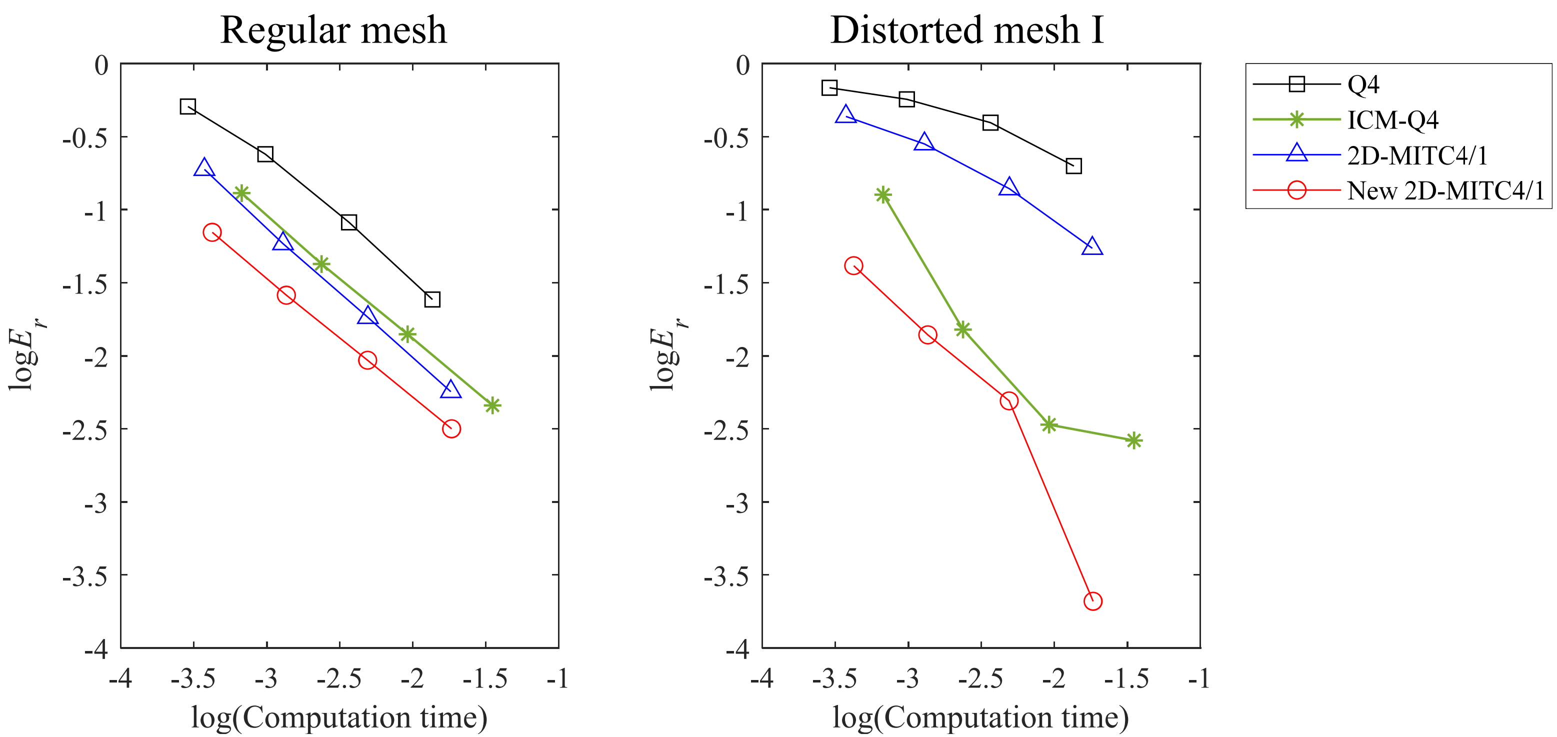


Numerical examples (Plane stress)

Computations are performed in a PC with Intel Core i7-8700, 3.20GHz CPU and 32GB RAM.
The CSR format is used for storing matrices and MATLAB is used.

❖ Cook's skew beam

- Computational efficiency curves



More accurate

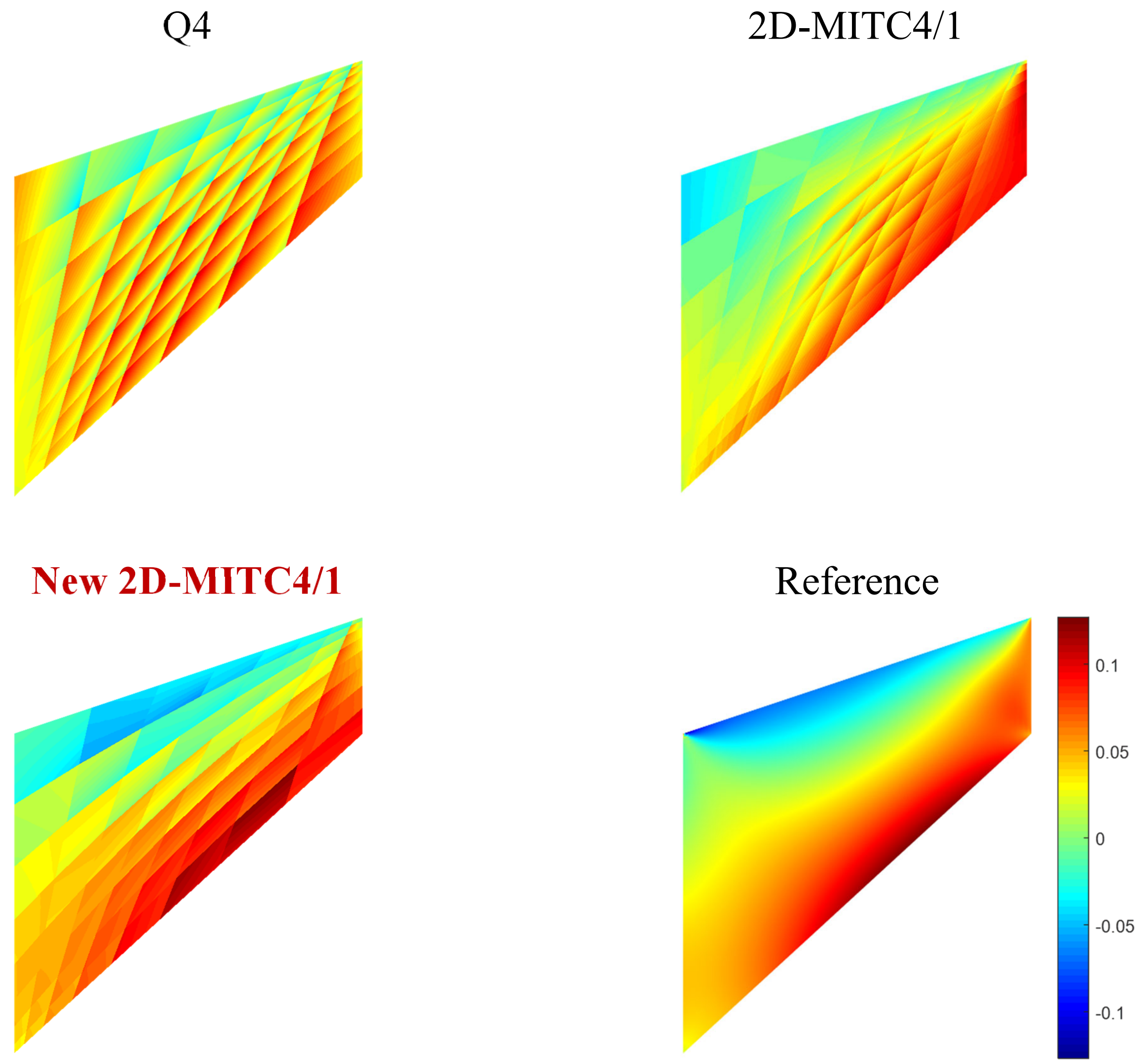
More computation time

- For the ICM-Q4 element, the additional DOFs are condensed out in an element level.
- The number and positions of non-zeros in the total matrix are identical to each other.
- Thus, computation times during the construction of the total stiffness matrix are measured.

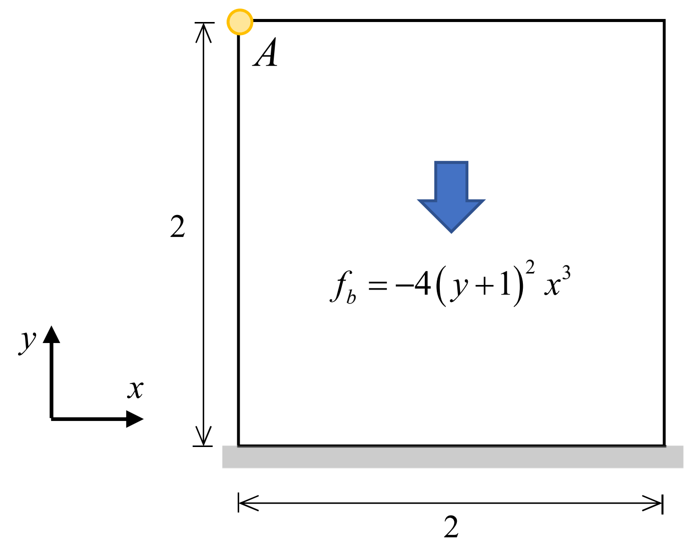
Numerical examples (Plane stress)

❖ Cook's skew beam

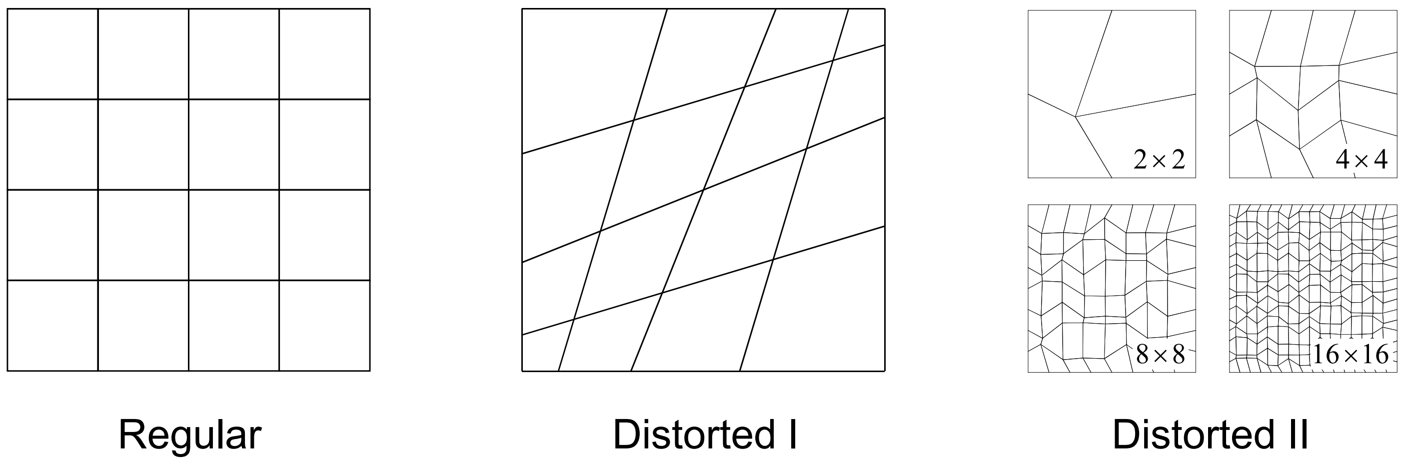
- Shear stress distribution for the distorted mesh (N=8)



❖ Block under body force



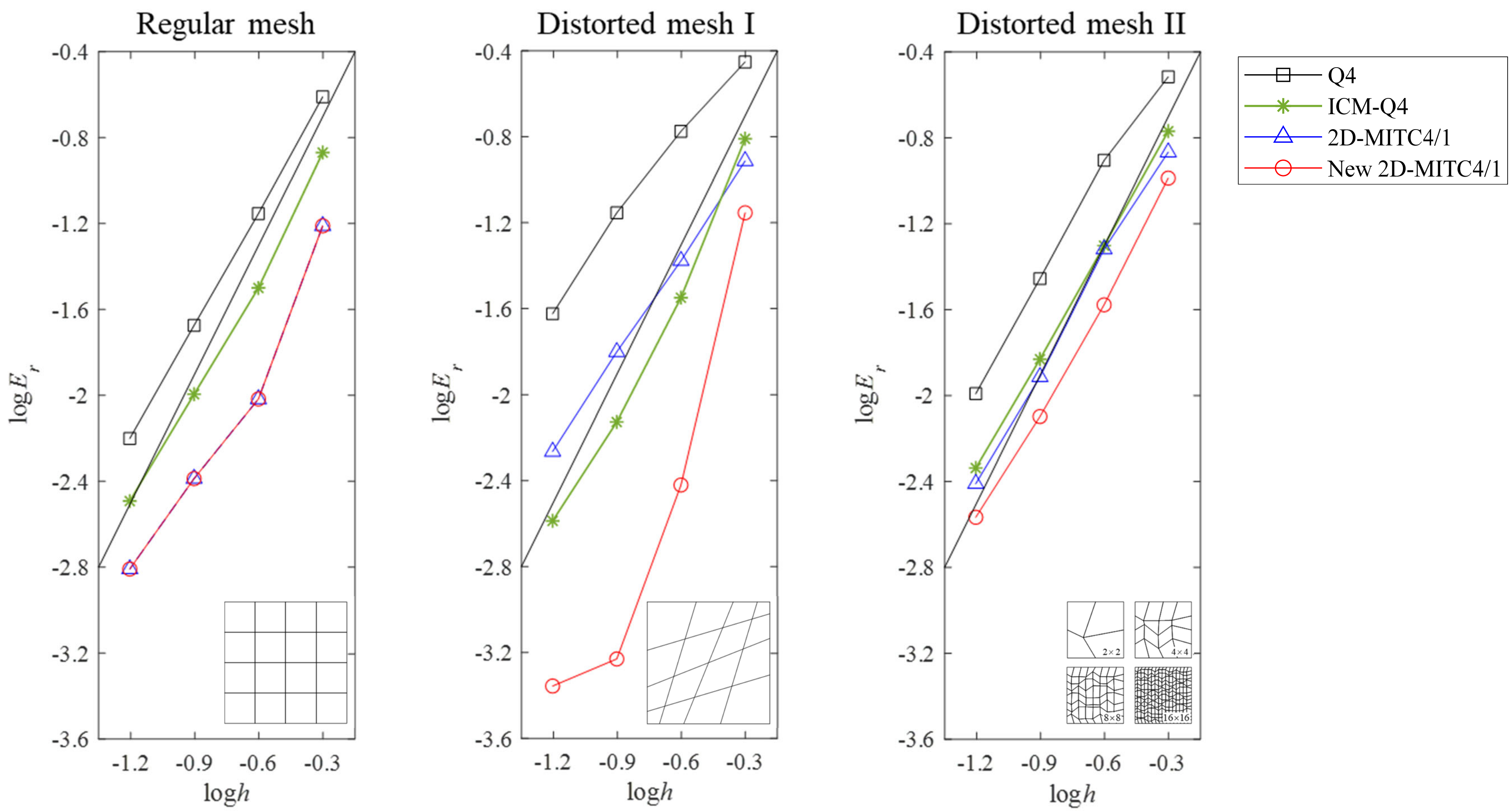
- **Force**
  - Body force $f_b = -4(y+1)^2 x^3$ (force per volume)
- **Boundary condition**
  - Clamed bottom side
- **Dimensions and properties**
  - Plane strain condition with $E = 2.0\times 10^7$, $\nu = 0.3$
- **Regular and two distorted meshes** $(N\times N)$
  - N=2, 4, 8, 16
- **Volumetric locking study**
  - Nearly incompressible material



Numerical examples (Plane strain)

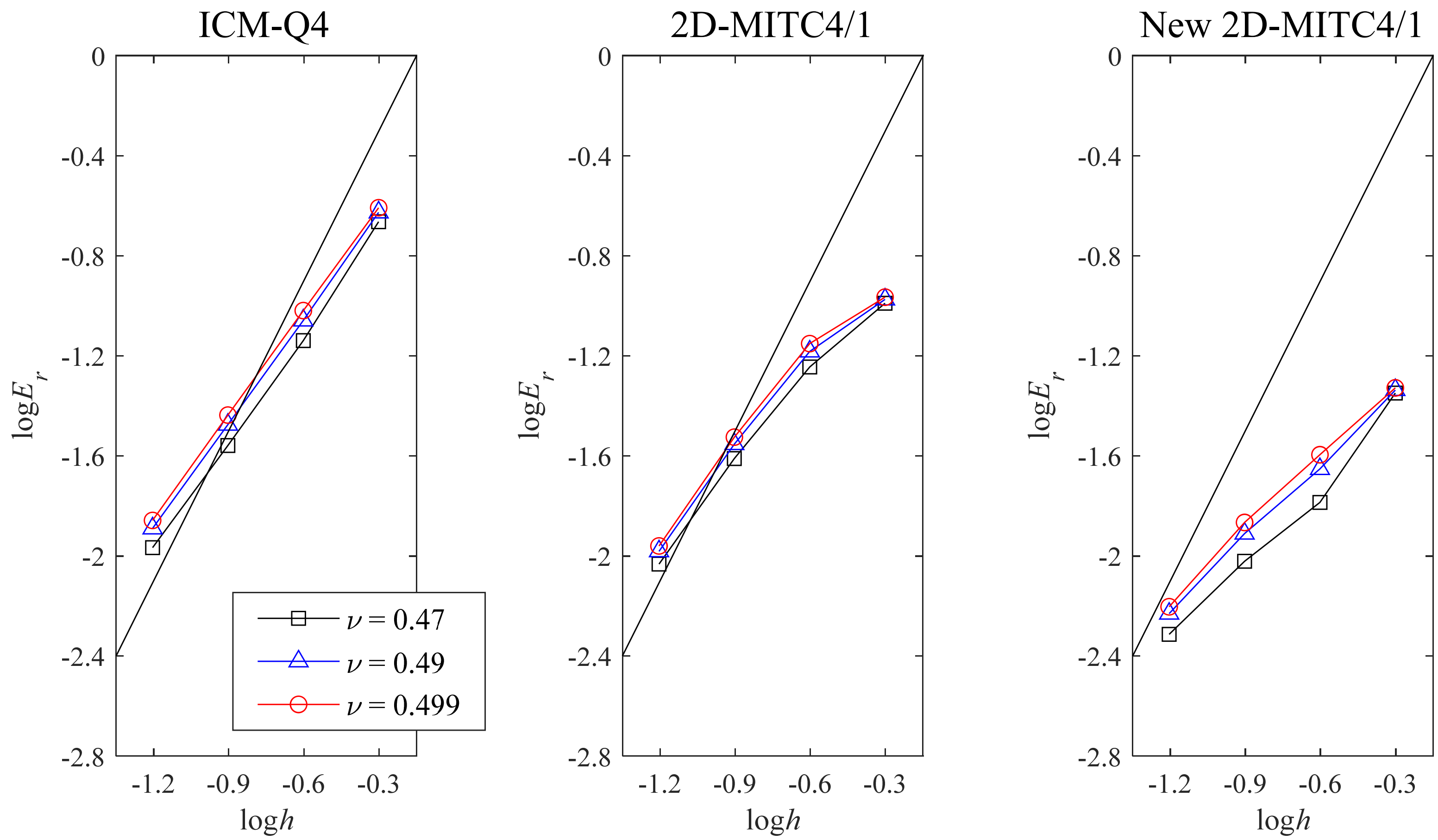
❖ Block under body force

- Convergence curves in plane strain problem with $\upsilon = 0.3$



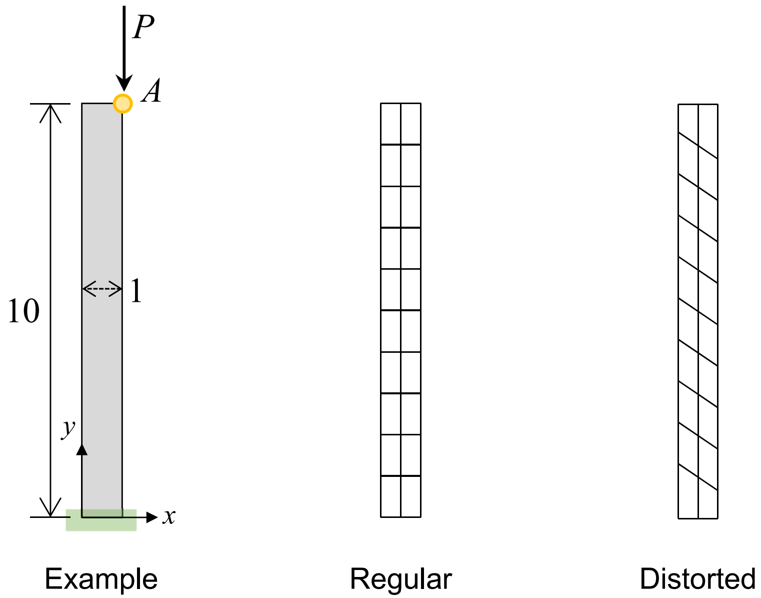
Numerical examples (Plane strain)

- **Block under body force (Distort mesh I)**
  - Convergence curves with nearly incompressible material (volumetric locking)



Numerical examples (Nonlinear)

❖ Column under a compressive load



- **Force**

  Compressive load $P_{max} = 5 \times 10^3$.

- **Boundary condition**

  Bottom edge is clamped.

- **Material property**

  Plane stress condition with $E = 1 \times 10^6$, $\nu = 0$.

- **Geometrically nonlinear analysis**

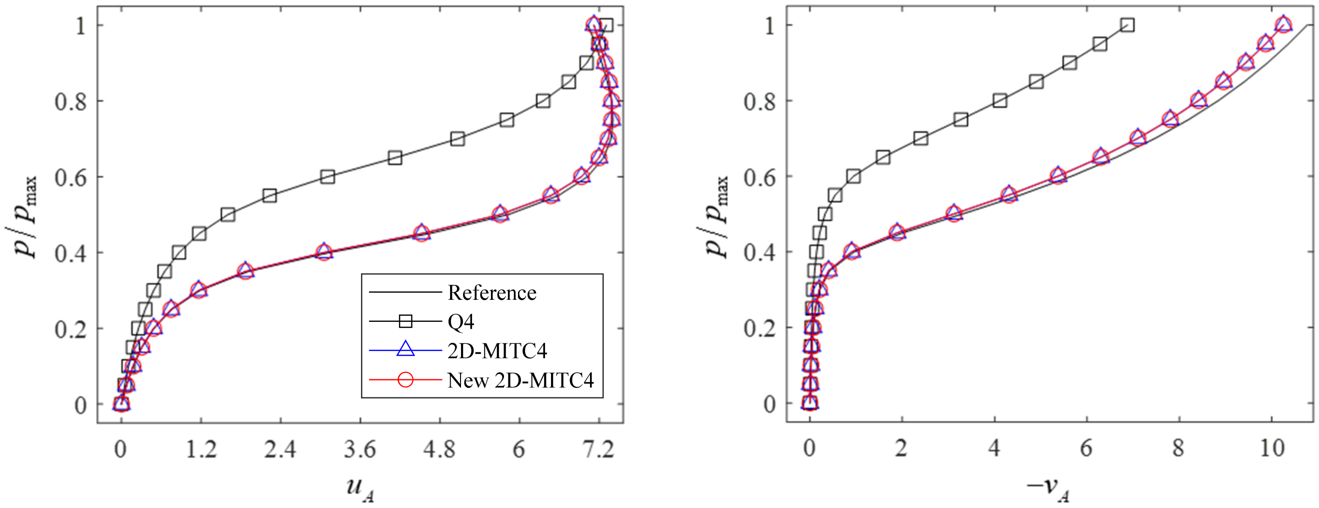
- **Regular and distorted meshes** ($N \times 5N$)

  $N = 2$.

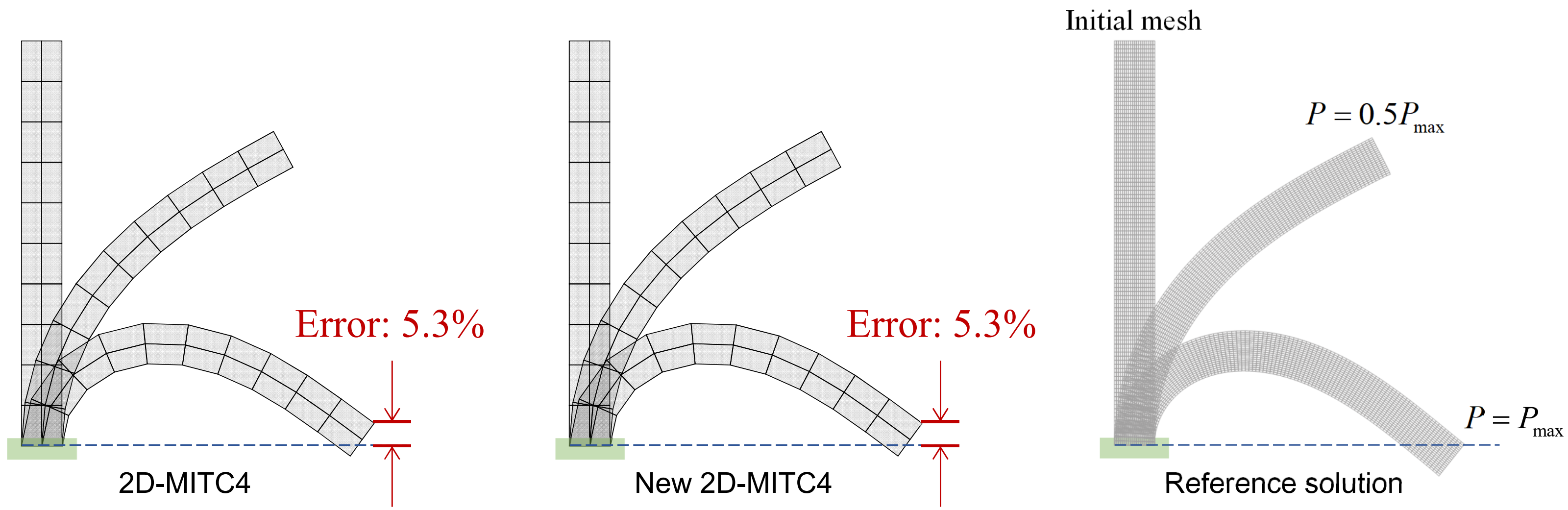
Numerical examples (Nonlinear)

❖ Column under a compressive load with <u>regular</u> mesh

- Load-displacement curves



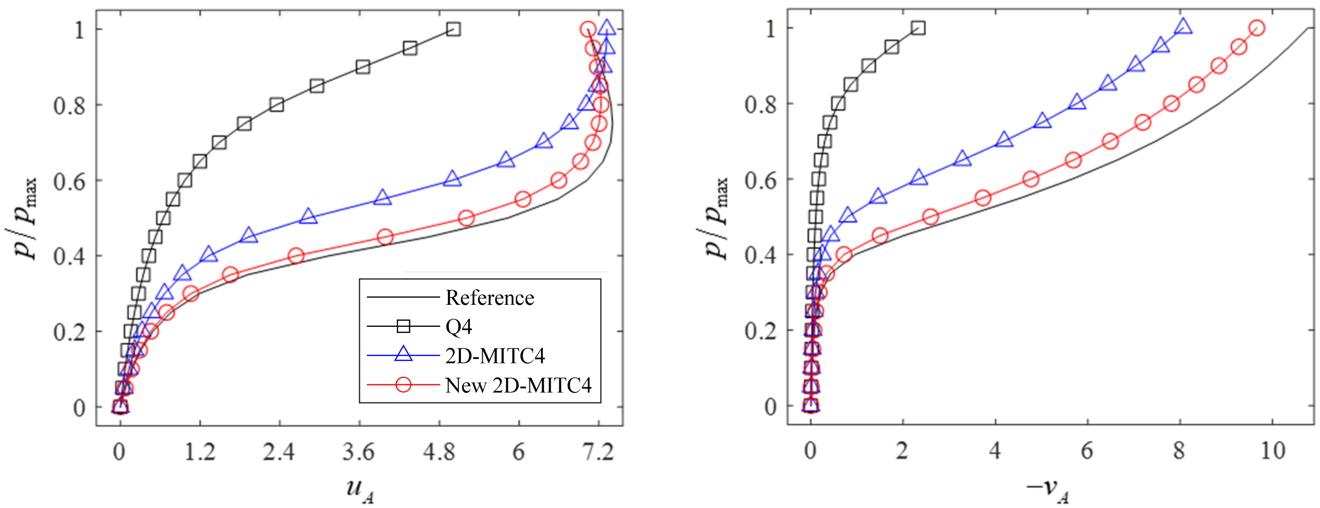
- Deformed configurations



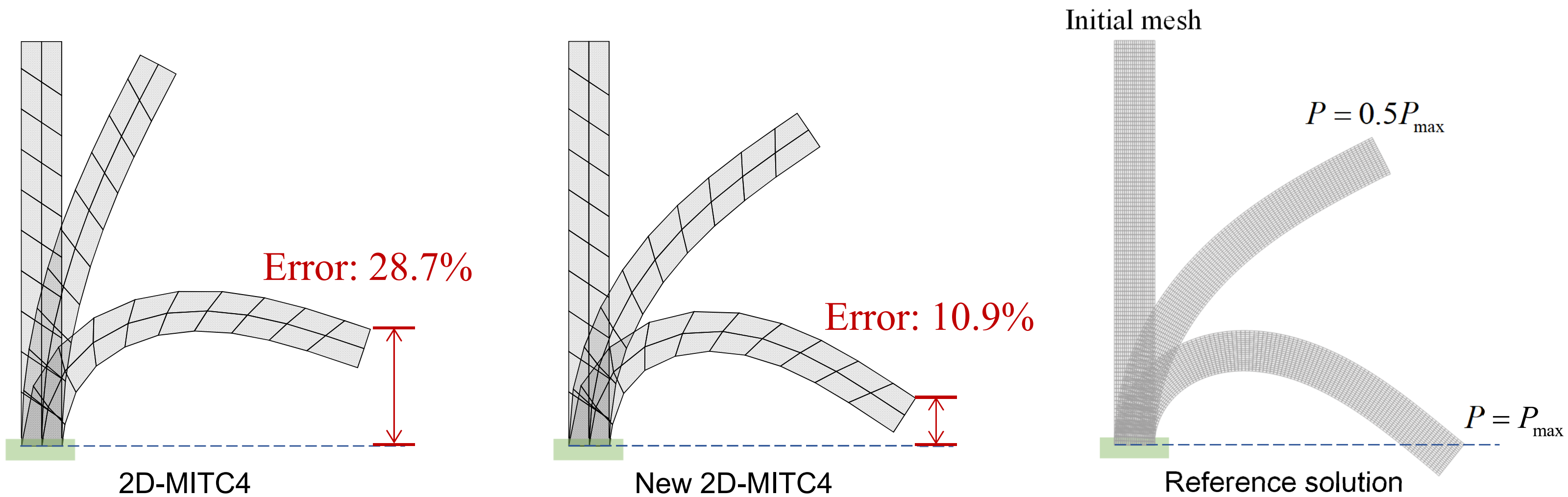
Numerical examples (Nonlinear)

❖ Column under a compressive load with <u>distorted</u> mesh

- Load-displacement curves

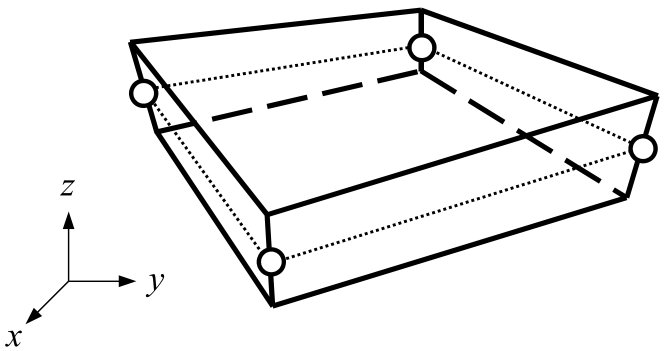


- Deformed configurations



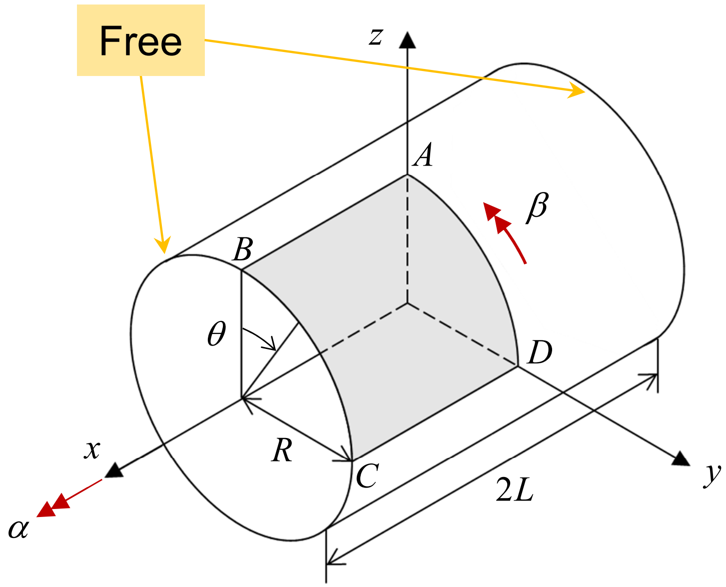
Topic 2

New MITC4+ shell element

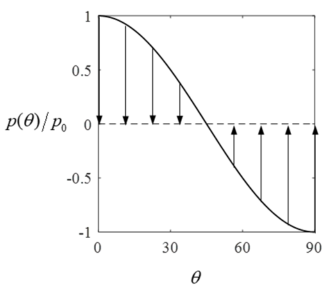


4-node quadrilateral shell element

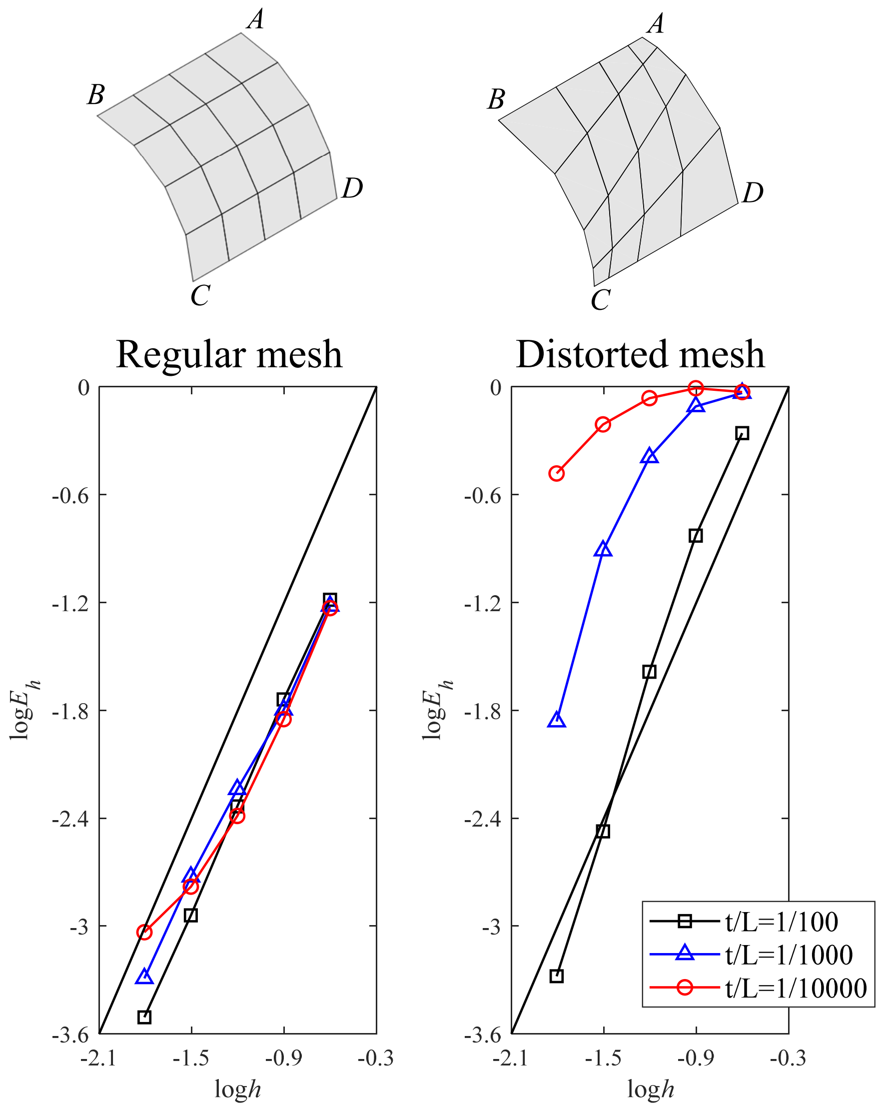
❖ MITC4 shell element



Cylindrical shell with **free ends**
(bending dominated)



Pressure distribution
Free ends
L=R=1
$E = 2.0 \times 10^5, \nu = 1/3$
1/8 model only
Reference : MITC9

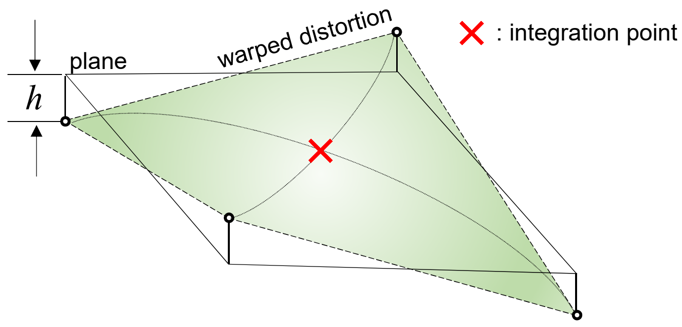


MITC4: 4-node shell element with the MITC method
- treatment for transverse shear locking.

Related studies

❖ Reduced integration

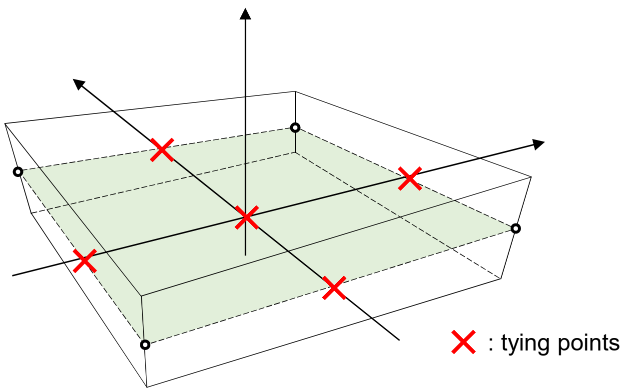
- **Zienkiewicz et al. (1971) / Belytshcko et al. (1982)**
  - The reduced integration technique
  - Spurious zero energy modes problem
- **Tsay et al. (1983) / Leviathan et al. (1994)**
  - Reduced integration with stabilization techniques
- **Rankin and Nour-Omid. (1988)**
  - Reduced integration with displacement projection



Reduced integration

❖ Assumed membrane strain

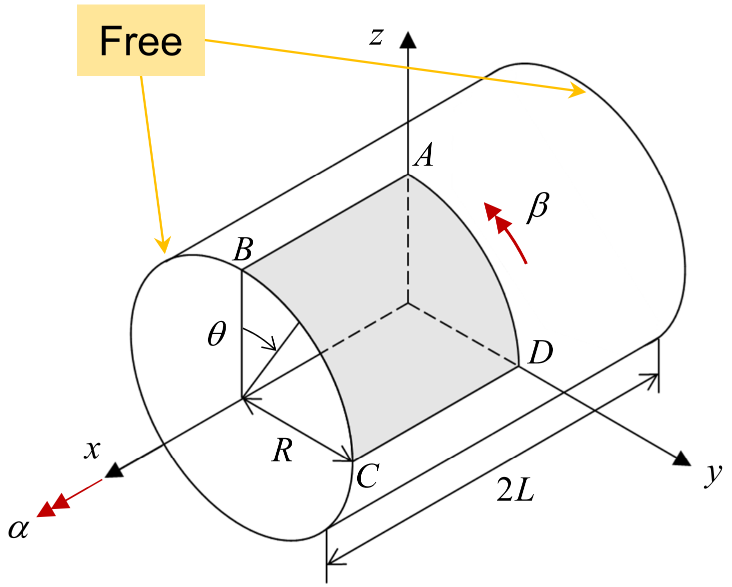
- **Park and Stanley. (1986) / Roh and Cho. (2004)**
  - Assumed natural strain(ANS) method
  - Patch test problem
- **Kulikov et al. (2010)**
  - Assumed natural strain(ANS) method
  - Exact geometry shell element
- **Ko et al. (2017) → MITC4+**
  - Mixed interpolated of tensorial component(MITC) method
  - Successfully alleviated the membrane locking



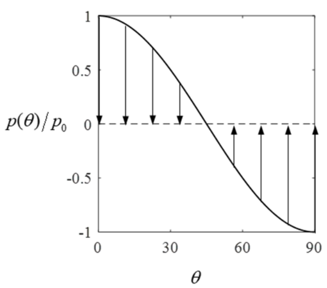
Tying points for assumed membrane strain

Research motivations

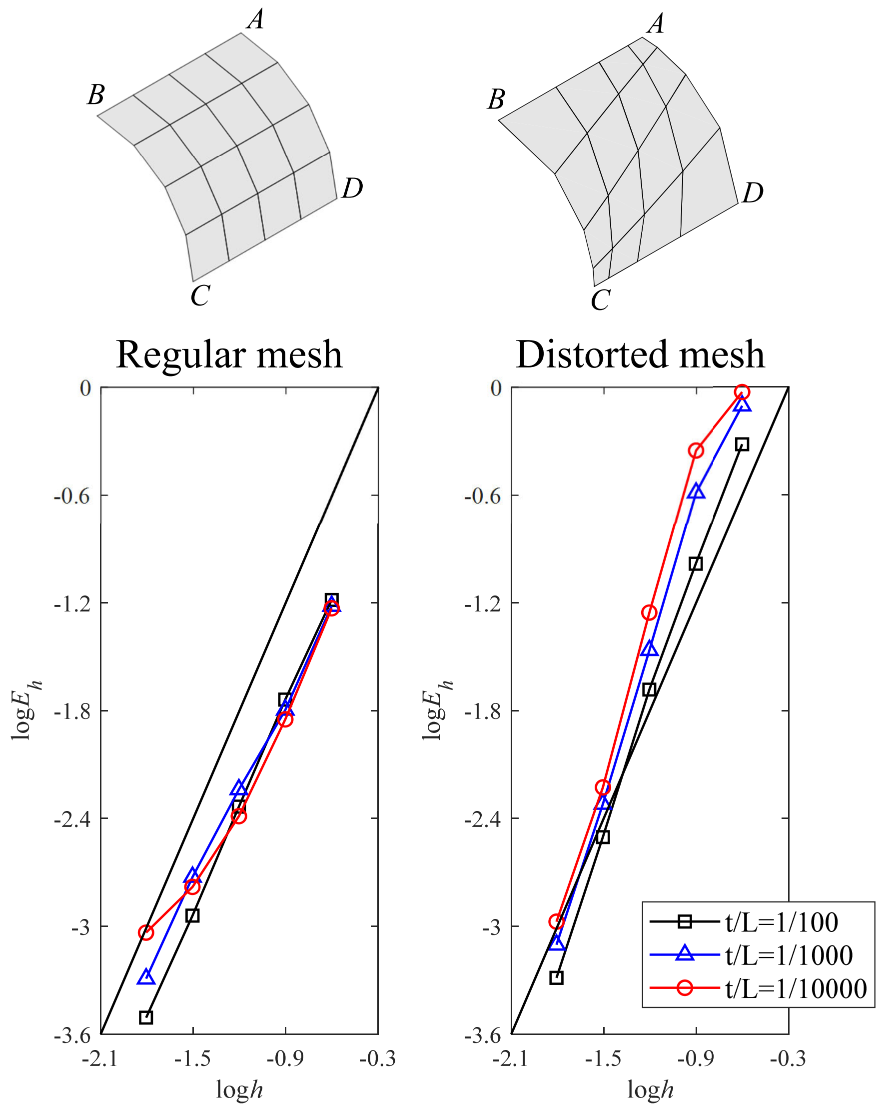
❖ MITC4+ shell element



Cylindrical shell with **free ends**
(bending dominated)



Pressure distribution
Free ends
L=R=1
$E = 2.0 \times 10^5, \nu = 1/3$
1/8 model only
Reference : MITC9

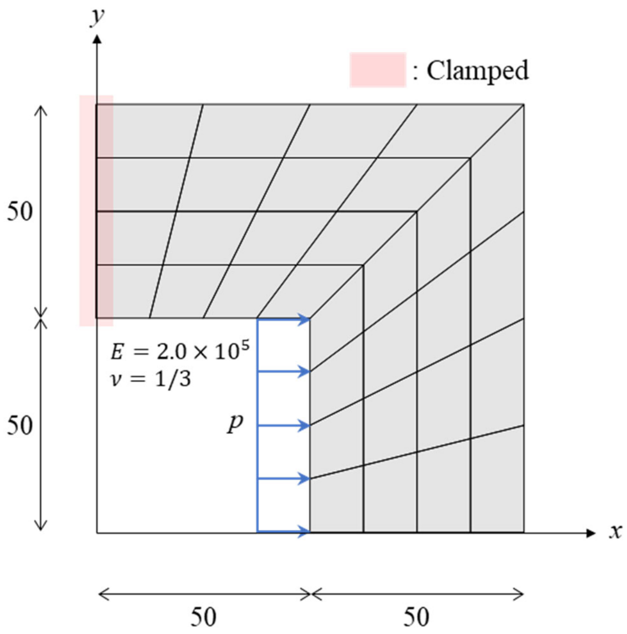


MITC4+: 4-node shell element with the MITC method
- treatment for transverse shear locking.
- treatment for membrane locking.

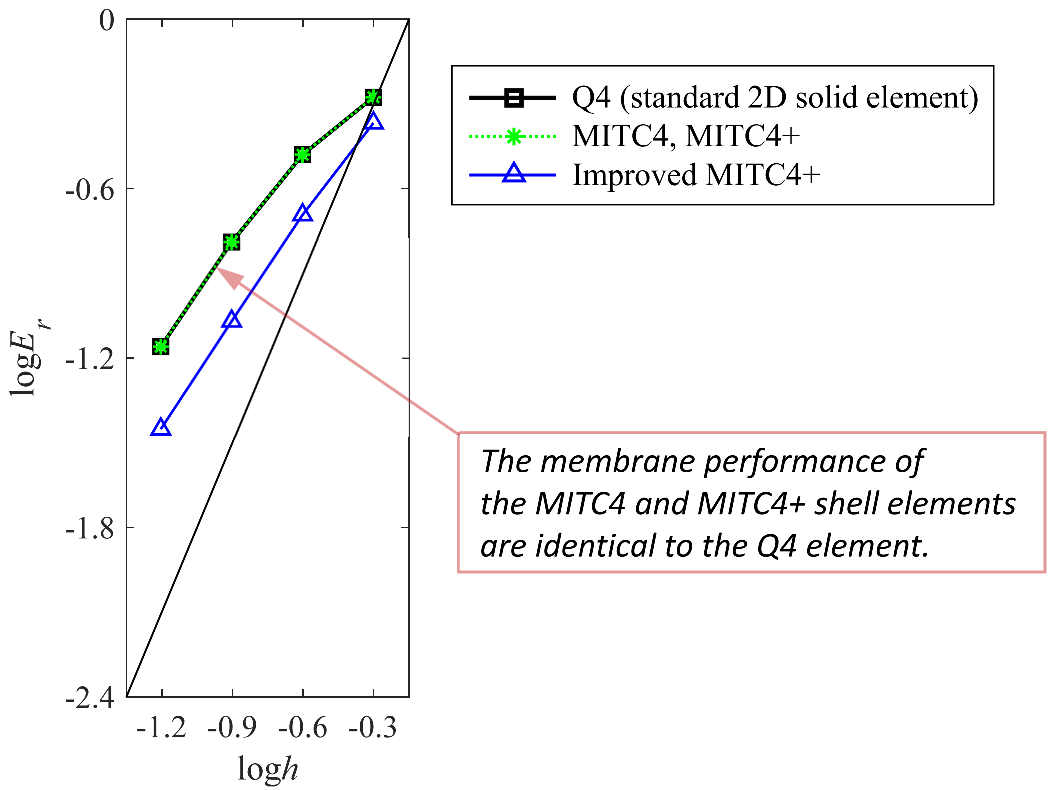
Research motivations

❖ Improved MITC4+ shell element

- The 2D-MITC4 solid element has been embedded into the MITC4+ shell element.
- The membrane performance was improved.



L-shaped structure
(2D plane stress problem)



Related studies

❖ Previous 4-node quadrilateral MITC shell elements

Element	Description
MITC4 (Dvorkin and Bathe, 1984)	▪ A continuum mechanics based 4-node shell element. ▪ The **transverse shear locking is alleviated** by constructing the assumed transverse shear strain field based on the <u>MITC approach</u>.
MITC4+ (Ko et al, 2017)	▪ The MITC4 shell element with alleviating membrane locking. ▪ The **membrane locking is alleviated** by <u>assuming the locking-causing term</u> as the linear combination of the strain coefficients.
Improved MITC4+ (Ko et al, 2017)	▪ The MITC4+ shell element with improved membrane behavior. ▪ The **membrane behavior is improved** by embedding the <u>previous 2D-MITC4</u> solid element.

Problems : complicated formulation & sensitivity to the mesh distortion

New MITC4+ (Proposed)	▪ The MITC4+ shell element with improved membrane behavior. ▪ The **formulation is simplified** by introducing the <u>new assumed membrane strain</u> field. ▪ To **further improve its performance**, the <u>geometry dependent Gauss integration</u> scheme is extended.

Previous MITC4+ shell element

$e_{11}=e_{rr}, e_{22}=e_{ss}, e_{12}=e_{rs}, e_{13}=e_{rt}, e_{23}=e_{st}$

❖ Geometry and displacement interpolations

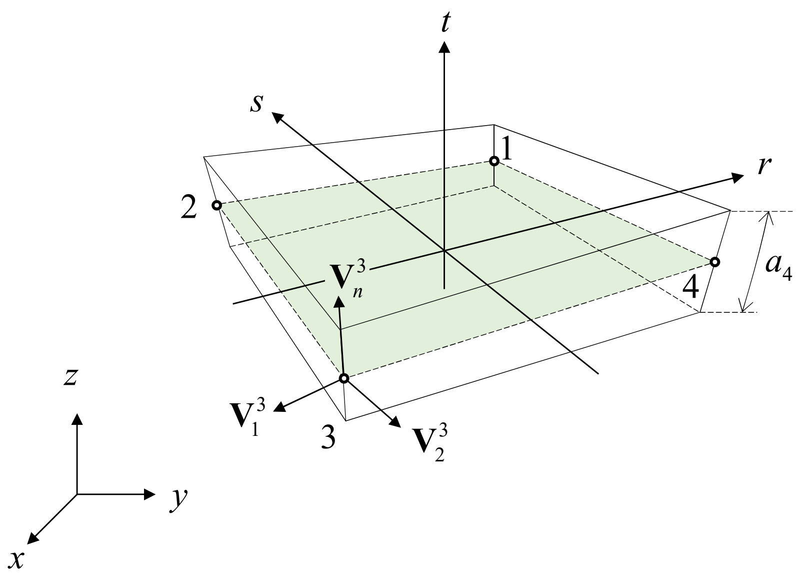
- $\mathbf{x}=\sum_{i=1}^{4} h_i(r,s)\mathbf{x}_i+\frac{t}{2}\sum_{i=1}^{4} a_i h_i(r,s)\mathbf{V}_n^i$ with $\mathbf{x}_i=\begin{bmatrix} x_i & y_i & w_i \end{bmatrix}^{\mathrm{T}}$

- $\mathbf{u}=\sum_{i=1}^{4} h_i(r,s)\mathbf{u}_i+\frac{t}{2}\sum_{i=1}^{4} a_i h_i(r,s)(-\alpha_i\mathbf{V}_2^i+\beta\mathbf{V}_1^i)$

  with $\mathbf{u}_i=\begin{bmatrix} u_i & v_i & w_i & \alpha_i & \beta_i \end{bmatrix}^{\mathrm{T}}$

- Shape functions: $h_i=\frac{1}{4}(1+\xi_i r)(1+\eta_i s)$

  with $\begin{bmatrix} \xi_1 & \xi_2 & \xi_3 & \xi_4 \end{bmatrix}=\begin{bmatrix} 1 & -1 & -1 & 1 \end{bmatrix}$
  $\begin{bmatrix} \eta_1 & \eta_2 & \eta_3 & \eta_4 \end{bmatrix}=\begin{bmatrix} 1 & 1 & -1 & -1 \end{bmatrix}$



A 4-node quadrilateral shell element

❖ Covariant strain components

- $e_{ij}=\frac{1}{2}(\mathbf{g}_i\cdot\mathbf{u}_{,j}+\mathbf{g}_j\cdot\mathbf{u}_{,i})$ with $i, j=1,2,3,$

  where $\mathbf{g}_i=\frac{\partial\mathbf{x}}{\partial r_i}$, $\mathbf{u}_{,i}=\frac{\partial\mathbf{u}}{\partial r_i}$ with $r_1=r$, $r_2=s$, $r_3=t$.

Out-of-plane: e_{rt}, e_{st}

In-plane: e_{rr}, e_{ss}, e_{rs}

Previous MITC4+ shell element

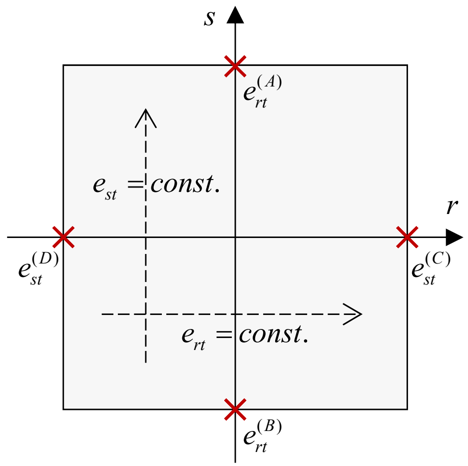
❖ Separation of covariant strain components

- Out-of-plane

Transverse shear strain:

$$\tilde{e}_{rt} = \frac{1}{2}(1+s)e_{rt}^{(A)} + \frac{1}{2}(1-s)e_{rt}^{(B)}$$

$$\tilde{e}_{st} = \frac{1}{2}(1+r)e_{st}^{(C)} + \frac{1}{2}(1-r)e_{st}^{(D)}$$



Tying points for assumed transverse shear strain

- In-plane

$$e_{ij} = e_{ij}^{m} + t e_{ij}^{b1} + t^2 e_{ij}^{b2}, \qquad i,j = 1,2$$

with $\mathbf{x}_m = \sum_{i=1}^{4} h_i(r,s)\mathbf{x}_i, \quad \mathbf{x}_b = \frac{1}{2}\sum_{i=1}^{4} a_i h_i(r,s)\mathbf{V}_n^i, \quad \mathbf{u}_m = \sum_{i=1}^{4} h_i(r,s)\mathbf{u}_i, \quad \mathbf{u}_b = \frac{1}{2}\sum_{i=1}^{4} a_i h_i(r,s)(-\alpha \mathbf{V}_2^i + \beta \mathbf{V}_1^i)$

Bending strain: $e_{ij}^{b1} = \frac{1}{2}\left(\frac{\partial \mathbf{x}_m}{\partial r_i}\cdot\frac{\partial \mathbf{u}_b}{\partial r_j} + \frac{\partial \mathbf{x}_m}{\partial r_j}\cdot\frac{\partial \mathbf{u}_b}{\partial r_i} + \frac{\partial \mathbf{x}_b}{\partial r_i}\cdot\frac{\partial \mathbf{u}_m}{\partial r_j} + \frac{\partial \mathbf{x}_b}{\partial r_j}\cdot\frac{\partial \mathbf{u}_m}{\partial r_i}\right), \quad e_{ij}^{b2} = \frac{1}{2}\left(\frac{\partial \mathbf{x}_b}{\partial r_i}\cdot\frac{\partial \mathbf{u}_b}{\partial r_j} + \frac{\partial \mathbf{x}_b}{\partial r_j}\cdot\frac{\partial \mathbf{u}_b}{\partial r_i}\right)$

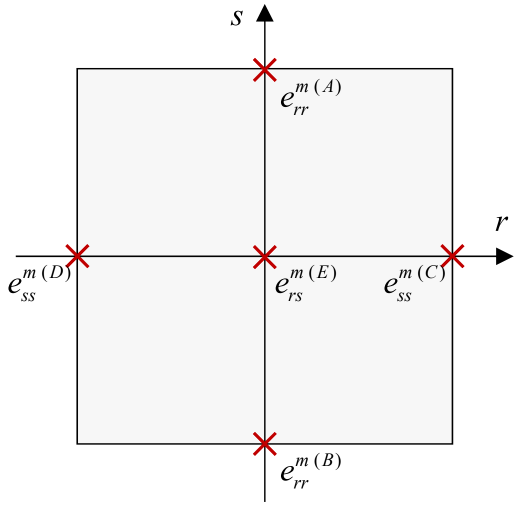
Membrane strain: $e_{ij}^{m} = \frac{1}{2}\left(\frac{\partial \mathbf{x}_m}{\partial r_i}\cdot\frac{\partial \mathbf{u}_m}{\partial r_j} + \frac{\partial \mathbf{x}_m}{\partial r_j}\cdot\frac{\partial \mathbf{u}_m}{\partial r_i}\right)$

Need to be modified

Previous MITC4+ shell element

❖ Assumed membrane strain field

- $$\hat{e}_{rr}^{m} = \frac{1}{2}(e_{rr}^{m(A)} + e_{rr}^{m(B)}) + \frac{1}{2}(e_{rr}^{m(A)} - e_{rr}^{m(B)}) \cdot s = (e_{rr}^{m}\big|_{con} + e_{rs}^{m}\big|_{bil}) + e_{rr}^{m}\big|_{lin} \cdot s$$
- $$\hat{e}_{ss}^{m} = \frac{1}{2}(e_{ss}^{m(C)} + e_{ss}^{m(D)}) + \frac{1}{2}(e_{ss}^{m(D)} - e_{ss}^{m(D)}) \cdot r = (e_{ss}^{m}\big|_{con} + e_{rs}^{m}\big|_{bil}) + e_{ss}^{m}\big|_{lin} \cdot r$$
- $$\hat{e}_{rs}^{m} = e_{rs}^{m(E)} + \frac{1}{2}e_{rr}^{m}\big|_{lin} \cdot r + \frac{1}{2}e_{ss}^{m}\big|_{lin} \cdot s \qquad = e_{rs}^{m}\big|_{con} + \frac{1}{2}e_{rr}^{m}\big|_{lin} \cdot r + \frac{1}{2}e_{ss}^{m}\big|_{lin} \cdot s$$



Tying points for assumed membrane strain

❖ *Additional* assumed for locking-causing term

- $$e_{rr}^{m} - \hat{e}_{rr}^{m} = (s^2 - 1)e_{rs}^{m}\big|_{bil}$$
- $$e_{ss}^{m} - \hat{e}_{ss}^{m} = (r^2 - 1)e_{rs}^{m}\big|_{bil}$$
- $$e_{rs}^{m} - \hat{e}_{rs}^{m} = e_{rs}^{m}\big|_{bil} \cdot rs$$

$$\tilde{e}_{rs}^{m}\big|_{bil} = \frac{c_r^2}{d}(e_{rr}^{m}\big|_{con} + e_{rs}^{m}\big|_{bil}) + \frac{c_s^2}{d}(e_{rr}^{m}\big|_{con} + e_{rs}^{m}\big|_{bil}) + \frac{2c_r c_s}{d}e_{rs}^{m}\big|_{bil} - \frac{c_r}{d}e_{rr}^{m}\big|_{lin} - \frac{c_s}{d}e_{ss}^{m}\big|_{lin}$$

$$c_r = \mathbf{x}_d \cdot \mathbf{m}^r, \quad c_s = \mathbf{x}_d \cdot \mathbf{m}^s, \quad d = c_r^2 + c_s^2 - 1, \quad \mathbf{m}^{r_i} \cdot \mathbf{x}_{r_j} = \delta_j^i$$

Proposed MITC4+ shell element

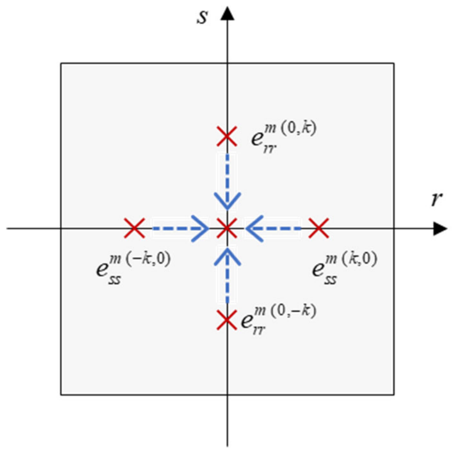
❖ Generalization of the assumed membrane strain field

- $$\widehat{e}_{rr}^{m} = \widehat{e}_{rr}^{m(0,0)} + \lambda(r,s)\frac{\widehat{e}_{rr}^{m(0,k)} - \widehat{e}_{rr}^{m(0,-k)}}{2k}s = e_{rr}^{m}\Big|_{con} + \frac{1}{1-k^2\alpha^2}[\lambda(r,s)(-2\alpha\, e_{rr}^{m}\Big|_{con} - 2\beta\, e_{rs}^{m}\Big|_{con} + e_{rr}^{m}\Big|_{lin})s]$$

- $$\widehat{e}_{ss}^{m} = \widehat{e}_{ss}^{m(0,0)} + \lambda(r,s)\frac{\widehat{e}_{ss}^{m(k,0)} - \widehat{e}_{ss}^{m(-k,0)}}{2k}r = e_{ss}^{m}\Big|_{con} + \frac{1}{1-k^2\beta^2}[\lambda(r,s)(-2\beta\, e_{ss}^{m}\Big|_{con} - 2\alpha\, e_{rs}^{m}\Big|_{con} + e_{ss}^{m}\Big|_{lin})r]$$

- $$\widehat{e}_{rs}^{m} = \widehat{e}_{rs}^{m(0,0)} = e_{rs}^{m}\Big|_{con}$$

When $k = 1/\sqrt{3}$, the strain field becomes identical to the previous 2D-MITC4 solid element



Generalization of the tying point with a variable 'k'

❖ New assumed membrane strain field ($k \rightarrow 0$)

- $$\widehat{e}_{rr}^{m} = e_{rr}^{m}\Big|_{con} + \lambda(r,s)(-2\alpha\, e_{rr}^{m}\Big|_{con} - 2\beta\, e_{rs}^{m}\Big|_{con} + e_{rr}^{m}\Big|_{lin})s$$

- $$\widehat{e}_{ss}^{m} = e_{ss}^{m}\Big|_{con} + \lambda(r,s)(-2\beta\, e_{ss}^{m}\Big|_{con} - 2\alpha\, e_{rs}^{m}\Big|_{con} + e_{ss}^{m}\Big|_{lin})r$$

- $$\widehat{e}_{rs}^{m} = e_{rs}^{m}\Big|_{con}$$

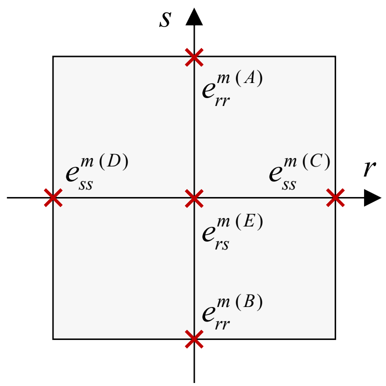
To compare the complexity of the previous and the new formulations, it is rewritten in the matrix form.

Proposed MITC4+ shell element

❖ Comparison of formulation

- $$\begin{bmatrix} \hat{e}_{rr}^{AS} \\ \hat{e}_{ss}^{AS} \\ \hat{e}_{rs}^{AS} \end{bmatrix} = \mathbf{MD}\begin{bmatrix} e_{rr}^{m(A)} & e_{rr}^{m(B)} & e_{ss}^{m(C)} & e_{ss}^{m(D)} & e_{rs}^{m(E)} \end{bmatrix}^{\mathrm{T}}$$

- $$a_A = \frac{\alpha(\alpha-1)}{2d},\ a_B = \frac{\alpha(\alpha+1)}{2d},\ a_C = \frac{\beta(\beta-1)}{2d},\ a_D = \frac{\beta(\beta+1)}{2d},\ a_E = \frac{2\alpha\beta}{d},\ d = \alpha^2 + \beta^2 - 1$$



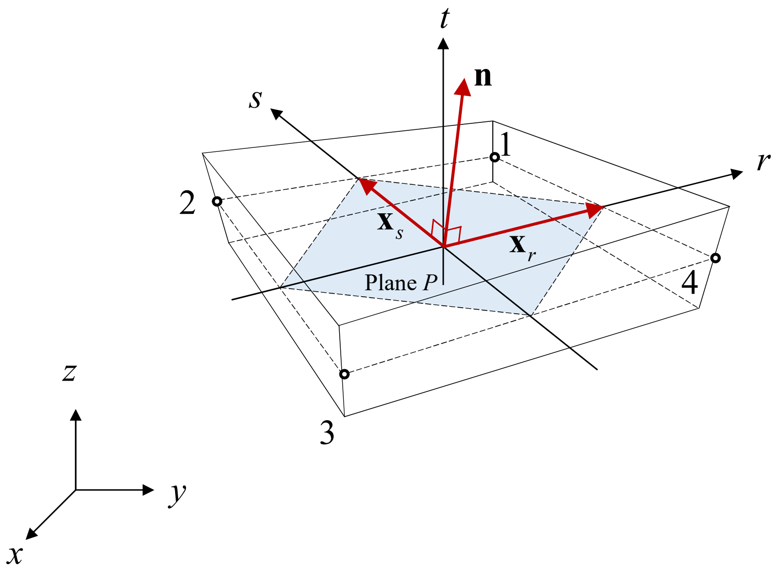
		Improved MITC4+ (previous)	New MITC4+ (proposed)
Membrane behavior improvement	**M**	$\lambda\begin{bmatrix} 1/\lambda+\sqrt{3}n_1 s & \sqrt{3}m_2 r & 0 \\ \sqrt{3}n_2 s & 1/\lambda+\sqrt{3}m_1 r & 0 \\ 2\sqrt{3}n_5 s & 2\sqrt{3}m_5 r & 1 \\ n_3 s & m_4 r & 0 \\ n_4 s & m_3 r & 0 \\ n_1 s/\sqrt{3} & m_1 r/\sqrt{3} & 0 \end{bmatrix}^{\mathrm{T}}$ $n_1 \sim n_5$ and $m_1 \sim m_5$ are calculated from the equations in 15p	$\lambda\begin{bmatrix} 1/\lambda-2\alpha s & 0 & 0 \\ 0 & 1/\lambda-2\beta r & 0 \\ -2\beta s & -2\alpha r & 1 \\ s & 0 & 0 \\ 0 & r & 0 \end{bmatrix}^{\mathrm{T}}$
Membrane locking alleviation	**D**	$\begin{bmatrix} 1/2-a_A & 1/2-a_B & -a_C & -a_D & -a_E \\ -a_A & -a_B & 1/2-a_C & 1/2-a_D & -a_E \\ 0 & 0 & 0 & 0 & 1 \\ 1/2 & -1/2 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & -1/2 & 0 \\ a_A & a_B & a_C & a_D & a_E \end{bmatrix}$	$\begin{bmatrix} 1/2-a_A & 1/2-a_B & -a_C & -a_D & -a_E \\ -a_A & -a_B & 1/2-a_C & 1/2-a_D & -a_E \\ 0 & 0 & 0 & 0 & 1 \\ 1/2 & -1/2 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & -1/2 & 0 \end{bmatrix}$

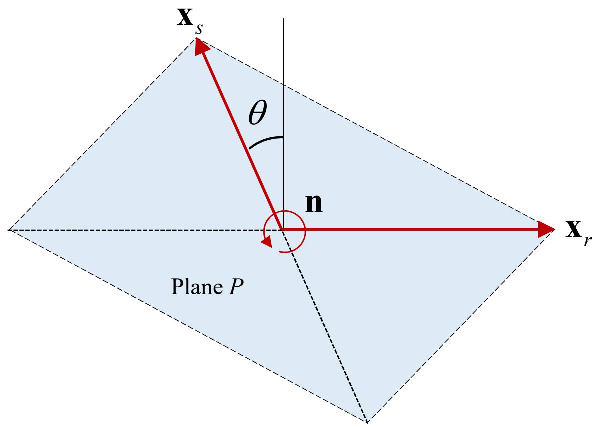
Proposed MITC4+ shell element

$$\mathbf{F}(r,s,t)=\mathbf{B}^{\mathrm{T}}(r,s,t)\mathbf{C}\mathbf{B}(r,s,t)\det(\mathbf{J}(r,s,t))$$

❖ Integration of stiffness matrix

- Standard : $\mathbf{K}=\sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2} w_i w_j w_j \mathbf{F}(\xi_i,\xi_j,\xi_k)$ with $\xi_1=\frac{1}{\sqrt{3}},\ \xi_2=-\frac{1}{\sqrt{3}}$ and $w_1=w_2=1$





- Modified : $\mathbf{K}=\sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2} w_i w_j w_j \mathbf{F}(\hat{\xi}_i,\hat{\xi}_j,\xi_k)$ with $\hat{\xi}_i=\mu\xi_i$ and $\mu=\cos^2(\theta)$

✓ *The skew angle is measured in the plane P*

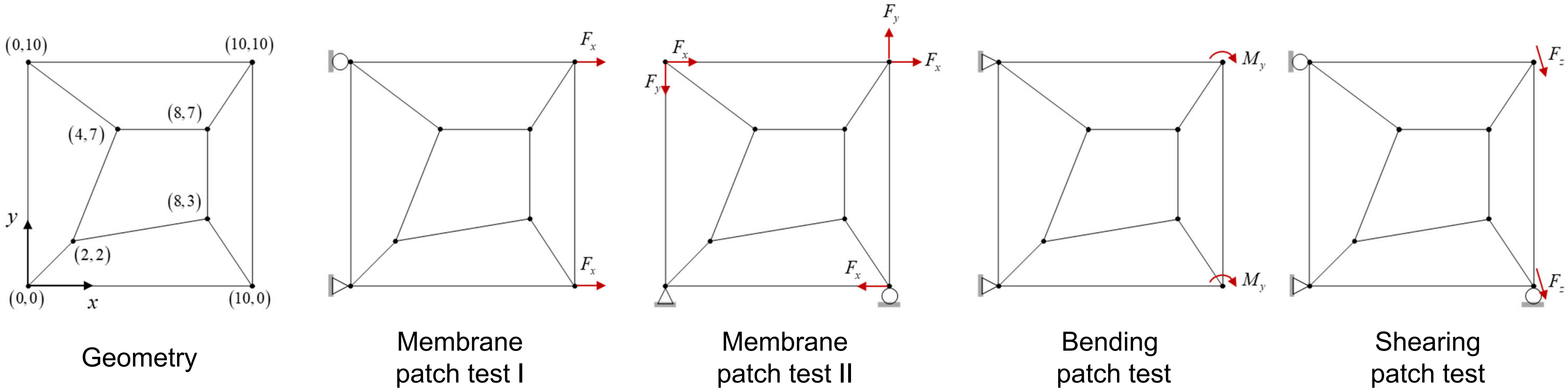
✓ *Geometry dependent Gauss integration in r-s plane*

✓ *Standard Gauss integration in thickness direction t*

Basic numerical tests

❖ Patch tests

- The minimum number of constraints are given to prevent rigid body motions.
- Proper loadings are applied to produce a constant stress field.
- In this tests, a constant stress field should be represented exactly.



❖ Zero energy mode test

- A single unsupported 4-node quadrilateral shell element should have only six zero energy modes.

❖ Isotropy test

- The behavior of finite elements should not be affected by the node numbering sequence.

Numerical examples (Shell)

❖ Finite elements considered

Element	Transverse shear locking	Membrane locking	Membrane behavior	Nonlinear formulation
MITC4 (Dvorkin and Bathe, 1984)	O	X	Standard Q4	O
MITC4+ (Ko et al, 2017)	O	O	Standard Q4	O
Improved MITC4+ (Ko et al, 2017)	O	O	Previous 2D-MITC4	-
New MITC4+ (proposed)	O	O	New 2D-MITC4	O

❖ Evaluation in linear analysis

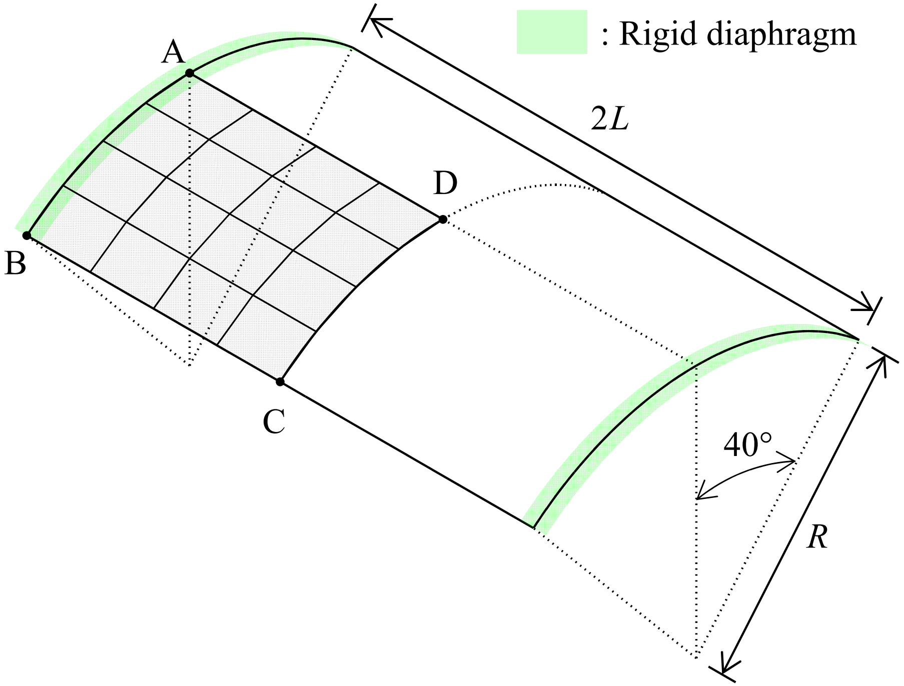
- Convergence of the relative error in s-norm $E_h = \left\| \mathbf{u}_{ref} - \mathbf{u}_h \right\|_s^2 / \left\| \mathbf{u}_{ref} \right\|_s^2$ with $\left\| \mathbf{u}_{ref} - \mathbf{u}_h \right\|_s^2 = \int_\Omega \Delta\boldsymbol{\varepsilon}^T \Delta\boldsymbol{\tau} d\Omega$
- According to decreasing in shell thickness $(t/L = 1/100,\ 1/1000,\ 1/10000)$
- Reference solutions are calculated using a <u>96×96</u> regular mesh of the <u>MITC9 shell elements</u>.

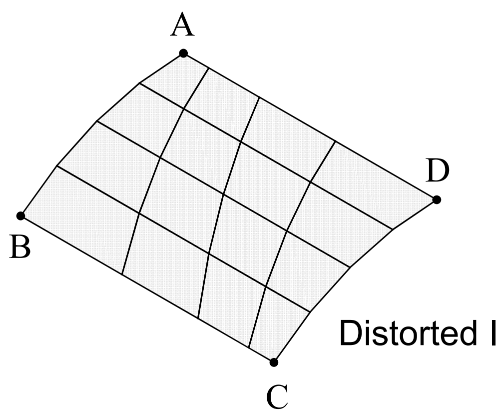
❖ Evaluation in nonlinear analysis

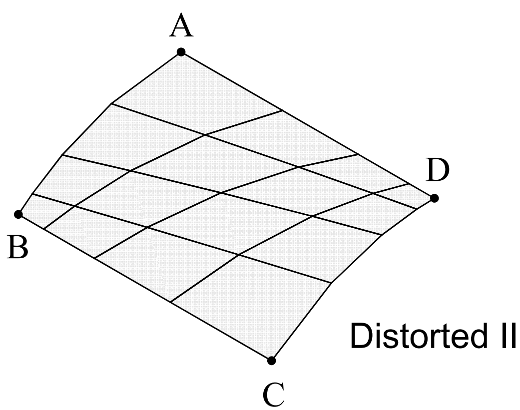
- Displacements at specific locations and deformed configurations

Numerical examples (Shell / Linear analysis)

❖ Scordelis-Lo roof



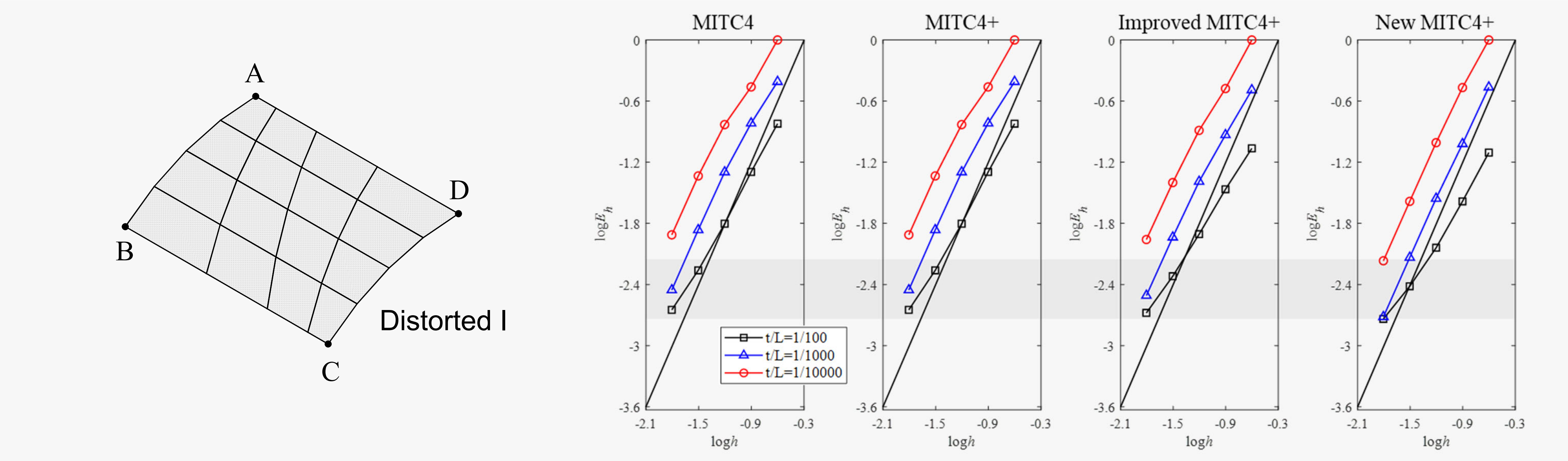


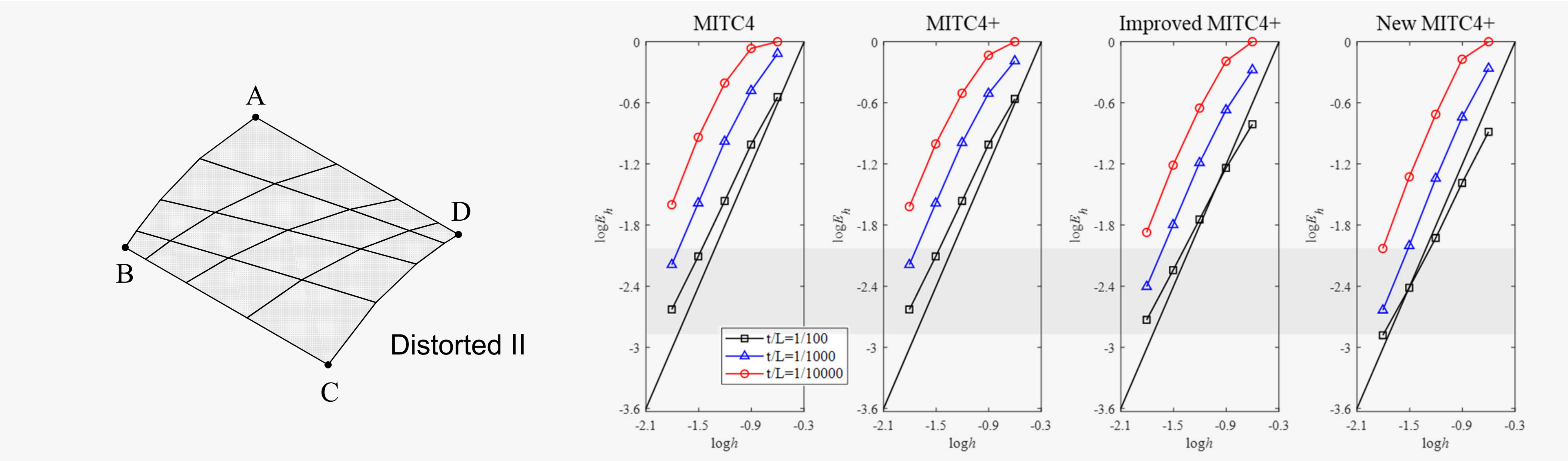


- **Force**
  - Self weight $f_z = -90$ (force per area)
- **Boundary condition**
  - Diaphragm
- **Dimensions and properties**
  - $R = L = 25.0,\ E = 4.32 \times 10^8,\ \nu = 0.0$
- **Two distorted meshes** ($N \times N$)
  - $N = 4, 8, 16, 32, 64$

❖ Scordelis-Lo roof

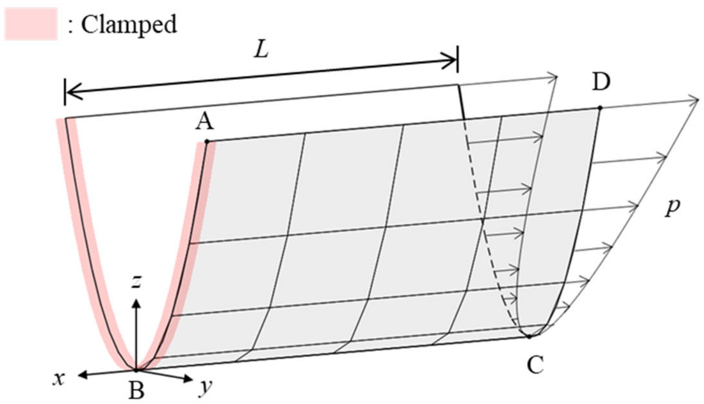
- Convergence behavior (mixed behavior)



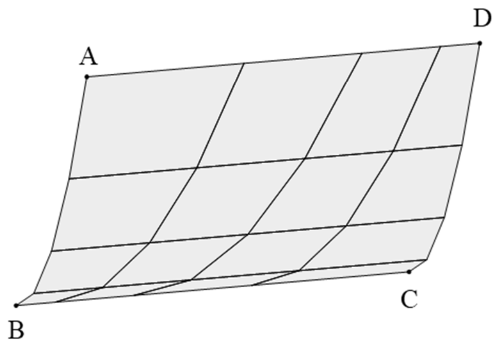


Numerical examples (Shell / Linear analysis)

❖ Hyperbolic cylinder



Regular mesh



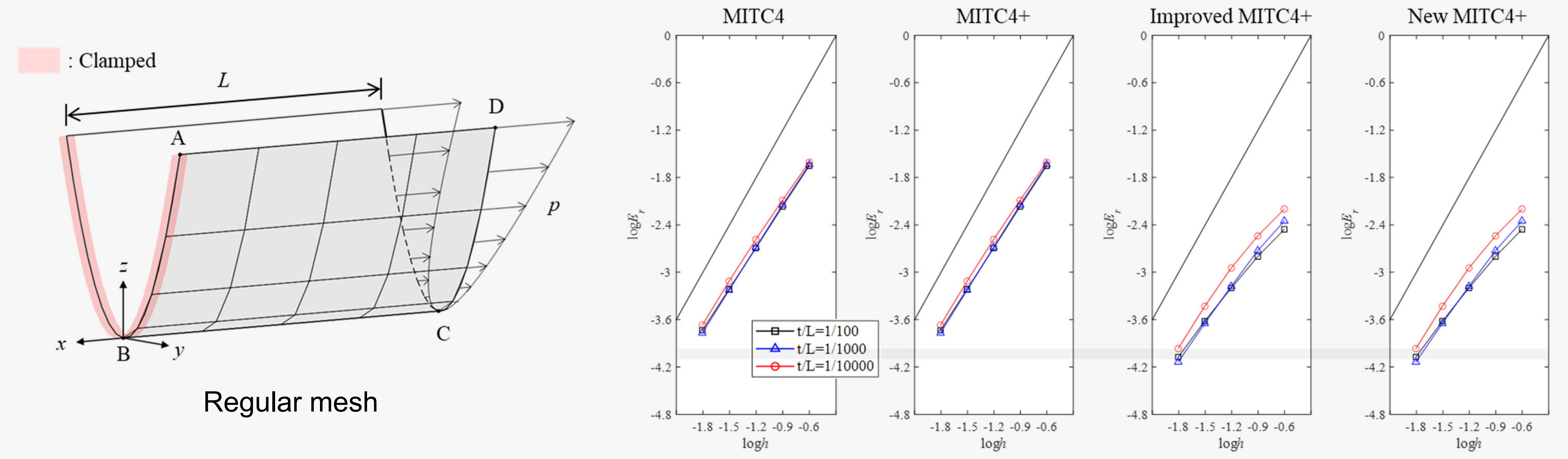
Distorted mesh

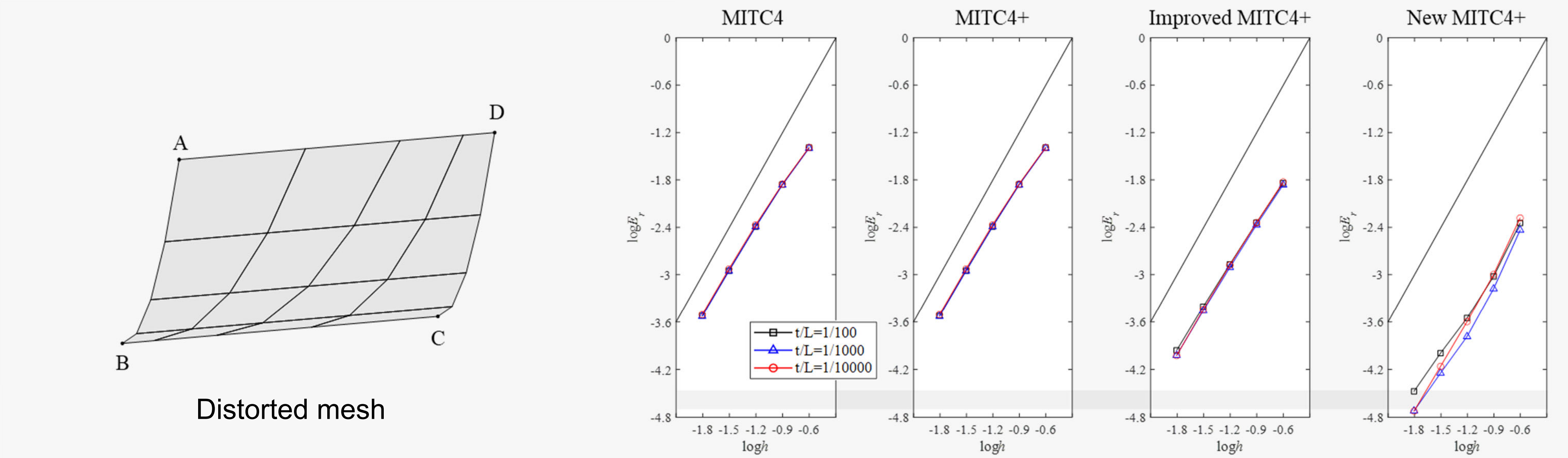
- **Force**
  - Tip distributed load $p_x = -z$ (force per length)
- **Boundary condition**
  - Clamed one side
- **Dimensions and properties**
  - $z = 4y^2,\ y \in [0, 0.5]$
  - $L = 2.0,\ E = 2.0\times10^{11},\ \nu = 1/3$
- **Regular and distorted meshes** ($N\times N$)
  - N = 4, 8, 16, 32, 64

Numerical examples (Shell / Linear analysis)

❖ Hyperbolic cylinder

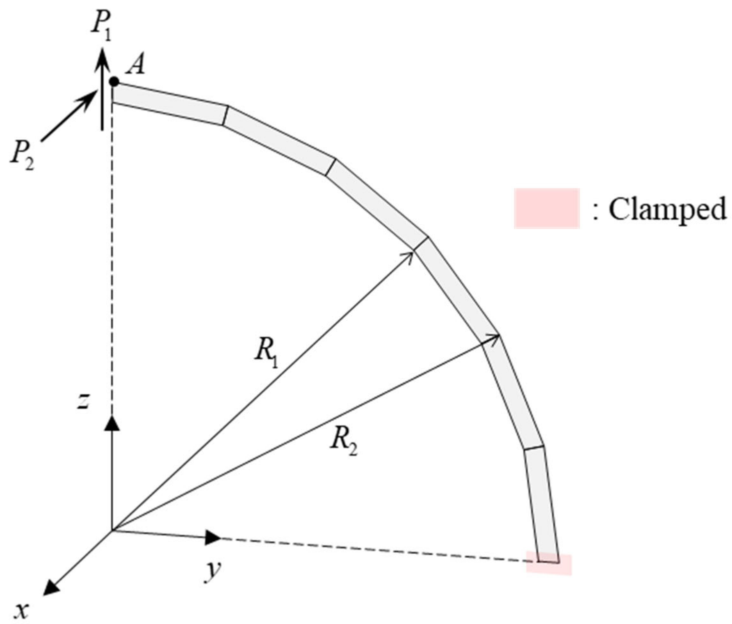
- Convergence behavior





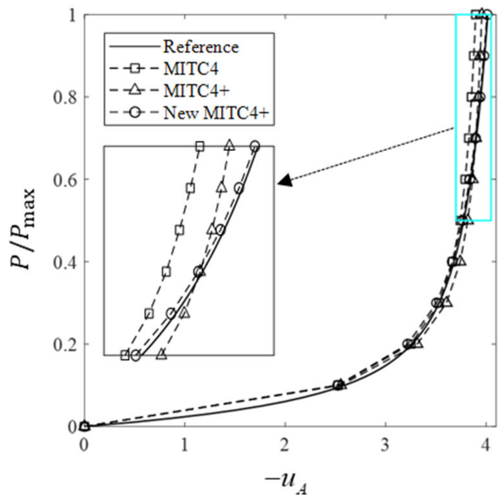
Numerical examples (Shell / Nonlinear analysis)

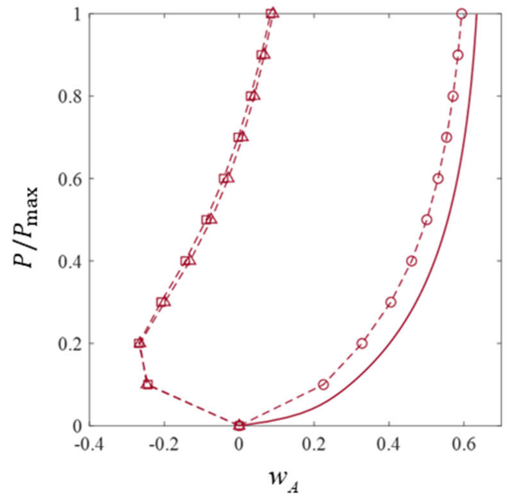
❖ Thin curved beam



- **Loading condition**
  - Tip forces $P = P_1 = P_2 = 100$ at free tip
- **Boundary condition**
  - Clamped bottom side
- **Dimensions and properties**
  - $R_1 = 4.12,\ R_2 = 4.32,\ t = 0.1$
  - $E = 1.0\times10^7,\ \nu = 0.25$
- **Regular mesh** ($N\times6N$)
  - $N = 1$ for finite element solution
  - $N = 2$ for reference solution (regular)

- Load-displacement curves at point A

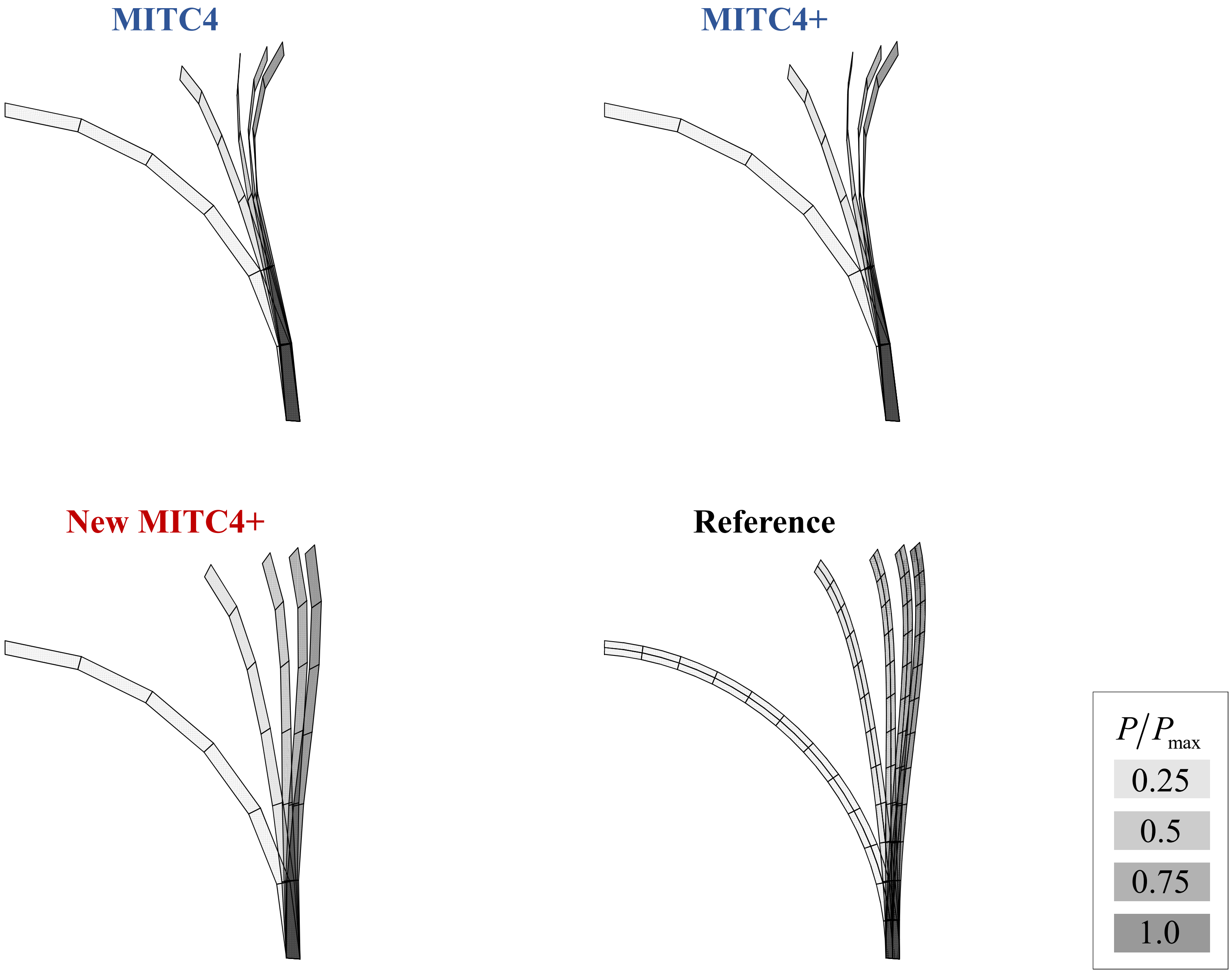




Numerical examples (Shell / Nonlinear analysis)

❖ Thin curved beam

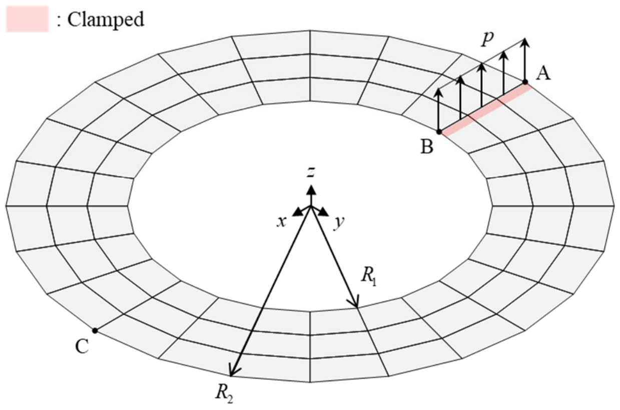
- Initial and deformed configurations at several load steps



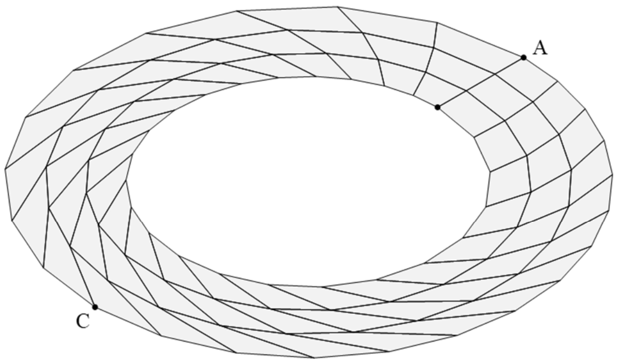
Numerical examples (Shell / Nonlinear analysis)

❖ Slit annular plate

- Problem description



Regular mesh

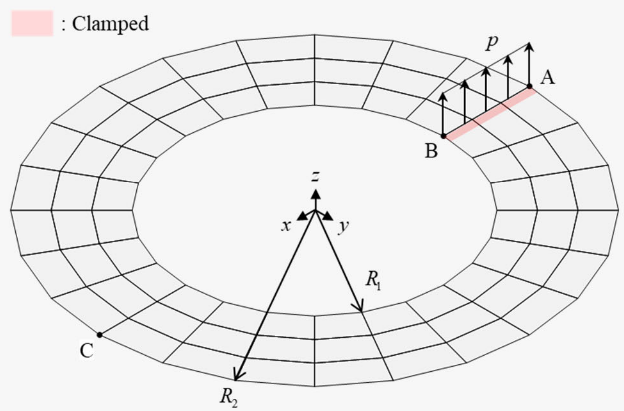


Distorted mesh

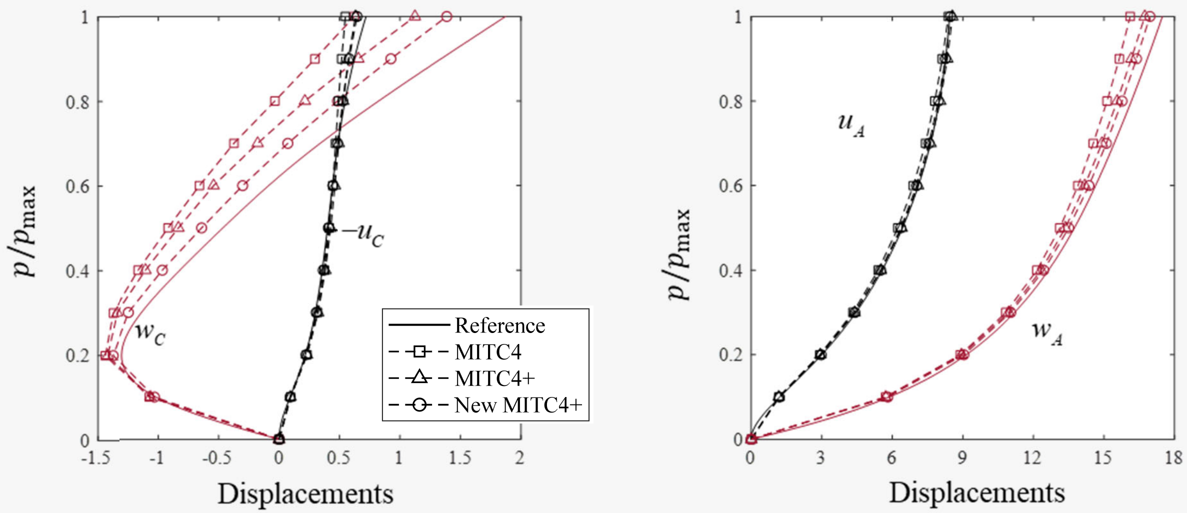
- **Loading condition**
  - Distributed forces $p = 0.8$ (force per length)
- **Boundary condition**
  - Clamped one side
- **Dimensions and properties**
  - $R_1 = 6.0,\ R_2 = 10.0,\ t = 0.03$
  - $E = 2.1\times10^7,\ \nu = 0.0$
- **Regular and distorted meshes** ($N\times 8N$)
  - $N = 3$ for finite element solution
  - $N = 12$ for reference solution (regular)

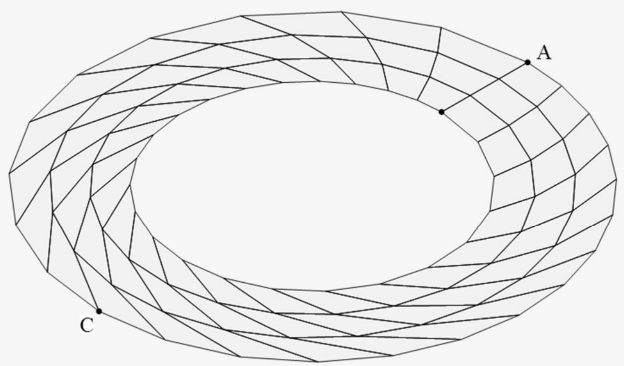
❖ Slit annular plate

- Load-displacement curves at points A and C

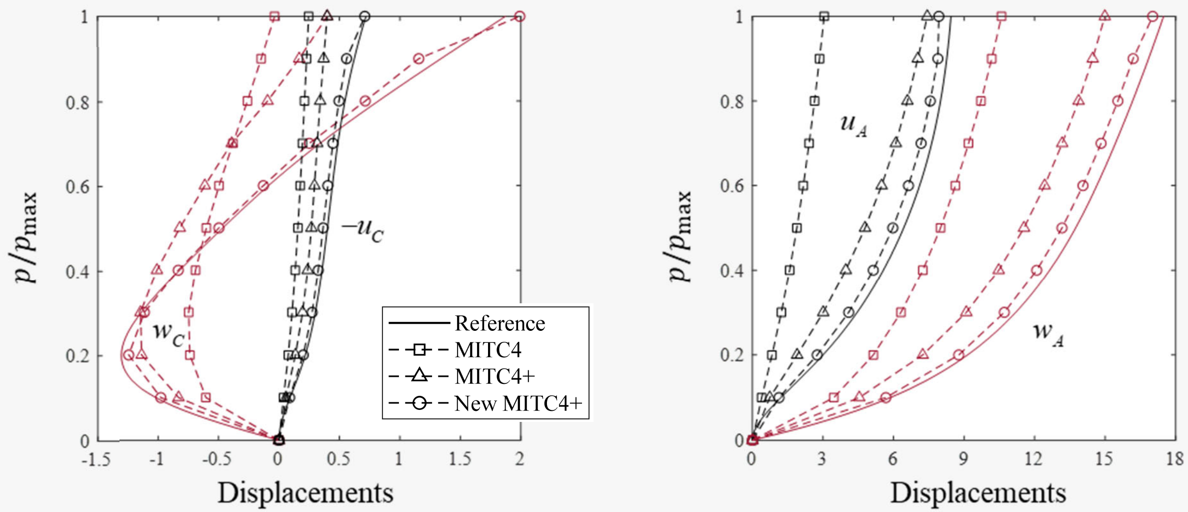


Regular mesh





Distorted mesh

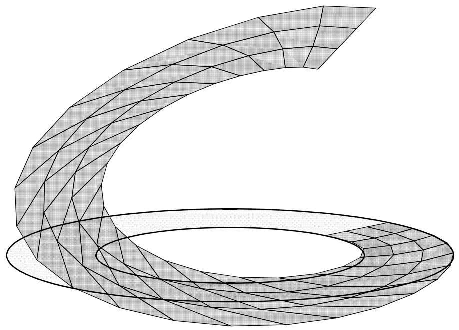


Numerical examples (Shell / Nonlinear analysis)

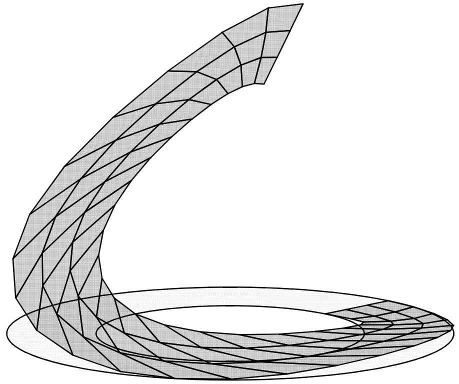
❖ Slit annular plate

- Final deformed configurations with the distorted mesh

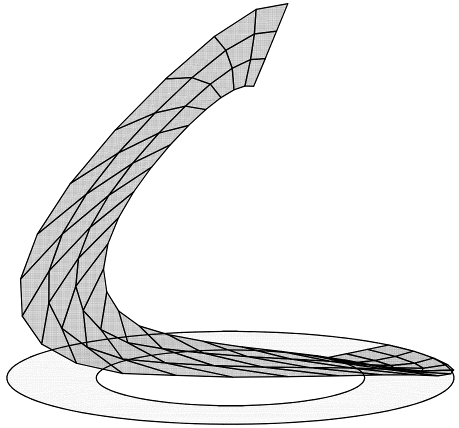
MITC4



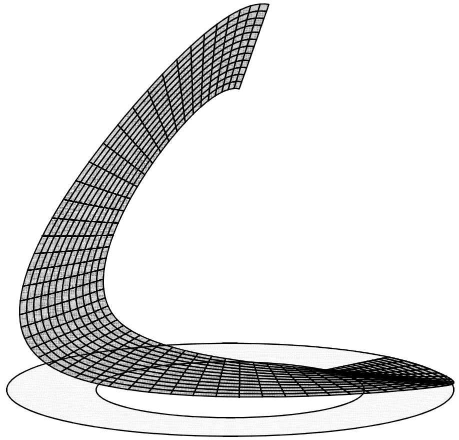
MITC4+



New MITC4+



Reference



4. Conclusions & Future works

Conclusions

❖ Topic 1 – New 2D-MITC4 solid element

- **Simplified formulation** is presented to investigate the behavior of the element.
- A **geometry dependent Gauss integration scheme** for the 2D-MITC4 solid element has been proposed.
- The proposed element provides **more accurate solutions** than the previous 2D-MITC4 solid element especially **in distorted meshes**.

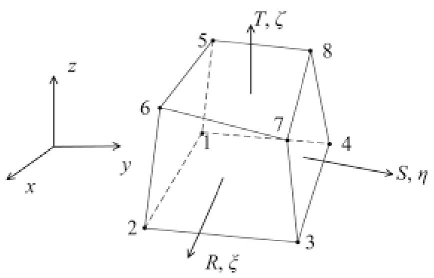
❖ Topic 2 – New MITC4+ shell element

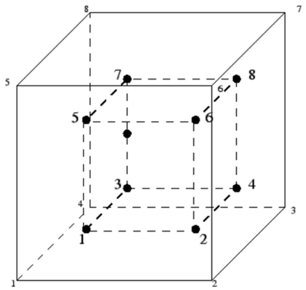
- A **new assumed membrane strain field** for the MITC4+ element has been proposed to simplify its formulation.
- The **geometry dependent Gauss integration scheme** is extended into the membrane strain field of the MITC4+ shell element.
- The membrane behavior of the proposed MITC4+ shell element has been **successfully improved** in both linear and nonlinear analysis.

Future works

1. Improve the performance of the 3D solid finite element

Solid



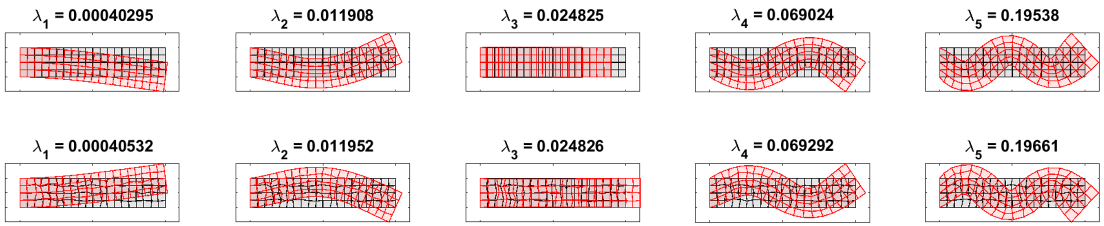


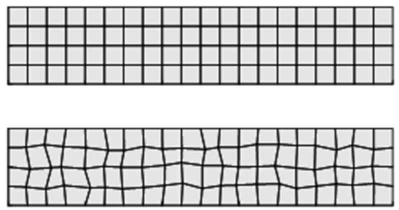
- ✓ Distortion measure
- ✓ Adjusting parameter
- ✓ Assumed strain field

2. Mass matrix for dynamic and modal analyses

Regular

Distorted





$$\mathbf{M}_e = \int_{V_e} \rho \mathbf{H}^{\mathrm{T}} \mathbf{H} \, dV_e$$

$$\mathbf{K}_e = \int_{V_e} \mathbf{B}^{\mathrm{T}} \mathbf{C} \mathbf{B} \, dV_e$$

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{K}\dot{\mathbf{U}} = \mathbf{R}$$

$$\mathbf{K}\boldsymbol{\phi} = \omega^2 \mathbf{M}\boldsymbol{\phi}$$

감사합니다.